Physics 2049 Exam 4 Solutions

1. A Gaussian surface in the shape of a right circular cylinder with end caps has a radius of 12.0 cm and a length of 80.0 cm. Through one end there is an inward magnetic flux of \(25 \times 10^{-6} \text{ T} \cdot \text{m}^2\). At the other end there is a uniform magnetic field of 1.60 mT, normal to the surface and directed outward. What is the net magnetic flux through the curved surface?

   \textbf{Answer:} \(47.4 \times 10^{-6} \text{ T} \cdot \text{m}^2\) inward

   \textbf{Solution:} Gauss’s Law for magnetic fields states that the net magnetic flux through a closed surface is zero (another way of saying that there are no magnetic monopoles). The flux through the ends is

   \[\Phi_{\text{ends}} = -25.0 \times 10^{-6} \text{ T} \cdot \text{m}^2 + (1.60 \times 10^{-3} \text{ T})\pi(0.12 \text{ m})^2 = 47.4 \times 10^{-6} \text{ T} \cdot \text{m}^2 \text{ (outward)} \quad (1)\]

   In order that the net flux through the entire cylinder be zero, the flux through the curved section must be \(47.4 \times 10^{-6} \text{ T} \cdot \text{m}^2\) inward.

2. A parallel plate capacitor is made of two disks of radius \(r = 1\) cm, separated by a distance of \(d = 1\) mm (the region between the plates is filled with air). The capacitor is connected in series with a 1.0 kΩ resistor, and the combination is connected across a 12 V battery. Initially a switch in the circuit is open and the capacitor is uncharged; the switch is then closed, charging the capacitor. What is the maximum magnitude of the magnetic field anywhere between the capacitor plates after closing the switch?

   \textbf{Answer:} \(2.4 \times 10^{-7} \text{ T}\)

   \textbf{Solution:} The induced magnetic field can be determined from the fourth Maxwell equation (the Ampère-Maxwell equation). In the region between the capacitor plates there is no current, so we have

   \[\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}. \quad (2)\]

   Consider a circular integration path of radius \(a < r\) which is concentric with the capacitor plates. The magnitude of the magnetic field on this path is constant, so the line integral is \(\oint \vec{B} \cdot d\vec{s} = B(2\pi a)\). The flux of the electric field through this circle is \(E(\pi a^2)\), so we have

   \[B(2\pi a) = \mu_0 \varepsilon_0 (\pi a^2) \frac{dE}{dt}. \quad (3)\]

   If the capacitor plates have a charge \(\pm Q\), then the magnitude of the electric field between the plates is (neglecting fringing effects) \(E = Q/(\varepsilon_0 \pi r^2)\). We therefore have

   \[B = \frac{\mu_0}{2\pi r^2} \frac{a}{dQ}{dt}. \quad (4)\]

   The largest field is at \(a = r\) (at the edge of the capacitor). The time dependence of the charge is \(Q(t) = \varepsilon C(1 - e^{-t/RC})\), so that \(dQ/dt = (\varepsilon /R)e^{-t/RC}\). The maximum current is at \(t = 0\) with \(dQ/dt = \varepsilon /R\), so that the maximum magnetic field is

   \[B_{\text{max}} = \frac{\mu_0}{2\pi r} \frac{\varepsilon}{R} = 2.4 \times 10^{-7} \text{ T}. \quad (5)\]
3. In the circuit shown below we close the switch and wait long enough so that the charge on the capacitor reaches a steady value. We then open the switch. What is the frequency of oscillation of the charge on the capacitor?

![Circuit diagram]

**Answer:** 398 Hz

**Solution:** After the capacitor has charged and we open the switch, the capacitor will begin discharging, and will execute damped oscillations of frequency

\[
f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R}{2L}}.
\]

(6)

In this case the two resistors are in series, so that \( R = 200 \, \Omega \). Substituting the numbers, we obtain 398 Hz.

4. Consider a series LCR circuit, with \( R = 10 \, \Omega \), \( L = 1.0 \, \text{H} \), and \( C = 1.0 \, \mu\text{F} \). If the amplitude of the driving emf is \( \mathcal{E}_m = 10 \, \text{V} \), what is the amplitude of the potential difference across the inductor at resonance?

