Chapter 12
Equilibrium and elasticity
Part 1
Equilibrium

- A body is in equilibrium if:

\[ \vec{F}_{net} = \frac{d\vec{P}}{dt} = 0 \]

\[ \vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0 \]

- Conditions for equilibrium:
  - The net force acting on the body is ZERO
  - The net torque acting on the body is ZERO

- The COM moves with a constant speed on a straight line and the rotation of a rigid body is at a constant rate (constant angular velocity)
Static Equilibrium

- In static equilibrium the body does not move
  - No translation!
  - No rotation!

\[ \vec{P} = \text{constant} = 0 \]
\[ \vec{L} = \text{constant} = 0 \]

- Conditions for static equilibrium (the same):
  - The net force acting on the body is ZERO
  - The net torque acting on the body is ZERO
  - This should be true for any part of the system and for any axis of rotation.
A closer look!

- Balance of the forces (all as if applied at the COM)

\[ F_{net,x} = \sum F_x = 0 \]
\[ F_{net,y} = \sum F_y = 0 \]
\[ F_{net,z} = \sum F_z = 0 \]

- Balance of the torque (around any axis of rotation)

\[ \tau_{net,x} = \sum \tau_x = 0 \]
\[ \tau_{net,y} = \sum \tau_y = 0 \]
\[ \tau_{net,z} = \sum \tau_z = 0 \]
Simple case: co-planar forces

• The vectors of all acting forces are in one plane
  • Choose a coordinate system so that the forces have at most two components.
  • The resulting torques are perpendicular to the plane of the forces.

\[
\vec{F} = F_x \hat{i} + F_y \hat{j}
\]

\[
\vec{\tau} = \vec{r} \times \vec{F} \quad \Rightarrow \quad \vec{\tau} \perp \vec{r} \text{ and } \vec{\tau} \perp \vec{F} \quad \Rightarrow \quad \vec{\tau} = \tau_z \hat{k}
\]
Co-planar case

• Conditions for static equilibrium:
  • Two equations for the forces
  • One equation for the torques

\[ F_{net,x} = \sum F_x = 0 \]

\[ F_{net,y} = \sum F_y = 0 \]

\[ \tau_{net,z} = \sum \tau_z = 0 \]

\[ F_N - m_1g - m_2g = 0 \]

\[ m_1gx_1 - m_2gx_2 = 0 \]
Example: mobile

- The figure shows a mobile of penguins from a ceiling. Each crossbar is horizontal, has negligible mass and extends **three times** as far to the right of the wire supporting it as to the left. Penguin 1 has a mass of 48 g. Find the mass of penguin 4.
Solution

From the condition for torque balance

Bar 3

\[ xm_3 g - 3xm_4 g = 0 \]

\[ m_3 = 3m_4 \]

Bar 2

\[ xm_2 g - 3x(m_3 + m_4) g = 0 \]

\[ m_2 = 3(m_3 + m_4) = 12m_4 \]

Bar 1

\[ xm_1 g - 3x(m_2 + m_3 + m_4) g = 0 \]

\[ m_1 = 3(m_2 + m_3 + m_4) \]

\[ m_1 = 3(12m_4 + 3m_4 + m_4) = 48m_4 \]

\[ m_4 = m_1 / 48 = 1 \text{ kg} \]
HITT quiz question

- If the forces can be adjusted in magnitude and the bars are uniform, in which cases equilibrium is possible?

1) a
2) b
3) c
4) d
Example: a ladder against a wall

- What must the friction coefficient with the floor be so that the ladder does not fall down?

\[
\begin{align*}
\sum F_x &= 0 & f_s - F_{N2} &= 0 \\
\sum F_y &= 0 & F_{N1} - mg &= 0 \\
\sum \tau_z &= 0 & F_{N2}L \sin \theta - mg \frac{L}{2} \cos \theta &= 0 \\

f_s &= \mu_s F_{N1} = \mu_s mg
\end{align*}
\]

\[
\mu_s mg L \cos \theta - mg \frac{L}{2} \cos \theta = 0 \quad \mu_s \sin \theta - \frac{1}{2} \cos \theta = 0
\]

\[
\mu_s = \frac{1}{2 \tan \theta}
\]