Energy Stored in Magnetic Field

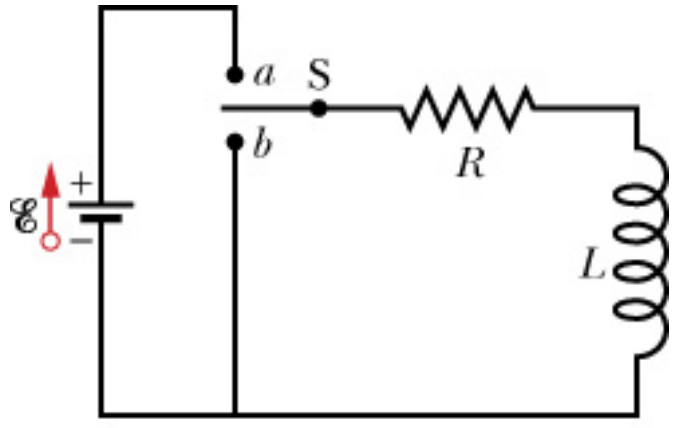
➔Just like electric fields, magnetic fields store energy

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$ ← Electric field energy density

$$u_B = \frac{B^2}{2\mu_0}$$ ← Magnetic field energy density

➔Let's see how this works

Energy of an Inductor



➔ How much energy is stored in an inductor when a current is flowing through it?

➔ Start with loop rule

$$\mathcal{E} = iR + L\frac{di}{dt}$$

➔ Multiply by i to get power equation

$$\mathcal{E}\, i = i^2 R + L\, i\, \frac{di}{dt}$$

Power produced = dissipated + stored

➔ Let P_L = power stored in inductor

$$P_L = \frac{dU_L}{dt} = L\, i\, \frac{di}{dt}$$

➔ Identify energy stored in inductor

$$U_L = \int Li\,di = \frac{1}{2}Li^2$$

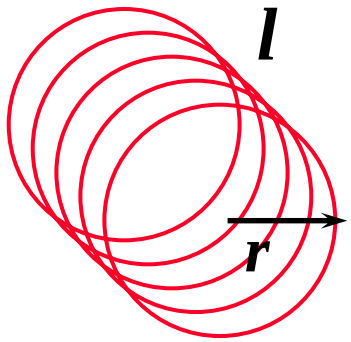
➔ Similar to capacitor:

$$U_C = \frac{q^2}{2C}$$

Energy in Magnetic Field (2)

➔Apply to solenoid (constant B field)

$$U_L = \tfrac{1}{2} L i^2 = \tfrac{1}{2}\left(\mu_0 n^2 l A\right) i^2$$



N turns

➔Use formula for B field: $B = \mu_0 n i$

$$U_L = \frac{B^2}{2\mu_0} lA$$

➔Calculate energy density: $u_B = \dfrac{U_L}{V} \quad V = Al$

$$u_B = \frac{B^2}{2\mu_0}$$ B field

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2$$ E field

➔This is generally true even if B is not constant

Energy Calculation Examples

➔Calculate u_B for earth field, B = 5 x 10^{-5} T

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(5\times10^{-5}\right)^2}{2\times4\pi\times10^{-7}} \simeq 10^{-3}\ \mathrm{J/m^3}$$

➔Calculate u_B for neutron star, B = 10^8 T

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(10^{8}\right)^2}{2\times4\pi\times10^{-7}} \simeq 4\times10^{21}\ \mathrm{J/m^3}$$

