Dual Resonance Models and Field/String Duality

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Outline

1. Introduction
2. Toward a string dual for large N QCD
3. Subcritical Neveu-Schwarz at One-loop:
   Holography and Unitarity
4. The Pomeron tachyon and UV divergences
5. Conclusions

Introduction

On this occasion, I would like to take a retro look at field/string duality from the perspective of the old days of dual resonance models: particularly the Neveu-Schwarz/Ramond model.

Not just to add historical perspective, but also to give us a new angle on the problem of finding a string dual for QCD. It is probably a long shot, but I think it worth a good look.

Let us begin with the DRM at the heart of the original Maldacena duality:

$\mathcal{N} = 4$ SUSY Yang-Mills $\equiv$ IIB String on $\mathrm{AdS}_5 \times \mathrm{S}^5$

That is, even G-parity Neveu-Schwarz bosons interacting with the Majorana-Weyl Ramond fermions:
the open superstring in 10 space-time dimensions,
with momenta restricted to 4 space-time dimensions.

The gauge group is incorporated via $U(N)$ Chan-Paton factors.

Multi-loop dual amplitudes determined by unitarity,
with momenta still restricted to 4d.
Notice: with this 4d setup, the nonplanar loops show a "pomeron" cut, reflecting closed string motion in the extra dimensions.

Aside: We used to think this cut was unitarity violating, but now we see it is just a reflection of extra dimensions, i.e. holography!

Notice: the Pomeron cut is absent in the planar loop approximation ('t Hooft's $N \to \infty$)

Thus no objection to considering planar open string theory in general dimensions (below the critical dimension of course).

In modern language the open strings described by this model have their ends fixed to a pair of N coincident 3-branes in 10 dimensional space-time (Polchinski).

This gives a nice physical interpretation both of the restriction of momenta to 4d and of the Chan-Paton factors.

The dual resonance model provides a pivot point for field/string duality. By taking $\alpha' \to 0$ in two different ways, one gets a heuristic indication of the duality.

I. Neveu and Scherk [Nuclear Physics B36 (1972) 155],
Low energy limit of open string theory is Yang-Mills theory (here $\mathcal{N} = 4$ SUSY YM with $U(N)$ gauge group in 4D).

The sum of planar open string multiloop diagrams should go to $N = \infty$ Yang-Mills as $\alpha' \to 0$
('t Hooft's limit, $\lambda = Ng^2/4\pi^2$ fixed).

Open string momentum is 4d. But the bulk of the open string vibrates in 10d. The oscillators $a_n^\mu, b_r^\mu, d_n^\mu$ are 10d: 8 gluon polarizations, the last 6 of which are 4D scalars.
These together with their superpartners, the particle content of $\mathcal{N} = 4$ SUSY Yang-Mills, are the only open string states that survive the $\alpha' \to 0$ limit.

This the "field" side of field/string duality.

II. On the other hand, as initially indicated by Neveu and Scherk (Phys.Rev.D1 (1970) 2355-2359) and further developed by many others, planar open string loops can be reinterpreted as closed string absorption by the vacuum.

In the planar approximation there are no closed string loops, whence this modification of the vacuum should be understandable at tree level closed string field theory.

Taking $\alpha' \to 0$ gives the other " string" side of field/string duality.

A tractable approximation to classical closed string field equation is $\alpha' \to 0$, naively a supergravity limit, in which the background is determined by solving 10d classical field equations in presence of a D3-brane source.

Unfortunately, the gluons are *on* the 3-branes at $r = 0$ where the background metric induced by the 3 branes is singular and can't be treated semi-classically.

However, the near horizon limit of the background *can* be treated classically at strong 't Hooft coupling $\lambda = g^2 N/4\pi^2 \gg 1$, when it is $\mathrm{AdS}_5 \times \mathrm{S}^5$ (Maldacena):

$$\left(1 + \frac{G\Lambda}{r^4}\right)^{-1} = \left(1 + \frac{N g^2 \alpha'^2}{r^4}\right)^{-1} \approx \frac{r^4}{N g^2 \alpha'^2}$$

when $\sqrt{\alpha'} \ll r \ll R \equiv (N g^2)^{1/4} \sqrt{\alpha'}$.

Maldacena's proposal: This near horizon physics *is* the strong 't Hooft coupling limit of $\mathcal{N} = 4$ Super Yang-Mills.

- $Ng^2 \to \infty$ justifies treating closed string by effective local fields even in near horizon limit ($\sqrt{\alpha'} \ll r \ll R$).

- This same limit is classical limit of AdS String

- Strong coupling $\mathcal{N} = 4$ described by classical string moving on $\mathrm{AdS}_5 \times \mathrm{S}^5$.

- Note that $Ng^2 \leq O(1)$, requires *at least* the fully quantum string on $\mathrm{AdS}_5 \times \mathrm{S}^5$. Similarly, a string dual for QCD would not have a semi-clssical limit, because it is asymptotically free.

