Student ID Number: 

PRELIMINARY EXAMINATION
DEPARTMENT OF PHYSICS
UNIVERSITY OF FLORIDA
Part A, 4 January 2005, 09:00 - 12:00

Instructions

1. You may use a calculator and CRC Math tables or equivalent. No other tables or aids are allowed or required. You may NOT use programmable calculators to store formulae.

2. All of the problems will be graded and will be tabulated to generate a final score. Therefore, you should submit work for all of the problems.

3. For convenience in grading please write legibly, use only one side of each sheet of paper, and work different problems on separate sheets of paper. The sheets for each problem will be stapled together but separately from the other two problems.

4. Your assigned student ID Number, the Problem Number, and the Page Number should appear in the upper right hand corner of each sheet. Do NOT use your name anywhere on the Exam.

5. All work must be shown to receive full credit. Work must be clear and unambiguous. Be sure that you hand your completed work to the Proctor.

6. Each problem is worth 10 points.

7. Following the UF Honor Code, your work on this examination must reflect your own independent effort, and you must not have given, nor received, any unauthorized help or assistance. If you have any questions, ask the Proctor.

University of Florida Honor Code: We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."

DO NOT OPEN EXAM UNTIL INSTRUCTED

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A1. Consider a particle moving in a three dimensional potential \( V(r, \theta, \phi) \), where

\[
V(r, \theta, \phi) = \frac{\kappa}{r^2} + \frac{1}{2} m \omega^2 r^2,
\]

and where \( \kappa \) is a constant and \( \kappa > 0 \). Solve the corresponding time-independent Schrödinger equation,

\[
\mathcal{H} \psi = E \psi,
\]

where the angular part is already known such that

\[
\psi(r, \theta, \phi) = R(r) Y_{\ell \ell} (\theta, \phi).
\]

(a) (3 points) What is the radial Schrödinger equation either in terms of \( R(r) \) or \( u(r) = r R(r) \)? Discuss the limits of \( r \to 0 \) and \( r \to \infty \). In particular, show that \( u(r) \) has the form

\[
u(r) = r R(r) = r^\beta e^{-\gamma r} g(r).
\]

Determine both \( \beta \) and \( \gamma \).

(b) (3 points) From the above work, derive a differential equation for \( g(r) \). Solve this differential equation by a series expansion. Show that the sum

\[
g(r) = \sum_{\mu} a_\mu r^\mu
\]

must end at a finite \( \mu \).

(c) (4 points) Which value of \( r \) maximizes the probability density function of the ground state? Does this value correspond to the minimum of the potential?
A2. An ideal capacitor is made of a pair of square metal plates with sides $a$ separated by distance $b$. A block of dielectric (with dielectric constant $\varepsilon > 1$) has dimensions $a \times a \times b$, so it can fit perfectly between the plates. The dielectric is inserted between the plates so the area of overlap is $a \times z$. The voltage between the plates is maintained at $V$ by a battery.

(a) (2 points) Find the energy stored in the capacitor. Neglect fringe electric fields (i.e. assume the electric field only has a $z$-component).

(b) (5 points) Suppose the distance of overlap changes by a small amount from $z$ to $z + \Delta z$. Find the change (both magnitude and sign) in the energy stored in the capacitor and in the energy stored in the battery.

(c) (3 points) What is the electrostatic force on the dielectric? Does this force tend to pull the dielectric into the plates or push it out of the plates?
A3. In a laboratory frame of reference, a long, straight current $i$ consists of linear densities $+\lambda_0$ and $-\lambda_0$, of positive and negative charge, moving in opposite directions each with a speed $u$. Note that

$$i = 2u\lambda_0.$$  

A single charge $q$ is a distance $r$ away from $i$ and is moving with a speed $v$ parallel to and in the same direction as $i$. Consider the case when $u$ and $v$ are not small compared to $c$, so Special Relativity will be needed to answer the first two parts of this question.

a. (3 points) In the frame of reference of $q$, what is the linear density $\lambda_+$ of the positive charges?

b. (2 points) In the frame of reference of $q$, what is the linear density $\lambda_-$ of the negative charges?

For the remainder of this problem, assume that $u$ and $v$ are both small when compared with $c$, and give your answers only in terms of $q$, $i$, $v$, $r$ and physical constants such as $c$, $\epsilon_0$ and $\mu_0$.

c. (3 points) In the frame of reference of $q$, a net electric field is observed which is proportional to $v$. What is this electric field at the position of $q$?

d. (2 points) What is the force on $q$, as measured in its frame of reference?