Subcritical String and Large N QCD

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Outline

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4. A subcritical string for large N QCD
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1 Introduction

- AdS/CFT shows string theory's practical side:
  As a tool to analyze nonperturbative features of QFT

- Can string theory illuminate confinement in QCD?

- Can we find a string dual of QCD?

  – One approach: In $\mathcal{N} = 4$ Super Yang-Mills/$AdS_5 \times S^5$ break SUSY so field side is pure Yang-Mills.

  – In this talk I discuss more direct constructions

1.1 AdS/CFT: Top-down

- Add N D3-branes to Type IIB Superstring
- Notice: Low energy limit of physics on the branes is $\mathcal{N} = 4$ SUSY YM with $U(N)$ gauge group in 4D
- Take 't Hooft limit $N \to \infty$, $\lambda = Ng^2/4\pi^2$ fixed.
  At low energy ($\alpha' \to 0$) this is
  planar diagram sum of gauge theory.
- String P.O.V.: $N \to \infty$ suppresses closed string loops:
  Find closed string trees in presence of D3-brane source
- Low energy limit described by a metric whose near horizon limit ($r \to 0$) is $\mathrm{AdS}_5 \times \mathrm{S}^5$.

$$\frac{G\Lambda}{r^4} \sim \frac{GN}{g^2 \alpha'^2 r^4} \sim \frac{Ng^2 \alpha'^2}{r^4} \equiv \frac{R^4}{r^4}$$

- $Ng^2 \to \infty$ justifies treating closed string by effective local fields even in near horizon limit ($\sqrt{\alpha'} \ll r \ll R$).

- This same limit is classical limit of AdS String

- Strong coupling $\mathcal{N} = 4$ described by classical string moving on $\mathrm{AdS}_5 \times \mathrm{S}^5$.

- $Ng^2 = O(1)$ requires *at least* the fully quantum string on $\mathrm{AdS}_5 \times \mathrm{S}^5$.

1.2 AdS/CFT: Bottom-up

- Notice $\mathcal{N} = 4$ SUSY YM is low energy limit of open superstring compactified to 4 dimensions.

- Or simply constrain open string ends to $R_{3,1} \subset R_{9,1}$: 6 scalars and their superpartners are zero modes of extra dimensions.

- Open string loops $\to$ closed strings moving in $D = 10$

- Sum planar open string loops $\equiv$ tree level closed string background source.

- $\alpha' \to 0$ get closed string background by solving effective field theory.

- For $\mathcal{N} = 4$ this effective field theory on the closed string side is IIB supergravity in D3-brane background.

2 Occam's Razor

"Entities should not be multiplied unnecessarily." (Wikipedia)

- Replacing particles with strings seems to multiply entities by infinity. But after $\alpha' \to 0$ redundant particles are removed, yet string remains (at least for large Ng^2.)

- $\mathcal{N} = 4 \equiv \mathrm{AdS}_5 \times \mathrm{S}^5$ (no redundancy on either side, and no contradiction).

- Replacing QCD with broken $\mathcal{N} = 4$ seems to contradict Occam (Unless $\mathcal{N} = 4$ necessary for field/string duality)

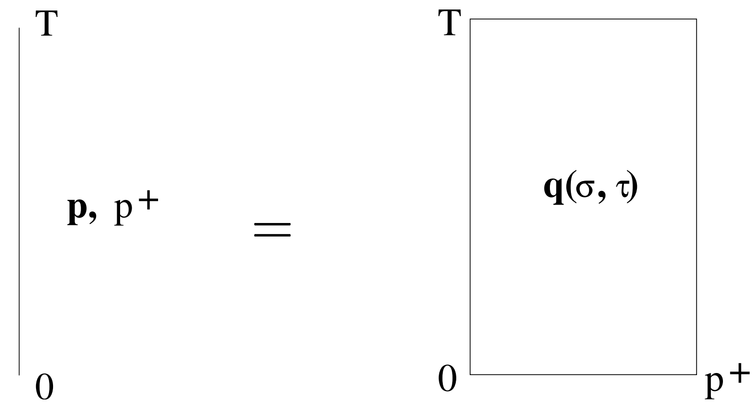
- I would like to explore more economical options

- Keep in mind that critical string may not exhaust all possible string theories.