➔Calculate u_B for magnetar, B = 10^{11} T

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(10^{11}\right)^2}{2\times4\pi\times10^{-7}} \simeq 4\times10^{27}\ \mathrm{J/m^3}$$

$$\rho = \frac{u_B}{c^2} \simeq 4\times10^{10}\ \mathrm{kg/m^3}$$

Equivalent mass density
From E = mc²

Web Sites on Neutron Stars, Magnetars

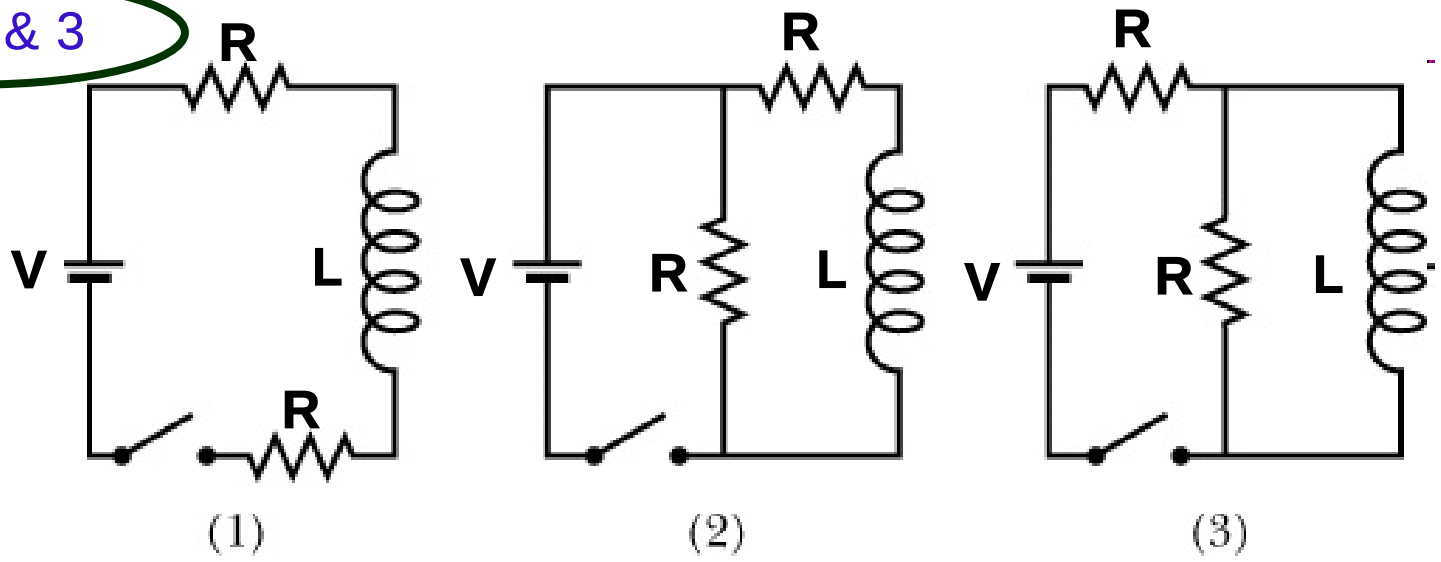
- Original magnetar discovery
  - http://science.nasa.gov/newhome/headlines/ast20may98_1.htm
  - http://www.firstscience.com/site/articles/solarflares.asp
- More recent magnetar discovery (Feb. 2005)
  - http://www.physorg.com/news3112.html
- Online articles on magnetars
  - http://solomon.as.utexas.edu/~duncan/magnetar.html
  - http://www.space.com/scienceastronomy/magnetar_formation_050201.html
  - http://apod.gsfc.nasa.gov/apod/ap041126.html
- Articles on neutron stars (second one has videos)
  - http://www.astro.umd.edu/~miller/nstar.html
  - http://antwrp.gsfc.nasa.gov/htmltest/rjn_bht.html

Quiz on Energy

➔Each circuit has identical values of V, R and L. After the switch has been closed for a long time, which circuit has the largest energy stored in the inductor L?

