The light bulbs in the circuits below are identical. Which configuration produces more light?

- (a) circuit I
- (b) circuit II
- (c) both the same

Circuit II has \( \frac{1}{2} \) current of each branch of circuit I, so each bulb is \( \frac{1}{4} \) as bright. The total power in circuit I is thus 4x that of circuit II.
Circuits

The three light bulbs in the circuit are identical. What is the brightness of bulb B compared to bulb A?

- a) 4 times as much
- b) twice as much
- c) the same
- d) half as much
- e) 1/4 as much

Use $P = I^2R$. Thus 2x current in A means it is 4x brighter.
Circuit Problem (1)

The light bulbs in the circuit are identical. What happens when the switch is closed?

- a) both bulbs go out
- b) the intensity of both bulbs increases
- c) the intensity of both bulbs decreases
- d) nothing changes

Before switch closed: \( V_a = 12 \text{V} \) because of battery. \( V_b = 12 \) because equal resistance divides 24V in half.

After switch closed: Nothing changes since (a) and (b) are still at same potential.
Circuit Problem (2)

The light bulbs in the circuit shown below are identical. When the switch is closed, what happens to the intensity of the light bulbs?

- a) bulb A increases
- b) bulb A decreases
- c) bulb B increases
- d) bulb B decreases
- e) nothing changes

Before switch closed: \( V_a = 12 \text{V} \) because of battery. \( V_b = 12 \) because equal resistance divides 24V in half.

After switch closed: Nothing changes since (a) and (b) are still at same potential.
Circuit Problem (3)

The bulbs A and B have the same R. What happens when the switch is closed?

- a) nothing happens
- b) A gets brighter, B dimmer
- c) B gets brighter, A dimmer
- d) both go out

Before: $V_a = 24$, $V_b = 18$. Bulb A and bulb B both have 18V across them.

After: $V_a = 24$, $V_b = 24$ (forced by the batteries). Bulb A has 12V across it and bulb B has 24V across it.
Kirchhoff’s Rules

Junction rule (conservation of charge)

- Current into junction = sum of currents out of it

\[ I = I_1 + I_2 + I_3 \]

Loop rule (conservation of energy)

- Algebraic sum of voltages around a closed loop is 0

1. \[ -I_3 R_1 - I_2 R_2 - E_2 - I_3 R_1 + E_2 = 0 \]
2. \[ -I_1 R_1 - E_1 - I_1 R_1 + E_2 + I_2 R_2 = 0 \]
Problem Solving Using Kirchhoff’s Rules

- Label the current in each branch of the circuit
  - Choice of direction is arbitrary
  - Signs will work out in the end (if you are careful!!)
  - Apply the junction rule at each junction
  - Keep track of sign of currents entering and leaving

- Apply loop rule to each loop (follow in one direction only)
  - Resistors: if loop direction matches current direction, voltage drop
  - Batteries: if loop direction goes through battery in “normal” direction, voltage gain

- Solve equations simultaneously
  - You need as many equations as you have unknowns
Kirchhoff’s rules

- Determine the magnitudes and directions of the currents through the two resistors in the figure below.

- Take two loops, 1 and 2, as shown

Use $I_1 = I_2 + I_3$

1. $+6 - 15I_3 = 0$
2. $-22I_2 + 9 + 15I_3 = 0$

$I_3 = 6/15 = 0.40$
$I_2 = 15/22 = 0.68$
$I_1 = I_2 + I_3 = 1.08$
Which of the equations is valid for the circuit shown below?

- a) $2 - I_1 - 2I_2 = 0$
- b) $2 - 2I_1 - 2I_2 - 4I_3 = 0$
- c) $2 - I_1 - 4 - 2I_2 = 0$
- d) $I_3 - 2I_2 - 4I_3 = 0$
- e) $2 - 2I_1 - 2I_2 - 4I_3 = 0$
Wheatstone Bridge

An ammeter A is connected between points a and b in the circuit below. What is the current through the ammeter?

