Student ID Number: __________

PRELIMINARY EXAMINATION
DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part C, 7 January 2003, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may NOT use programmable calculators to store formulae.

2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

4. Your assigned student ID Number, the Problem Number, and the Page Number should appear in the upper right hand corner of each sheet. Do NOT use your name anywhere on the Exam.

5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

6. Each problem is worth 10 points.

7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."

DO NOT OPEN EXAM UNTIL INSTRUCTED
C1. A square loop of wire, of side $a$, lies midway between two long wires, $3a$ apart, and in the same plane, as shown in the figure. (Actually the long wires are sides of a large rectangular loop, but the short ends are so far away that they can be neglected.) A clockwise current $I$ in the small square loop is gradually increasing: $\frac{dI}{dt} = k = \text{const}.$

(a) (7 points) Find the emf induced in the big loop.

(b) (3 points) Which way will the induced current flow? Explain.
C2. In many problems (for example, a charged pressed on to a surface by an electric field or the quantum version of a ball bouncing on a hard surface), one finds a particle of mass \( m \) in a linear potential well for \( z > 0 \) and a hard repulsive barrier at \( z = 0 \), where we can write
\[
-h^2 \frac{\partial^2 \psi}{\partial z^2} + V(z) \psi = E \psi \quad \text{and} \quad V(z) = Fz \quad \text{for} \quad z > 0.
\]

(a) (4 points) Use dimensional arguments to show that, if the \( n \)th stationary state wave function depends on \( z \) only in the dimensionless combination \( \alpha_n z \), then its energy is
\[
E_n \approx f_n \left( \frac{\hbar^2}{m} \frac{F^2}{z} \right)^{1/3} \quad \text{and} \quad \alpha^3 \approx g_n \frac{m}{\hbar^2} \frac{F}{z}.
\]
where \( f_n \) and \( g_n \) are numerical constants.

(b) (4 points) Suppose the ground state wave function is given by
\[
\psi(z) = \begin{cases} 
0 & z < 0 \\
A z \exp(-\alpha z) & z \geq 0
\end{cases}
\]
In a variational calculation, the lowest possible energy may be found by calculating \( E(\alpha) \) and then minimizing \( E \) with respect to \( \alpha \). Show that the minimum energy, \( E_0 \), and the value of \( \alpha \) for which \( E \) is a minimum, \( \alpha_0 \), may be written as
\[
E_0 = \left( \frac{3}{2} \right)^{5/3} \left( \frac{\hbar^2}{m} \frac{F^2}{z} \right)^{1/3} \quad \text{and} \quad \alpha_0 = \frac{3}{2} \frac{mE}{\hbar^2}.
\]

(c) (2 points) Instead of the variational solution, consider a WKB approach. The turning points for the energy \( E \) are \( z_1 = 0 \) and \( z_2 = E/F \). The WKB formula for a linear potential may be written as
\[
\frac{1}{\hbar} \int_{z_1}^{z_2} p(z) \, dz = \pi \left( n + \frac{3}{4} \right) \quad \text{where} \quad p(z) = \sqrt{2m[E - V(z)]}.
\]
Using this formula, show that
\[
E_n = \frac{1}{2} \left( \frac{9\pi}{4} \frac{\hbar^2}{m} \right)^{1/3} \left( n + \frac{3}{4} \right)^{2/3}.
\]
Show that for \( E_0 \), the coefficient of \( (\hbar^2 F^2/m)^{1/3} \) is given to be 1.97 by the variational method and 1.84 by the WKB approach.
C3. This question tests your understanding of elementary solid state physics, and your ability to remember a few quantities that help you make qualitative arguments quantitative. Imagine three chunks of solid: one is bright and shiny; one is dark and shiny, and one is yellow and almost transparent.

(a) (3 points) Describe each in terms of its electrical characteristics, using the modern theory of electrons in crystalline solids.

(b) (3 points) Put limits on the gap of each solid, referring to the visible spectrum, but giving the answers in approximate eV.

(c) (4 points) What would happen to the electrical characteristics of these three solids as they were cooled significantly close to absolute zero?