Strongly Nonlinear Response of Fractal clusters

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Introduction

Naturally existing and artifically made inhomogenous materials have been the interests of physists for years. These inhomogeneous materials can be divided into two classes according to their scale of inhomogenities. When inhomogenities are of atomic scale, for instance, in form of impurities, quantum mechanics must be applied together with classical electromagnetic theory to due with both microscopic and macroscopic behaviors. On the other hand, when inhomogenities are caused by regions which are small but much larger than atomic scale, classical physics can be applied to both the host material and the regions that cause the inhomogenities. In these media, the connections between microscopic and macroscopic properties are much stronger. Some examples are composites, granular and porus materials. The transport and optical properties of such kind of nonlinear media have received much attention in recent years because of their potential applications in engineering and physics [1–4].

Established theories are available in the weakly nonlinear case in which the nonlinearity can be treated as a small perturbation [5,6]. Over the past few years, substantial progress has been made to calculate the effective nonlinear response for random nonlinear composites in which a small volume fraction of nonlinear material is *randomly* embedded in a host medium. An effective-medium approximation (EMA)

[7] for weakly nonlinear composites was proposed. The scaling form of the weakly nonlinear response has also been extracted from the EMA [8]. Attention has also been paid to a class of strongly nonlinear conducting composite media with a power-law nonlinearity which occurs when a sufficiently strong field is applied to condensed matters [9]. For this composite system, the inclusion and the host medium obey a local current-field (**J**–**E**) relation of the form :

$$\mathbf{J} = \chi |\mathbf{E}|^{2\beta} \mathbf{E}, \tag{1}$$

where $\beta > 0$. For such a nonlinear relation, the conventional perturbation method [6] fails and a variational method have been developed to obtain the dilute-limit expressions for the effective response of a small volume fraction of spherical inclusions embedded in a host medium [10], valid for $\beta = 1$.

The above approach is valid only for *truly* random composites in the dilute limit. In fact, many growth and fabrication processes may produce spatial correlations in realistic composites. In particular, a fractal clustering will be generated via various aggregation processes [11–13]. The fractal geometry should have an observable effect on the nonlinear as well as the linear properties [14–16].

Objective and Results of the Project

The general objective of the project is to study the effective nonlinear response of composites. Starting from May 94, I studied the weakly nonlinear response of fractal clusters[1] until I turned my attention to this more interesting study on the strongly nonlinear response of fractal clusters in August.

We concentrated on the class of strongly nonlinear materials obeying equation (1). To study the spacial correlations of the clusters, we have to study the dilute-

[1] The results of this part are unpublished.

mixing problem first. We generalized the results of Yu and Gu [10] to an arbitary d-dimension space and for arbitary power β. For two types of nonlinear materials with nonlinear coefficients χ_i and χ_m mixed in a ratio of $p : 1-p$, with $p \ll 1$, we showed that the effective nonlinear response is given by :

$$\chi_e = \chi_m + p(\chi_m Q_\beta(b) + \chi_i(1-b)^{2+2\beta}), \tag{2}$$

where b is called the dipolar factor and is given by different means in different models, and

$$Q_\beta(b) = \sum_{i=0}^{\infty} q_i(\beta) b^i, \tag{3}$$

where $q_i(\beta) = -1, 2(1+\beta), (1+\beta)(d-1)(2+4\beta+d)/(2+d), \ldots$ for $i = 0, 1, 2, \ldots$ respectively[2].

Modeling the Clustering Effect :

1. Differential Effective Medium Approximation

The effect of fractal clusters on the weakly nonlinear system [17] has been studied by Hui and Stroud using a differential effective medium approximation (DEMA). This study concluded that there is a large enhancement on the effective nonlinear response, which also means that the strongly nonlinear response of fractal cluters should be considered. However, the above DEMA is not valid for the strongly nonlinear response.

The idea of DEMA goes like this : to study the effect of spacial correlations of a cluster, we start by building the cluster 'core' with material 1. A bigger cluster is then built by adding to the 'core' a thin 'shell' of the mixture of material 2 and

[2] For simplicity, the lengthy general formula is not presented here.

material 1 in a certain rato. Since the amount added is small, the dilute-mixing expression can be applied to obtain the effective response of the bigger cluster. By repeating the above procedures, the effective nonlinear response of a cluster of any size can by found.

We insisted that the idea of DEMA may still be applied if the dilute-limit expression for strongly nonlinear material [10] is used instead. For such a cluster, we showed that the effective response χ_e satisfies [18]:

$$\exp\left[-\int_{\chi_1}^{\chi_e} \frac{d\chi_e}{Q_\beta(b)\chi_e + (1-b)^{2+2\beta}\chi_2}\right] = f, \tag{4}$$

where f is the volume fraction of material 1[3].

On the other hand, numerical simulations in deterministic fractal clusters are believed to be very accurate [20–22]. We performed the numerical simulations on conductance networks and compared the results to our theory, excellent agreements were found [Fig.1][4]. We also proposed and confirmed the scaling form numerically for the DEMA theory.

