Homework 2
15 + 8 (bonus) = 23 points

Problem 1 (4 points)
Find how thick a target of liquid Hydrogen should be to give 50-50 chances for scattering:
• 10 GeV protons (energy typical for cosmic rays)—2 points
• 10 MeV neutrinos (~ the highest energy solar neutrino)—2 points (express the answer in light years)
The total cross-section of pp scattering is $\sigma_{\text{pp}} \sim 40$ mb in a very wide range of energies. The total cross-section of $\nu p$ interaction is $\sigma_{\nu p} \sim 10^{-40}$ cm$^2$ for neutrinos with about 10 MeV energy.

Problem 2 (2 points)
The matrix element of weak interactions can be well approximated by a constant in a wide range of energies. How should the weak interaction cross section for process $\nu + n \rightarrow e^- + p$ depend on neutrino energy $E$ in the lab frame where the proton target is at rest?

Problem 3 (2 points)
Typical width of excited states of proton and neutron is $\Gamma \sim 100$ MeV.
What lifetime does it correspond to? (1 point)
How far can light travel during this time? Compare the answer with the proton radius of 1 fm. (1 point)

Problem 4 (3 points)
Imagine a particle X decaying into three different di-lepton channels with the following branching ratios:
$\text{BR}(X \rightarrow e^+e^-) = 0.01$
$\text{BR}(X \rightarrow \mu^+\mu^-) = 0.09$
$\text{BR}(X \rightarrow \tau^+\tau^-) = 0.90$

An experiment detects many events with X-particle decays. One graduate student looks at a distribution of invariant masses of two muons in the $\mu^+\mu^-$-decay channel and finds that the distribution has a peak that she can fit with the following formula ($A$ is some constant and mass $m$ is in GeV):
$$dN / dm = \frac{A}{(m-100)^2 + 9}$$

What is the lifetime of the X particle (express the answer in seconds)? (1 point)

Another student studies a distribution of invariant masses of two electrons. What is your prediction for the shape and normalization of this distribution? (2 points)
Problem 5 (4 points)

Non-relativistic protons with an initial momentum $p$ scatter elastically off a gold thin-film target in particular direction ($\theta, \phi$) within a range of stereo angles $d\Omega$ with a cross section $d\sigma$. The cross section is dominated by interactions with the gold nuclei and the role of electrons can be neglected. Now consider a scattering of $\alpha$-particles of the same initial momentum in the same stereo-angle angle direction and range. An $\alpha$-particle is a nucleus of helium and has **twice the charge** and **four times the mass** of a proton. In both cases, one can assume that the gold nuclei, being so heavy, are stationary.

First, by what factor are the matrix elements $m_\alpha$ for the two scatterings different? (2 points)
Second, taking into account all other factors, what is the $\alpha$-particle cross section in terms of the proton cross section $d\sigma$? (2 points)

Problem 6 (8 bonus points)

Assume that protons are infinitely heavy. As you learned, if protons were positive point-like charges, the matrix element and cross section for elastic scattering of electrons on protons would be:

$$m_0(q) = \frac{e^2}{q^2} \quad \text{and} \quad \frac{d\sigma_0}{d\Omega} = \frac{1}{4\pi^2} \frac{e^4 p^2}{q^4 v^2}$$

Find how these expressions would get modified, if protons were not point-like and had the spatial charge distributed in space as follows:

$$\rho(r) = A e^{-r/r_0}.$$  

Looking at the factors modifying the scattering cross section and assuming that $r_0 \sim 1$ fm, argue how large electron energy must be to allow one detecting the proton structure, if one could measure cross sections with $\delta \sim 10\%$ precision.

Hints:
1) The normalization constant $A$ must be such that the proton charge integrated over volume would be equal to $e$. Find this constant—1 points.
2) Calculate potential $V(r)$ due to the charge distribution $\rho(r)$—2 points
3) Calculate matrix element $M(q)$ for electron scattering off the potential $V(r)$—2 points
4) Define at what $q$ the difference in cross sections would become larger that $\delta \sim 10\%$. At what minimum electron energy such $q$ would be possible?—2 points

Note: Such deviation in electron scattering cross sections were seen in 1950s, which allowed to directly measure the proton size for the first time.