- a) $I / 2$
- b) $I / 4$
- c) zero
- d) need more information

Before ammeter is added:
- The top branch divides the voltage evenly, so $V_a = V/2$.
- The bottom branch also divides the voltage evenly, so $V_b = V/2$.
- Thus $V_a = V_b$ and current is 0 across ammeter.
Wheatstone Bridge

An ammeter $A$ is connected between points $a$ and $b$ in the circuit below. What is the current through the ammeter?

a) $I/2$

b) $I/4$

c) zero

d) need more information

Same analysis. Before ammeter is added:

• The top branch divides the voltage evenly, so $V_a = V/2$.
• The bottom branch also divides the voltage evenly, so $V_b = V/2$.
• Thus $V_a = V_b$ and current is 0 across ammeter.
All batteries are 4V
All resistors are 4Ω

Find current in R
Hint: follow the batteries
Res-Monster Maze (p. 725)

All batteries are 4V
All resistors are 4Ω

Find current in R
Hint: follow the batteries
Problem solving

Find the value of $R$ that maximizes power emitted by $R$.

\[ P_2 = I_2^2 R \]

\[ R_{\|} = \frac{12R}{12 + R} \]

\[ I_T = \frac{18}{6 + \frac{12R}{12 + R}} = \frac{12 + R}{4 + R} \]

\[ \frac{I_2}{I_T} = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{12}} = \frac{12}{12 + R} \implies I_2 = \frac{12}{4 + R} \]

\[ P_2 = I_2^2 R = \frac{144R}{(4 + R)^2} \]

Maximize

\[ R = 4\Omega \quad P_2 = 9 \text{ W} \]
Light Bulbs

A three-way light bulb contains two filaments that can be connected to the 120 V either individually or in parallel.

- A three-way light bulb can produce 50 W, 100 W or 150 W, at the usual household voltage of 120 V.
- What are the resistances of the filaments that can give the three wattages quoted above?

Use $P = \frac{V^2}{R}$

- $R_1 = \frac{120^2}{50} = 288 \Omega$ (50W)
- $R_2 = \frac{120^2}{100} = 144 \Omega$ (100W)
Problem

What is the maximum number of 100 W light bulbs you can connect in parallel in a 100 V circuit without tripping a 20 A circuit breaker?

- (a) 1
- (b) 5
- (c) 10
- (d) 20
- (e) 100

Each bulb draws a current of 1A. Thus only 20 bulbs are allowed before the circuit breaker is tripped.
RC Circuits

⇒ Charging a capacitor takes time in a real circuit
  ◆ Resistance allows only a certain amount of current to flow
  ◆ Current takes time to charge a capacitor

⇒ Assume uncharged capacitor initially
  ◆ Close switch at \( t = 0 \)
  ◆ Initial current is \( i = \frac{E}{R} \) (no charge on capacitor)

⇒ Current flows, charging capacitor
  ◆ Generates capacitor potential of \( \frac{q}{C} \)

\[
i = \frac{E - q / C}{R}
\]

⇒ Current decreases continuously as capacitor charges!
  ◆ Goes to 0 when fully charged
Analysis of RC Circuits

- Current and charge are related

\[ i = dq / dt \]

- So can recast previous equation as “differential equation”

\[ i = \frac{E - q / C}{R} \quad \Rightarrow \quad \frac{dq}{dt} + \frac{q}{RC} = \frac{E}{R} \]

- General solution is \( q = EC + Ke^{-t/RC} \)
  - (Check and see!)
  - \( K = -EC \) (necessary to make \( q = 0 \) at \( t = 0 \))

- Solve for charge \( q \) and current \( i \)

\[ q = EC \left(1 - e^{-t/RC}\right) \quad \Rightarrow \quad \frac{dq}{dt} = \frac{E}{R} e^{-t/RC} \]
Charge and Current vs Time
(For Initially Uncharged Capacitor)

\[ q(t) = q_0 \left(1 - e^{-t/RC}\right) \]

\[ i(t) = i_0 e^{-t/RC} \]
Exponential Behavior

$\Rightarrow t = RC$ is the “characteristic time” of any RC circuit
- Only $t / RC$ is meaningful

$\Rightarrow t = RC$
- Current falls to 37% of maximum value
- Charge rises to 63% of maximum value

$\Rightarrow t = 2RC$
- Current falls to 13.5% of maximum value
- Charge rises to 86.5% of maximum value

$\Rightarrow t = 3RC$
- Current falls to 5% of maximum value
- Charge rises to 95% of maximum value

$\Rightarrow t = 5RC$
- Current falls to 0.7% of maximum value
- Charge rises to 99.3% of maximum value
Discharging a Capacitor

Connect fully charged capacitor to a resistor at t = 0

General solution is \( q = Ke^{-t/RC} \)

\( K = VC \) (necessary to make have full charge at t = 0)

Solve for charge q and current i

\[ q = VCe^{-t/RC} \quad i = \frac{dq}{dt} = -\frac{V}{R}e^{-t/RC} \]
Charge and Current vs Time
(For Initially Charged Capacitor)

\[ q(t) = q_0 e^{-t/RC} \]

\[ i(t) = i_0 e^{-t/RC} \]