Exam 3 Solutions
(First answer is correct)

1. (WebAssign 30.15) As shown in the figure, a stiff wire bent into a semicircle of radius 5 cm is rotated at constant frequency 55 Hz in a uniform magnetic field of 2.4 T. The resistance $R$ is 16Ω. What is the amplitude of the emf (in volts) generated in the circuit?

(1) 3.26
(2) 0.52
(3) 0.41
(4) 0.20
(5) 6.51

The emf is given by Faraday’s law $\mathcal{E} = \frac{d\Phi}{dt}$. The semicircular area is $\frac{1}{2} \pi r^2$ and, taking into account the rotation of the semicircle, the total flux is $\Phi_B = \frac{1}{2} \pi r^2 \cos \omega t$. Taking the derivative yields the emf $\mathcal{E} = -\frac{1}{2} \pi r^2 \omega B \sin \omega t$, so the amplitude of the emf is $\mathcal{E}_m = \frac{1}{2} \pi r^2 \omega B = 3.26$ V.

2. (WebAssign 31.43) Consider an RLC circuit with a driving emf of amplitude $\mathcal{E}_m = 20$ V, $R = 25\Omega$, $L = 0.06$ H and $C = 2.0\mu$F. What is the amplitude of the voltage (in volts) across the inductor at resonance?

(1) 139
(2) 220
(3) 455
(4) 345
(5) 197

At resonance only $R$ contributes to the impedance, so the current amplitude is $I_m = \frac{200}{250} = 0.8$ A. The voltage amplitude across the inductor can be found from the inductive reactance to be $V_L = X_L I_m = \omega LI_m$. Using $\omega = \sqrt{1/LC}$ at resonance and substituting values yields 139 V.

3. (Homework 31.24) A single loop circuit has $R = 9\Omega$, $C = 16\mu$F and $L = 4$H. Initially, the capacitor has charge $12\mu$C and the current is zero. What is the charge (in $\mu$C) on the capacitor 10 complete cycles later?
In an RLC circuit the maximum charge on the capacitor (as well as the current amplitude) decreases as \( e^{-tR/2L} \). The oscillation angular frequency is \( \omega' = \sqrt{1 / LC - (R / 2L)^2} \approx 125 \text{ rad/sec} \). The total time for 10 cycles is \( t = 10 \times 2\pi / \omega = 0.503 \text{ s} \). The exponential term is thus \( e^{-tR/2L} = 0.568 \) and the charge is \( 12 \times 0.568 = 6.82 \mu\text{C} \).

4. (WebAssign 32.10) In the figure shown, a uniform electric field is directed out of the page within a circular region of radius \( R = 4.80 \text{ cm} \). The magnitude of the electric field is given by \( E = (4.5 \times 10^8 \text{ V/m/sec})t \), where \( t \) is in seconds. What is the magnitude of the induced magnetic field (in T) at a radial distance of 1.60 cm and time \( t = 12 \text{ sec} \)?

\[
\begin{align*}
(1) & \quad 4.0 \times 10^{-11} \\
(2) & \quad 1.2 \times 10^{-10} \\
(3) & \quad 4.8 \times 10^{-10} \\
(4) & \quad 1.3 \times 10^{-11} \\
(5) & \quad 1.4 \times 10^{-9}
\end{align*}
\]

The induced B field is found most easily from \( B = \mu_0 i_d / 2\pi r \), where \( i_d \) is the displacement current. Here \( i_d = \varepsilon_0 d\Phi_E / dt = \varepsilon_0 \pi r^2 dE / dt \). Note that since \( E \) is proportional to \( t \), \( dE/dt \) is constant. Substituting values yields \( B = 4.0 \times 10^{-11} \text{ T} \).

5. (WebAssign 33.24) A laser emits light at power 6.0 milliwatts and wavelength 633 nm. The beam diameter is smaller than the diameter of a spherical particle placed in its path. The particle is perfectly absorbing and has mass \( 5 \times 10^{-15} \text{ kg} \). What is the magnitude of the acceleration (in \( \text{m/s}^2 \)) from the laser radiation pressure acting on the particle?

\[
\begin{align*}
(1) & \quad 4.0 \times 10^3 \\
(2) & \quad 2.0 \times 10^{-11} \\
(3) & \quad 1.2 \times 10^{12} \\
(4) & \quad 1.8 \times 10^6 \\
(5) & \quad 2.5 \times 10^4
\end{align*}
\]

The force from the laser beam is given by \( F = I / c \), where \( I \) is the intensity. Since the beam is totally absorbed (force is in a single direction), the acceleration is \( a = I / mc = 4.0 \times 10^3 \text{ m/s}^2 \).
6. *(Homework 30.Q8, checkpoint 6, 2 previous exams)* In the LR circuit shown, resistance \( R_1 \) is twice as large as resistance \( R_2 \). The switch is closed at \( t = 0 \) and, after a long time later, it is re-opened. Which of the following statements is true?