2. Cluster Effective Medium Approximation

Based on the insights given by the DEMA, we developed a second model called the cluster effective medium approximation [19]. We introduced, for a cluster of size L, a cluster energy functional $C_\beta(L,b)$ on which the variatinal principal is applied to obtain physical properties. The effective response $\chi_e(L)$ is then given by the equation $\chi_e(L) = C_\beta(L, \tilde{b}(L))$, where $\tilde{b}$ is the value of b when C_β achieves its minimum. After

[3]For a fractal cluster of size L, core size ρ, fractal dimesion d_f, $f \approx (L/\rho)^{-(d-d_f)}$

[4]The apparent mismatches in the data for large f is *not* an deflect of our model. It is due to the fact that for large f, the fractal cluster in the simulation has not yet reached the fractal limit.

imposing conditions which account for the spacial correlations, we showed that :

$$C_\beta(L,b) = \chi_1 f^{-Q_\beta(b)} + \frac{\chi_2(1-b)^{2+2\beta}}{Q_\beta(b)}(f^{-Q_\beta(b)}-1). \tag{5}$$

Our results were also compared with numerical simulations in deterministic fractal clusters, excellent agreements were found again [Fig.2].

In February 95, the cluster EMA model was submitted to Physics Letter A and the DEMA model together with the generalized dilute-limit expression was submitted to Physcal Review B. They were published in May and August respectively [18-19].

Possible Future Studies

Looking forward, the project may be continued in many directions. The generalized dilute-limit results allow us to study the dimensional crossover effect in which the dimension is not an integer. Moreover, we may also study the more general and realistic case that β is not the same for all composites. Further numerical studies by using different kinds of fractals can improve the simulations. On the other hand, the theorectical results may be compared with possible experiments, such as a recent one on laser-radiated polymers [23], where a power-law current-voltage characteristic of the form $I \approx V$ has been observed even in small applied voltage V.

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References

[1] D. J. Bergman and D. Stroud, in *Solid State Physics*, edited by H. Ehrenreich and D. Turnbull, vol. **46**, p.147 (Academic, New York, 1992).

[2] See the articles in *Proc. Third International Conference on Electrical Transport and Optical Properties in Inhomogeneous Media* [Physica A **207**, 1994].

[3] See the articles in *Breakdown and Nonlinearity in Soft Condensed Matter*, edited A. Hansen, Springer Lecture Notes on Physics series, to appear 1995.

[4] J. W. Haus, R. Inguva, and C. M. Bowden, Phys. Rev. A **40**, 5729 (1990); M. J. Bloemer, P. R. Ashley, J. W. Haus, N. Kalyaniwalla, and C. R. Christensen, IEEE J. Quan. Elec. **26**, 1075 (1990).

[5] D. Stroud and P. M. Hui, Phys. Rev. B **37**, 8719 (1988).

[6] K. W. Yu, Y. C. Wang, P. M. Hui and G. Q. Gu, Phys. Rev. B **47**, 1782 (1993); G. Q. Gu and K. W. Yu, Phys. Rev. B **46**, 4502 (1992).

[7] X. C. Zeng, D. J. Bergman, P. M. Hui and D. Stroud, Phys. Rev. B **38**, 10970 (1988); X. C. Zeng, D. J. Bergman, P. M. Hui and D. Stroud, Physica A **157**, 192 (1989).

[8] K. W. Yu, Y. C. Chu and E. M. Y. Chan, Phys. Rev. B **50**, 7984 (1994).

[9] R. Blumenfeld and D. J. Bergman, Phys. Rev. B **44**, 7378 (1991).

[10] K. W. Yu and G. Q. Gu, Phys. Lett. **193**A, 311 (1994).

[11] C. Y. Chang, L. C. Kuo and P. M. Hui, Phys. Rev. B **46**, 14505 (1992).

[12] P. Meakin, in *Phase Transition and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic, New York, 1988), Vol. 12, p.355.

[13] See the articles in *On Growth and Form: Fractal and Non-Fractal Patterns in Physics*, edited by H. E. Stanley and N. Ostrowsky (Martinus Nijhoff, Amsterdam,

1986).

[14] B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, San Francisco, 1982).

[15] S. Alexander and R. Orbach, J. Phys. (Paris) Lett. **43**, L625 (1982).

[16] For a recent review, see T. Nakayama, K. Yakubo and R. Orbach, Rev. Mod. Phys. **66**, 381 (1994).

[17] P. M. Hui and D. Stroud, Phys. Rev. B **49**, 11729 (1994).

[18] T. S. Choy and K. W. Yu, Phys. Rev. B, (1995).

[19] T. S. Choy and K. W. Yu, Phys. Lett. A, (1995).

[20] J. P. Clerc, G. Giraud, J. M. Laugier and J. M. Luck, Adv. Phys. **39**, 191 (1990).

[21] O. Levy and D. J. Bergman, J. Phys. Cond. Mat. **5**, 7095 (1993); C.S. Yang and P. M. Hui, Phys. Rev. B **44**, 12559 (1991).

[22] J. P. Straley and S. W. Kenkel, Phys. Rev. B **29**, 6299 (1984); S. W. Kenkel and J. P. Straley, Phys. Rev. Lett. **49**, 767 (1982).

[23] Z. Ball, H. M. Phillips, D. L. Callahan and R. Sauerbrey, Phys. Rev. Lett. **73**, 2099 (1994).