B1. An electron with mass $m$ and energy $E = 0.1\ V_o$ is incident on a potential step with $V_o = 2\ eV$ as shown in the figure. (This is the order of magnitude of the work function for electrons at the surface of metals.)

(a) (3 points) Sketch the wave functions in the regions $x < 0$ and $x > 0$. Indicate all characteristic length scales in terms of $m$, $E$, $V_o$, and $\hbar$.

(b) (4 points) Calculate the relative probability of the electron penetrating the step and plot it as a function of $x$ for $x > 0$ up to a distance of $x \approx 10^{-9}\ m$, roughly five atomic diameters. [You may wish to note that the mass of the electron is $m_e = 0.511\ MeV/c^2$ and that $\hbar c = 1240\ eV\ nm$.]

(c) (3 points) Note that the kinetic energy of the electron for $x > 0$ is negative. Argue how this is consistent with your results of part (b).
B2. An uncharged metal sphere of radius $R$ is placed in an otherwise uniform electric field $E = E_0 \hat{z}$. Answer the following questions to analyze this solution. Assume $E_0 > 0$.

(a) (1 point) What charge will be pushed to the "northern" hemisphere and what charge will be pushed to the "southern" hemisphere?

(b) (1 point) How will the induced charge distort the field in the neighborhood of the sphere? Draw and label a suitable diagram.

(c) (3 points) Find the potential in the region on ($r = R$), and outside ($r > R$), the sphere.

(d) (2 points) For the potential calculated in (c), what is the contribution attributable to the induced charge?

(e) (3 points) Calculate the induced charge density on the sphere.
B3. Three point masses, each having mass $M/3$, are attached to a massless string of length $L$ at positions $\frac{1}{4}L, \frac{1}{2}L,$ and $\frac{3}{4}L$, i.e. they are equidistant from each other and from the ends of the string. The string is held at a constant tension $T$ and clamped at both ends. Now assume that the masses are allowed to oscillate in one of the directions orthogonal to the string and with such small amplitudes that the tension remains approximately constant.

(a) (6 points) Find all the angular frequencies for the normal modes of this system in terms of $\omega^2 = T/ML$.

(b) (4 points) Compare the first three angular frequencies found in (a) to the first three angular frequencies of a string of mass $M$ and length $L$, held under constant tension $T$ and clamped at both ends. Express your answer in terms of $\omega^2 = T/ML$. 