(1) Just after the switch is reopened, the current through \( R_1 \) is the same as the current through \( R_2 \)
(2) Just after the switch is closed, \( R_2 \) carries twice as large a current as \( R_1 \)
(3) Just after the switch is closed, no current flows through any of the resistors
(4) Just before the switch is reopened, \( R_1 \) carries zero current
(5) None of these statements are true

*When the switch is first closed, there is zero current in the left hand branch because of the inductor. After a long time, the current in the left branch reaches its full value and the inductor has no effect. When the switch is reopened, the left hand branch maintains its current and since the resistors are now in series the current in \( R_1 \) is the same as in \( R_2 \).*

7. *(Chapter 31 new)* A uniform magnetic field of 3 T is present in a spherical volume of 2 m radius. What is the total energy (in J) of the magnetic field in this volume?

(1) \( 1.2 \times 10^8 \)
(2) \( 6.0 \times 10^5 \)
(3) \( 3.6 \times 10^2 \)
(4) \( 7.4 \times 10^{11} \)
(5) \( 4.1 \times 10^{-1} \)

The \( B \) field energy density is \( u_B = B^2 / 2 \mu_0 \). Using the formula for the volume of a sphere, this yields a total energy of \( U_B = \left( 4\pi r^3 / 3 \right) B^2 / 2 \mu_0 = 1.2 \times 10^8 \) J.

8. *(Webassign 32.32)* An electron is placed in a magnetic field \( B \) that is directed along a \( z \) axis. The energy difference between parallel and antiparallel alignments of the \( z \) component of the electron's spin magnetic moment with \( B \) is \( 5.20 \times 10^{-25} \) J. What is the magnitude of \( B \) in Tesla?

(1) 0.028
(2) 0.11
(3) \( 5.6 \times 10^{-3} \)
(4) \( 1.7 \times 10^{-4} \)
(5) \( 3.3 \times 10^{-6} \)
The difference in potential energies of an electron in a magnetic field \( B \) is \( \Delta E = 2 \mu_B B \), where \( \mu_B \) is the Bohr magneton. The above values yield \( B = 0.028 \, T \).

9. (Sample problem 33-3) A beam of unpolarized light is sent through a system of three polarizing sheets. The polarizing direction of the third sheet makes an angle of \( \theta_3 = 90^\circ \) with respect to that of the first sheet. The polarization direction of the second sheet, placed between sheets 1 and 3, makes an angle \( \theta_2 \) with respect to that of the first sheet. Which of the following statements is false?

(1) After the light goes through the first sheet, it becomes 50% polarized.
(2) If \( \theta_2 \) is 90°, no light is transmitted through the system.
(3) If \( \theta_2 \) is 0°, no light is transmitted through the system.
(4) If \( \theta_2 \) is 45°, the intensity of the light transmitted through the system is 12.5% of the initial intensity.
(5) If the second sheet is removed, no light is transmitted through the system.

After light of any kind goes through a polarizing filter, it becomes 100% polarized. Answers (2) – (5) are all true.

10. (Chapter 33 new) A ray of light from a source placed at the bottom of a pool of water \( (n = 1.333) \) impinges on the water surface with an angle of incidence of 30°. Which of the following statements is true?

(1) The angle of refraction is 41.8°.
(2) The ray is totally reflected at the surface.
(3) The angle of refraction is 90°.
(4) The angle of reflection is 57.2°.
(5) The angle of reflection depends on the wavelength of the light.

The angle of refraction is given by \( n \sin \theta_1 = \sin \theta_2 \), where \( n = 1.333 \), \( \theta_1 = 30^\circ \) and \( \theta_2 \) is the angle of the refracted ray. This yields \( \theta_2 = 41.8^\circ \).