At the end of this process, after $\alpha' \to 0$, the dual resonance model, which motivated the field string field equivalence, has vanished.

But something important has been left behind...

Alice to the Cheshire Cat:

"I wish you wouldn't keep appearing and vanishing so suddenly: you make me quite giddy!"

This time it vanished quite slowly, beginning with the end of the tail, and ending with the grin, which remained some time after the rest of it had gone.

"Well! I've often seen a cat without a grin," thought Alice; "but a grin without a cat! It's the most curious thing I ever saw in my life!"

Alice in Wonderland
Lewis Carroll

Toward a String Dual for large N QCD

- Simplest DRM with pure Yang-Mills low energy limit: NS open string with odd G-parity states removed in 4 spacetime dimensions: No fermions! No extra dimensions!

- For large N QCD keep only planar graphs. Ditto for the string model.

- Can now find closed string interpretation by examining planar multi-loop string graphs. Since we are subcritical usual closed string theory won't do!

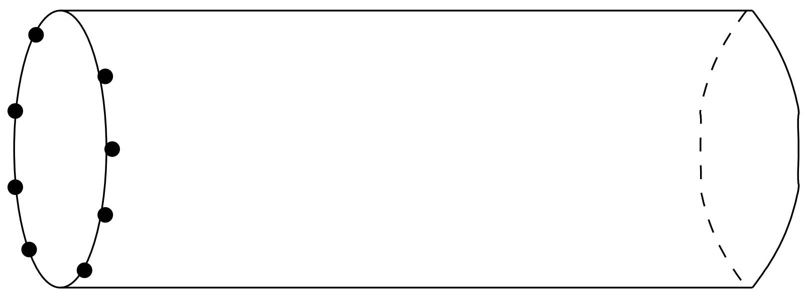
- Note that, as is well-known to string theorists of a certain age, the tree theorem fixes multi-loop diagrams correctly by unitarity.
  This is an important methodology since string path integrals are poorly understood when $D < D_{crit} = 10$.

NS DRM at 1 loop, $D < 10$

For $D < 10$ Super-Virasoro conditions eliminate fewer states:

- $n > 1$: only one component of a_n^μ and one component of $b_{n-1/2}^\mu$ are removed. (for D=10 two are removed)
- a_1^μ and $b_{1/2}^\mu$ both have two components removed.
- Thus the measure factor in the one open string loop integrand is (cf Brower-Thorn and Goddard-Waltz):

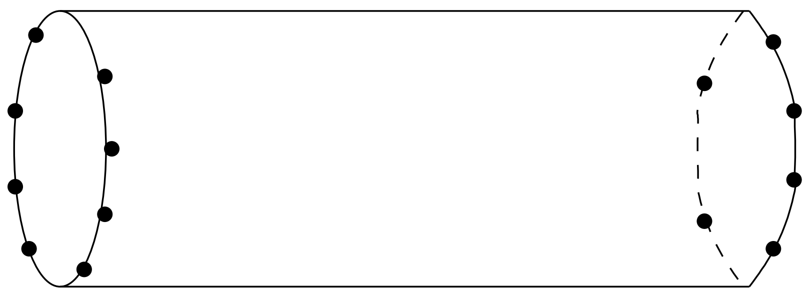
$$\frac{1-w}{1+w^{1/2}} \frac{\prod_r (1+w^r)^{D-1}}{\prod_n (1-w^n)^{D-1}} = (1-w^{1/2}) \frac{\prod_r (1+w^r)^{D-1}}{\prod_n (1-w^n)^{D-1}}$$



$$\mathcal{M} = \frac{1}{2}(\mathcal{M}^+ - \mathcal{M}^-)$$

projects out odd G-parity particles from the loop

To discover closed string spectrum examine nonplanar 1 loop



- $q^{(D-1)/8} \to q^{(D-1)/8+\alpha' K^2/2}$
- $\mathcal{M}^{+}_{\mathrm{NP}}$ has closed string cut starting at $\alpha' K^2 = (D-1)/4$, but $\mathcal{M}^{-}_{\mathrm{NP}}$ closed string cut starts at $K^2 = 0$.
- No massless graviton for $D = 4$!

Interpreting the closed string cut

C. B. Thorn, Phys.Lett.B**242**(1990)364

- Factors not present for critical superstring $D = 10$:

$$\sqrt{\frac{-\pi}{\ln q}}(w^{(D-9)/16} \mp w^{(D-1)/16})$$

$$= \int \frac{d\mu}{2} q^{\mu^2/4} \left(\cosh \mu \sqrt{\frac{9-D}{16}} \mp \cos \mu \sqrt{\frac{D-1}{16}} \right)$$

$$= \int d\mu q^{\mu^2/4} \begin{cases} \sinh \frac{\mu\gamma_+}{2} \sinh \frac{\mu\gamma_-}{2} \\ \cosh \frac{\mu\gamma_+}{2} \cosh \frac{\mu\gamma_-}{2} \end{cases}$$

$$\gamma_\pm = \sqrt{\frac{9-D}{16}} \pm i\sqrt{\frac{D-1}{16}}$$

- Open strings are "Dp-branes" in $D+1$ dimensional closed string theory, with $p = D - 1$.