Lightcone worldsheet Bardakci-Thorn(NPB626:287,2002)

String picture from Yang-Mills with no extra "entities"

Master Formula for Massless Propagator:



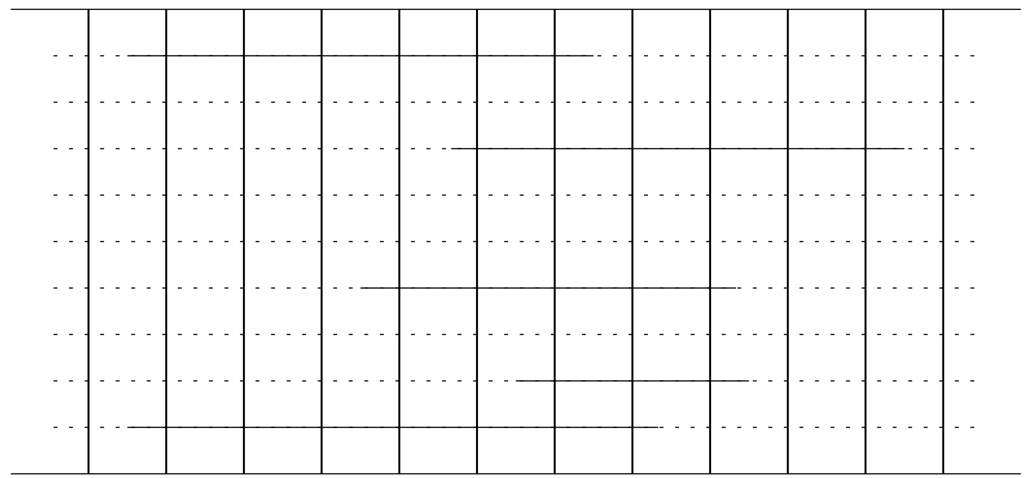
$$\exp\left\{-\frac{T}{2p^+}\boldsymbol{p}^2\right\} = \int_{\substack{\boldsymbol{q}(0,\tau)=0 \\ \boldsymbol{q}(p^+,\tau)=\boldsymbol{p}}} DcDbD\boldsymbol{q}\; e^{iS_0}$$

$$\exp\left\{-\frac{T}{2p^+}\boldsymbol{p}^2\right\} = \int_{\substack{\boldsymbol{q}(0,\tau)=0 \\ \boldsymbol{q}(p^+,\tau)=\boldsymbol{p}}} DcDbD\boldsymbol{q}\ e^{iS_0}$$

$$iS_0 = \int_0^T d\tau \int_0^{p^+} d\sigma \left(b'c' - \frac{1}{2}\boldsymbol{q}'^2\right)$$

- Dirichlet b.c.'s. Cf. string in momentum space
- Represent a field quantum as a composite of String Bits
- Total $p^+ =$ (Number of bits)$\times m$.

Worldsheet Diagram on x^+, p^+ Grid:
$\sigma = im$, $\tau = ja$



- Ising-like spin at each site: $s_i^j = \pm 1$
- $P_i^j \equiv \frac{1+s_i^j}{2} = 0, 1$
- Internal boundaries (solid lines) correspond to a row of $+$ spins. Bulk (dotted lines) is a sea of $-$ spins.

- For example

−	−	−	−	−	−	−	−	−	−	−	−	−	−
−	−	−	−	−	−	−	−	−	−	−	−	−	−
−	−	−	−	+	+	+	+	−	−	−	−	−	−
−	−	−	−	−	−	−	−	−	−	−	−	−	−
−	−	−	−	−	−	+	+	+	+	−	−	−	−
−	−	−	−	−	−	−	−	−	−	−	−	−	−
−	−	−	−	−	−	−	−	−	−	−	−	−	−

  is a two loop diagram.

- A factor of coupling g for each spin flip along a horizontal line.

$$\text{Number of flips} = \sum_{ij} \frac{1 - s_i^j s_i^{j+1}}{2}$$

- Dirichlet b.c.'s on solid lines: add term $\frac{P_i^j P_i^{j-1} a}{2m\epsilon}(\boldsymbol{q}_i^j - \boldsymbol{q}_i^{j-1})^2$, then $\epsilon \to 0$ forces b.c.'s.

- Can identify an effective dynamical tension

$$\frac{P_i^j P_i^{j-1} a^2}{m^2 \epsilon} \sim T_{eff}^{-2}(s),$$

(Ising spin dependent) string tension.