- (1) 1 only
- (2) 2 only
- (3) 3 only
- (4) 1 & 2
- (5) 2 & 3

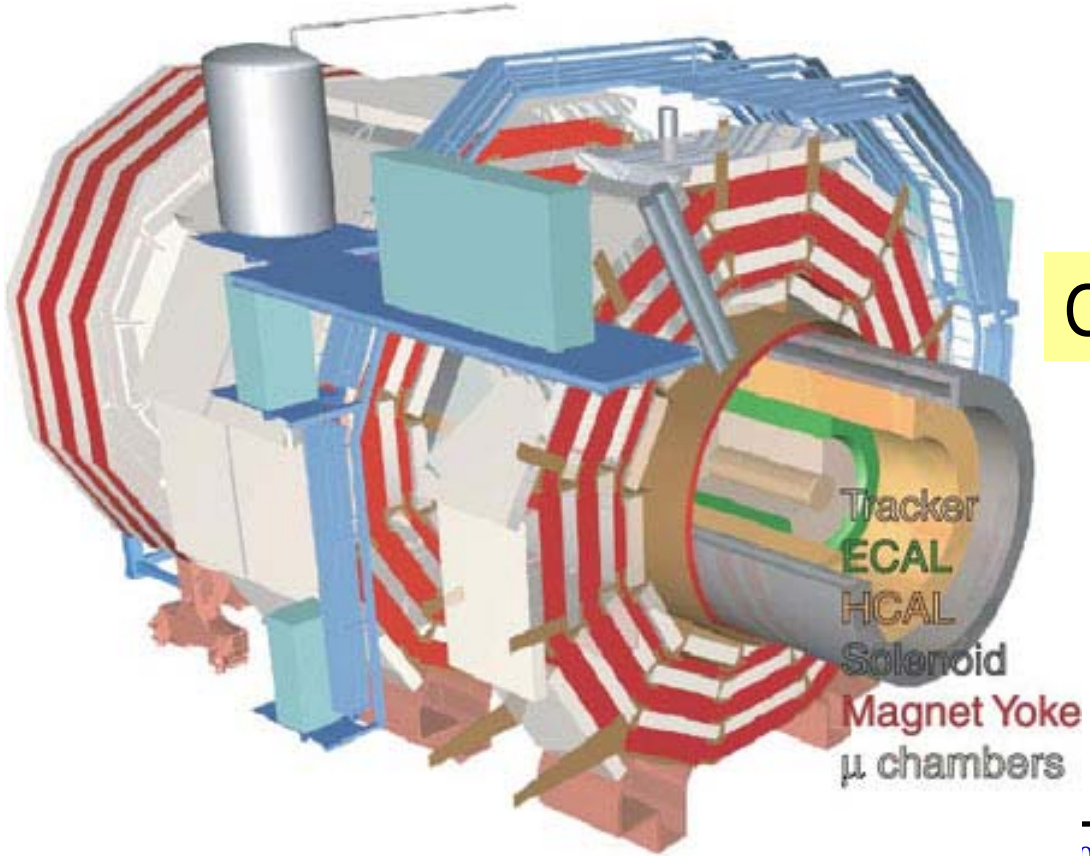
Answer is modified from what was given in class.
Answers 2 and 3 were accepted