Holographic low energy closed string spectrum

- Tachyon pole at $\alpha'(K^2+\mu^2)/2 = (D-1)/8$ in $\mathcal{M}^+$.
- No massless graviton poles
- Massless RR (5D) closed string states in $\mathcal{M}^-$

$$S, \qquad A_{\mu\nu}, \qquad A_\mu$$

- Planar diagram sum should resolve the IR issues connected to the tachyon and the RR massless states.

Closed string tachyon and UV divergences

A. Neveu and J. Scherk, Nuclear Physics B36 (1972) 317-331

- Neveu and Scherk obtain a counterterm to remove UV divergences in one open bosonic string loop diagram by continuation of the Pomeron form factor $\mathcal{F}(p_1, \ldots, p_N, p)$ to $p = 0$.

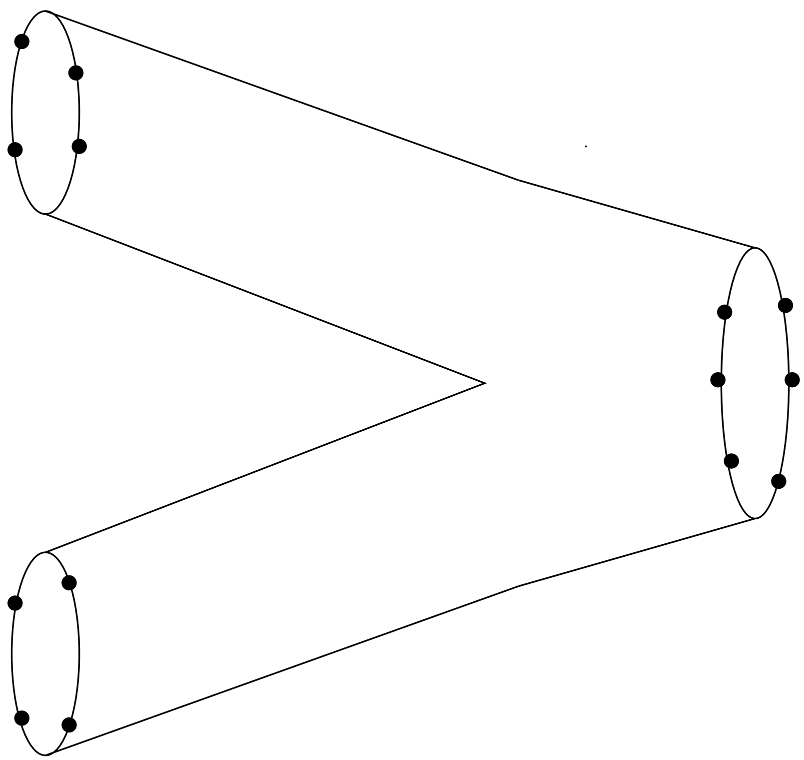
- They show that divergence is $(N-2) \times B_N$, where B_N is N particle tree amplitude.

- This N dependence consistent with absorbing ∞'s in coupling renormalization.

- Should be applicable to the even G-parity NS model as well: P. Goddard and R. Waltz, Nuclear Physics B34 (1971) 99-108.

Open string divergences and QFT divergences
R.R. Metsaev and A.A. Tseytlin, Nuclear Physics B298 (1988) 109-132, show how field theory limit of bosonic string loops reproduce Yang-Mills uv divergences. This same procedure should work for even G-parity NS. (In progress)

To discover closed string cubic vertex examine 2 loop:



Ditto for quartic and higher closed string vertices

Subcritical String

- $L_n = i\alpha n a_n^5 + \hat{L}_n$, $G_r = 2i\alpha r b_r^5 + \hat{G}_r$, $L_0 = \frac{\alpha^2}{2} + \hat{L}_0$
  (Fairlie, Thorn, 1971)

- $c = D + 1 + 8\alpha^2 = 10$, $(L_0)_{min} = \frac{\alpha^2}{2} = \frac{9-D}{16}$

- $\frac{\alpha' M_G^2}{4} = (L_0)_{min} - \frac{1}{2} = -\frac{D-1}{16}$

- Liouville field theory
  (Polyakov, Curtright-Thorn, Gervais-Neveu)

- Deserves much closer attention now

Conclusions

- We do not have the luxury of a ready-made closed string theory to teach us about the closed string background.
- Can try to extract closed string dynamics from open string multi-loop diagrams.
- Expectation: 5th dimension is Polyakov's Liouville field
- Challenge: Find closed string tree amplitudes for 4D Neveu-Schwarz.
- Armed with closed string trees, determine effective field theory for massless and tachyonic closed string states
- Find closed string background: Does closed string tachyon drive theory to a confining vacuum?

We shall not cease from exploration
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time.

- T. S. Eliot