Cf. AdS radius ($r = R^2/z$) dependence of tension in AdS/CFT correspondence on light-cone:

$$\begin{aligned} S_E &= \frac{1}{2}\int d\tau \int_0^{p^+} d\sigma \left(\boldsymbol{q}'^2 + \dot{\boldsymbol{q}}^2 \frac{R^4}{r^4 T_0^2} + T_0^2 r'^2 + \dot{r}^2 \frac{R^4}{r^4} \right) \\ T_{\rm eff}(r) &= \frac{r^2}{R^2} T_0 \end{aligned}$$

- LCWS construction for a given matrix theory is minimal
- Seems to work order by order in pert th.
- Drawbacks

  1. Can't see closed string reinterpretation in pert th.
  2. Fixed gauge pert th. requires extra counterterms to fix "benign anomalies" due to UV divergences
  3. Having no *a priori* method to generate C.T.'s handicaps nonperturbative calculations.

- For 1-Loop Yang-Mills local w.s. interpretation of C.T.'s achievable by introducing extra dimensions
- For $\mathcal{N} = 4$ most of these extra C.T.'s are zero at 1 loop. (This point argues for seeking QCD in a broken $\mathcal{N} = 4$.)

3 String for large N QCD

Let's follow the bottom up path:

- Simplest string model with pure Yang-Mills low energy limit: NS open string with odd G-parity states removed in 4 space-time dimensions
- For large N QCD keep only planar graphs. Ditto for the string model.
- Can now find closed string interpretation by examining planar multi-loop string graphs.
- Note that tree theorem fixes multi-loop diagrams correctly by unitarity.
- This is an important method because string path integrals are poorly understood when $D < D_{crit} = 10$.

4 NS at 1 loop, $D < 10$

For $D < 10$ Super-Virasoro conditions eliminate fewer states:

- $n > 1$: only one component of a_n^μ and one component of $b_{n-1/2}^\mu$ are removed. (for D=10 two are removed)
- a_1^μ and $b_{1/2}^\mu$ both have two components removed.
- Thus the measure factor in the one open string loop integrand is (cf Brower-Thorn and Goddard-Waltz):

$$\frac{1-w}{1+w^{1/2}}\frac{\prod_r(1+w^r)^{D-1}}{\prod_n(1-w^n)^{D-1}} = (1-w^{1/2})\frac{\prod_r(1+w^r)^{D-1}}{\prod_n(1-w^n)^{D-1}}$$

One loop planar NS amplitude with no odd G-parity in loop:

$$\mathcal{M} = \frac{1}{2}(\mathcal{M}^+ - \mathcal{M}^-)$$

where, in cylinder variables, $\ln q = 2\pi^2/\ln w$,

$$\mathcal{M}^+ = \left(\frac{1}{8\pi^2\alpha'}\right)^{D/2} \int \prod_{k=2}^{N} d\theta_k \int_0^1 \frac{dq}{q}\sqrt{\frac{-\pi}{\ln q}}\, q^{-(D-1)/8}\left(w^{(D-9)/16} - w^{(D-1)/16}\right)$$
$$\frac{\prod_r (1+q^{2r})^{D-1}}{\prod_n (1-q^{2n})^{D-1}} \prod_{l<m} \left[\psi(\theta_m - \theta_l, q)\right]^{2\alpha' k_l\cdot k_m}$$
$$\langle k_1 \cdot H(\theta_1) k_2 \cdot H(\theta_2) \cdots k_N \cdot H(\theta_N)\rangle^+$$

$$\psi(\theta, q) = \sin\frac{\theta}{2}\prod_n \frac{(1-q^{2n}e^{i\theta})(1-q^{2n}e^{-i\theta})}{(1-q^{2n})^2}$$

$$\ln w = \frac{2\pi^2}{\ln q}$$

where the average over H fields is evaluated via Wick's theorem for free fermi fields with contractions:

$$\langle k\cdot H(\delta)k'\cdot H(0)\rangle^{+} \equiv k\cdot k'\chi^{+}(\delta,q) =$$

$$\frac{k\cdot k'}{\sin(\delta/2)}\prod_{n}\frac{(1-q^{n})^{2}(1+q^{2n})^{2}(1+q^{2n-1}e^{i\delta})(1+q^{2n-1}e^{-i\delta})}{(1-q^{2n}e^{i\delta})(1-q^{2n}e^{-i\delta})}$$