Gigajoule Magnet at CERN

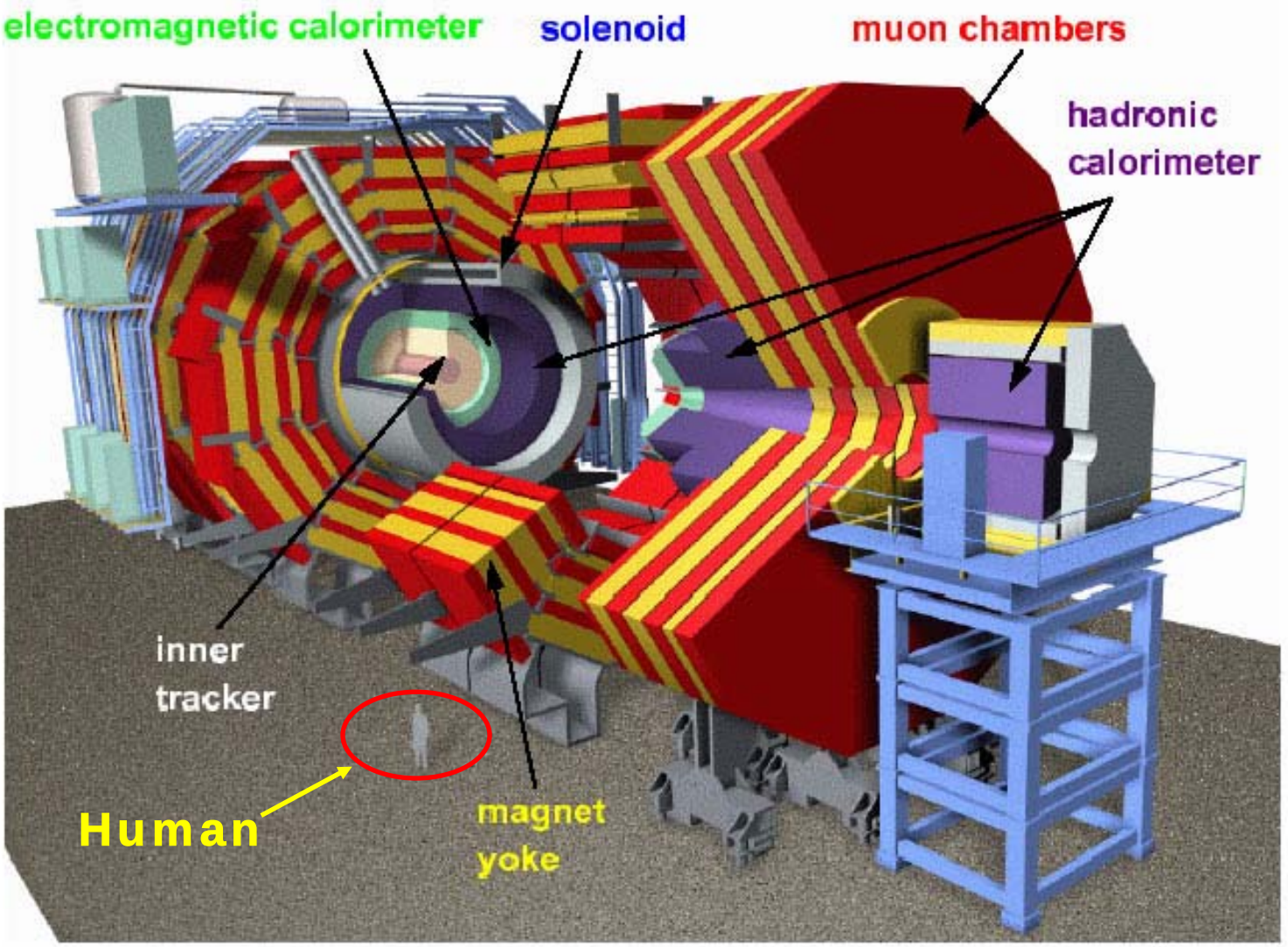
➔CMS experiment at CERN

- p-p collisions at world's highest energy in 2009
- Hope to discover new particles, find the origin of mass and new fundamental forces



Compact Muon Solenoid

Compact Muon Solenoid



CMS Experiment Magnet

➔Large central solenoid magnet to study particle production

- B = 4T, R = 3.15 m, L = 12.5 m
- U_B = 2.6 x 10^9 J = 2.6 gigajoules!!

$$U_B = \frac{B^2}{2\mu_0} lA = \frac{4^2}{2 \times 4\pi \times 10^{-7}} \left(\pi \times 3.15^2\right)(12.5)$$

http://www.spacedaily.com/news/energy-tech-04b.html

Articles on CMS and LHC

- Large Hadron Collider at CERN
  - http://public.web.cern.ch/public/en/LHC/LHC-en.html
- Home page and picture of CMS experiment
  - http://cms.cern.ch/
- Wikipedia articles
  - http://en.wikipedia.org/wiki/Compact_Muon_Solenoid
  - http://en.wikipedia.org/wiki/Large_Hadron_Collider