Student ID Number: _________

PRELIMINARY EXAMINATION
DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part B, 21 August 2007, 14:00–17:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may NOT use programmable calculators to store formulae.

2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

4. Your assigned student ID Number, the Problem Number, and the Page Number should appear in the upper right hand corner of each sheet. Do NOT use your name anywhere on the Exam.

5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

6. Each problem is worth 10 points.

7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: “On my honor, I have neither given nor received unauthorized aid in doing this assignment.”

DO NOT OPEN EXAM UNTIL INSTRUCTED
B1. Consider an electron with mass $m_e$ confined within an infinite square well defined by

\[
V(x) = \begin{cases} 
0, & \text{for } 0 < x < L, \\
\infty, & \text{otherwise.}
\end{cases}
\]

(a) (2 points) Using Schrödinger’s equation calculate the allowed stationary-state eigenfunctions $\psi_n(x)$, where the complete wavefunctions are given by $\Psi_n(x, t) = \psi_n(x) e^{-iE_n t/\hbar}$. Normalize the eigenfunctions so that the probability of finding the electron somewhere in the box is one.

(b) (2 points) Show that the wavefunctions $\Psi_n(x, t)$ correspond to states with definite energy (i.e. show that $\Delta E = \sigma_E = 0$).

(c) (2 points) Calculate the allowed energy levels, $E_n$, of the system. Express your answer in terms of the Compton wavelength of the electron, $\lambda_e \equiv \hbar/m_e c$, and the rest mass energy of the electron, $m_e c^2$. What is the ground state energy (in MeV) for the case $L = \lambda_e$? (Note that $m_e c^2 = 0.511$ MeV.)

(d) (4 points) Suppose the electron in this infinite square well has a wave function at $t = 0$ which is given by

\[
\Psi(x, 0) = \frac{2}{\sqrt{L}} \sin(\pi x/L) \cos(3\pi x/L).
\]

If you measure the energy of this particle, what are the possible values you might get, and what is the probability of getting each of them? What is the expectation value of the energy for this state (i.e. average energy)? What is $\Psi(x, t)$ and what is the expectation value of $x$, $\langle x \rangle$, for the state $\Psi(x, t)$? Does $\langle x \rangle$ depend on time?
B2. A mass \( m \) is initially at rest at point A on a track.

As indicated by the diagram (a side view, not to scale), the left half of the track takes the shape of a quarter of a circle with radius \( R_1 \), while the right half takes the shape of three quarters of a circle radius \( R \). The mass \( m \) slides down the track and collides with a second mass \( 2m \) at rest at the bottom of the track. After the collision, the two masses are stuck together. The composite mass continues to move up the right half of the track, with speed \( 3\sqrt{gR} \) immediately after the collision. Ignore friction and air resistance throughout this problem.

(a) (2 points) Find the speed \( V_1 \) of \( m \) immediately before the collision, in terms of \( g \) and \( R \).

(b) (2 points) Find \( R_1 \), in terms of \( V_1 \), \( g \) and \( m \).

(c) (2 points) Find the magnitude and direction of the normal force exerted by the track on the composite mass immediately after the collision, in terms of \( R \), \( g \) and \( m \).

(d) (4 points) Find the magnitude and direction of the normal force exerted by the track on the composite mass when it reaches point B, in terms of \( R \), \( g \) and \( m \).
B3. A uniform line charge $\lambda$ is placed on an infinite straight wire, a distance $a$ above an infinite grounded conducting plane.

Let’s orient the axes so that the conducting plane is in the $xy$ plane, and the wire runs parallel to the $x$ axis and directly above it, intersecting the $z$ axis at the point $z = a$ as shown in the figure.

(a) (6 points) Find the electrostatic potential $\varphi(x, y, z)$ in the region above the plane.
(b) (4 points) Find the charge density $\sigma(x, y)$ induced on the plane.