and the range of integration is

$$0=\theta_1<\theta_2<\cdots<\theta_N<2\pi$$

The index r ranges over positive half odd integers,
the index n over positive integers,
and $l,m\in[1,\cdots,N]$

$$\mathcal{M}^- = \left(\frac{1}{8\pi^2\alpha'}\right)^{D/2} \int \prod_{k=2}^{N} d\theta_k \int_0^1 \frac{dq}{q} \sqrt{\frac{-\pi}{\ln q}} 2^{(D-1)/2} (w^{(D-9)/16} + w^{(D-1)/16})$$

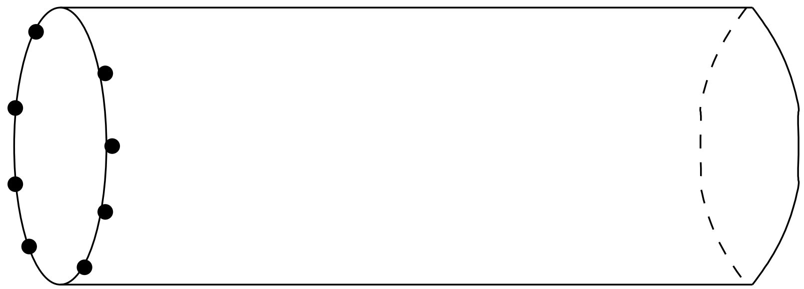
$$\frac{\prod_n (1+q^{2n})^{D-1}}{\prod_n (1-q^{2n})^{D-1}} \prod_{l<m} [\psi(\theta_m - \theta_l, q)]^{2\alpha' k_l \cdot k_m}$$

$$\langle k_1 \cdot H(\theta_1) k_2 \cdot H(\theta_2) \cdots k_N \cdot H(\theta_N)\rangle^-$$

where now the Wick contractions are:

$$\langle k \cdot H(\delta) k' \cdot H(0)\rangle^- \equiv k \cdot k' \chi^-(\delta, q) =$$

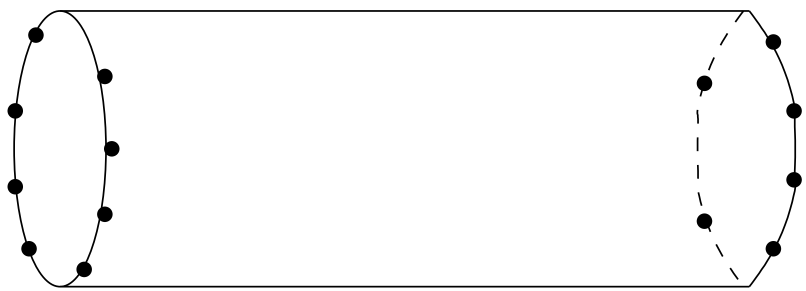
$$\frac{k \cdot k' \cos\delta}{\sin(\delta/2)} \prod_n \frac{(1-q^n)^2 (1+q^{2n-1})^2 (1+q^{2n}e^{i\delta})(1+q^{2n}e^{-i\delta})}{(1-q^{2n}e^{i\delta})(1-q^{2n}e^{-i\delta})}$$



$$\mathcal{M} = \frac{1}{2}(\mathcal{M}^{+} - \mathcal{M}^{-})$$

projects out odd G-parity particles from the loop

To discover closed string spectrum examine nonplanar 1 loop



- $q^{(D-1)/8} \to q^{(D-1)/8+\alpha' K^2/2}$
- $\mathcal{M}^{+}_{\mathrm{NP}}$ has closed string cut starting at $\alpha' K^2 = (D-1)/4$, but $\mathcal{M}^{-}_{\mathrm{NP}}$ closed string cut starts at $K^2 = 0$.

