Collisions in one dimension that conserve kinetic energy and momentum

Fall 2014

Instructor: Steven Detweiler

Assume that we have two masses \( M_1 \) and \( M_2 \) moving with initial speeds \( v_{1i} \) and \( v_{2i} \) in the same direction—if the objects are actually moving towards each other then just assume that \( v_{2i} \), for example, is negative.

Our goal is to find the final speeds of \( M_1 \) and \( M_2 \) after they collide with each other. The conservation of momentum implies that

\[
M_1 v_{1i} + M_2 v_{2i} = M_1 v_{1f} + M_2 v_{2f}
\]

where \( v_{1f} \) and \( v_{2f} \) are the speeds of the masses after the collision.

The conservation of kinetic energy implies that

\[
\frac{1}{2} M_1 v_{1i}^2 + \frac{1}{2} M_2 v_{2i}^2 = \frac{1}{2} M_1 v_{1f}^2 + \frac{1}{2} M_2 v_{2f}^2.
\]

First we change these equations by putting the all of the \( M_1 \) terms on the left-hand side and the \( M_2 \) terms on the right-hand side to obtain

\[
M_1(v_{1i} - v_{1f}) = M_2(v_{2f} - v_{1f}) \quad (1)
\]

\[
M_1(v_{1i}^2 - v_{1f}^2) = M_2(v_{2f}^2 - v_{2i}^2). \quad (2)
\]

Now, we divide the first of these equations by the second so that the masses cancel each other out, and we have

\[
\frac{v_{1i}^2 - v_{1f}^2}{v_{1i} - v_{1f}} = \frac{v_{2f}^2 - v_{2i}^2}{v_{2i} - v_{2f}}. \quad (3)
\]

But, we know that \( a^2 - b^2 = (a - b)(a + b) \), so our equation above is equivalent to

\[
v_{1i} + v_{1f} = v_{2f} + v_{2i}. \quad (4)
\]

At this point the algebra gets challenging. But, we choose any two of the three equations (1), (2) and (4) to solve for the two unknowns, \( v_{1f} \) and \( v_{2f} \).

For example first use

\[
v_{2f} = v_{1i} + v_{1f} - v_{2i}, \quad (5)
\]

from equation (4), and use this equation to substitute for \( v_{2f} \) into equation (1), and then solve for \( v_{1f} \)

\[
v_{1f} = \frac{M_1 - M_2}{M_1 + M_2} v_{1i} + \frac{2M_2}{M_1 + M_2} v_{2i}. \quad (6)
\]

By switching the subscripts 1 and 2, this becomes

\[
v_{2f} = \frac{M_2 - M_1}{M_1 + M_2} v_{2i} + \frac{2M_1}{M_1 + M_2} v_{1i}. \quad (7)
\]

All of the above equations, become much easier to interpret, if the situation that you have has one of the masses initially at rest.