Interpreting the closed string cut

- Factors not present for critical superstring $D = 10$:

$$
\begin{aligned}
&\sqrt{\frac{-\pi}{\ln q}}(w^{(D-9)/16} \mp w^{(D-1)/16}) \\
&\quad = \int \frac{d\mu}{2} q^{\mu^2/4} \left(\cosh \mu\sqrt{\frac{9-D}{16}} \mp \cos \mu\sqrt{\frac{D-1}{16}} \right) \\
&\quad = \int d\mu q^{\mu^2/4} \begin{cases} \sinh \frac{\mu\gamma_+}{2} \sinh \frac{\mu\gamma_-}{2} \\ \cosh \frac{\mu\gamma_+}{2} \cosh \frac{\mu\gamma_-}{2} \end{cases} \\
\gamma_\pm &= \sqrt{\frac{9-D}{16}} \pm i\sqrt{\frac{D-1}{16}}
\end{aligned}
$$

- Open strings are "Dp-branes" in $D+1$ dimensional closed string theory, with $p = D - 1$.

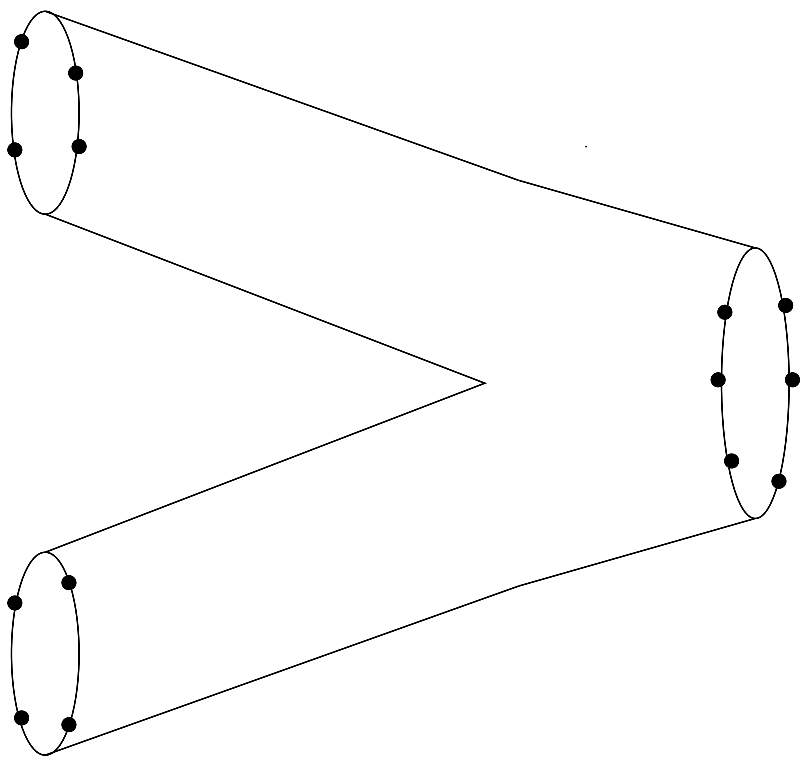
Holographic low energy closed string spectrum

- Tachyon pole at $\alpha'(K^2 + \mu^2)/2 = (D-1)/8$ in $\mathcal{M}^+$.
- No massless graviton poles
- Massless RR (5D) closed string states in $\mathcal{M}^-$

$$S, \qquad A_{\mu\nu}, \qquad A_\mu$$

- Planar diagram sum should resolve the IR issues connected to the tachyon and the RR massless states.

To discover closed string cubic vertex examine 2 loop:



Ditto for quartic and higher closed string vertices

Subcritical String

- $L_n = i\alpha n a_n^5 + \hat{L}_n$, $G_r = 2i\alpha r b_r^5 + \hat{G}_r$, $L_0 = \frac{\alpha^2}{2} + \hat{L}_0$
  (Fairlie, Thorn, 1971)

- $c = D + 1 + 8\alpha^2 = 10$, $(L_0)_{min} = \frac{\alpha^2}{2} = \frac{9-D}{16}$

- $\frac{\alpha' M_G^2}{4} = (L_0)_{min} - \frac{1}{2} = -\frac{D-1}{16}$

- Liouville field theory
  (Polyakov, Curtright-Thorn, Gervais-Neveu)

- Deserves much closer attention now

5 Conclusions

- We do not have the luxury of a ready-made closed string theory to teach us about the closed string background.
- Can try to extract closed string dynamics from open string multi-loop diagrams.
- Expectation: 5th dimension is Polyakov's Liouville field
- Challenge: Find closed string tree amplitudes for 4D Neveu-Schwarz.
- Armed with closed string trees, determine effective field theory for massless and tachyonic closed string states
- Find closed string background: Does closed string tachyon drive theory to a confining vacuum?