**Answer:** 1000 V

**Solution:** At resonance the maximum current in the circuit is \( I_m = \mathcal{E}_m/R = 1 \, \text{A} \). The amplitude of the emf across the inductor is \( \mathcal{E}_L = \omega LI_m \); since we’re at resonance, \( \omega = 1/\sqrt{LC} \), so that \( \mathcal{E}_L = \sqrt{L/C} I_m = 1000 \, \text{V} \).

5. A transformer has 600 primary turns and 3600 secondary turns. The output from the secondary is connected across a 1 kΩ resistor. If the primary is connected to a household electrical outlet with an rms voltage of 120 V, what is the maximum current (not the rms current) through the resistor?

**Answer:** 1.0 A

**Solution:** This is a 6:1 step-up transformer, so that the rms voltage across the secondary is 720 V, and the peak voltage is \( 720 \, \text{V} \times \sqrt{2} = 1020 \, \text{V} \). The peak current is therefore \( 1020 \, \text{V}/(1000 \, \Omega) = 1.0 \, \text{A} \).
6. A vertical dipole antenna (parallel to the $z$-axis) is emitting electromagnetic waves. A receiving coil which consists of a planar loop of wire (to detect the magnetic field) is placed to the right of the antenna, as shown in the figure. How should we orient the plane of the loop antenna in order to optimize the reception?

**Answer:** The plane of the loop should be parallel to the $y-z$ plane.

**Solution:** A changing magnetic flux through the loop antenna will induce an emf in the loop. The magnetic field produced by the dipole antenna will be perpendicular to the plane of the paper, so that to produce maximum magnetic flux we’ll want to orient the loop to be parallel to the plane of the paper; i.e., parallel to the $y-z$ plane.

7. The electric field of a plane-polarized electromagnetic wave is given by $E_z = E_0 \cos(kx + \omega t)$. What is the corresponding magnetic field? (The right-handed $xyz$ coordinate system is shown below.)

**Answer:** $B_y = (E_0/c) \cos(kx + \omega t)$

**Solution:** This wave is propagating in the $-x$ direction. Since an EM wave propagates in the direction of $\vec{E} \times \vec{B}$, and $\vec{E}$ is in the $z$-direction, $\vec{B}$ must be in the $y$-direction, and in-phase with the electric field. The magnitude of the magnetic field is magnitude of the electric field divided by $c$, so $B_y = (E_0/c) \cos(kx + \omega t)$.

8. What is the amplitude of the electric field oscillations at a distance of 1 m from a 60 W source of electromagnetic radiation? Assume that the radiation is a plane wave.

**Answer:** 60 V/m

**Solution:** The intensity $I$ is the power per unit area: $I = P/A$. We also know that the intensity is proportional to the square of the electric field, so that $I = E_0^2/(2\mu_0c)$. Solving for the electric field, we have

$$E_0 = \sqrt{\frac{2\mu_0cP}{A}} = 60 \text{ V/m.} \quad (7)$$
9. A point source of light is 80 cm below the surface of a body of water (of index of refraction \( n = 1.33 \)). Find the diameter of the circle at the surface through which light emerges from the water.

**Answer:** 180 cm

**Solution:** The geometry is illustrated in the figure below. From Snell’s Law, \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \); since we have total internal reflection, \( \theta_2 = 90^\circ \), and therefore \( \sin \theta_c = n_2 / n_1 = 0.75 \), so that \( \theta_c = 48.8^\circ \). The radius \( R \) of the circle is given by \( R = h \tan \theta_c = 91 \text{ cm} \), so that the diameter of the circle is \( d = 182 \text{ cm} \). 

10. A beam of unpolarized light is sent through a system of two polarizing sheets. The polarization axes of the two sheets make an angle of \( \theta \) with each other. If the intensity of the light transmitted through the second sheet is 10\% of the incident intensity, what is the angle \( \theta \)?

**Answer:** 63.4°

**Solution:** Since the incident light is completely unpolarized, the transmitted intensity through the first polarizer is \( I_1 = I_0 / 2 \). The light which is incident upon the second polarizer is now linearly polarized, so that the transmitted intensity through the second polarizer is \( I_2 = I_1 \cos^2 \theta = (I_0 / 2) \cos^2 \theta \). Since \( I_2 = 0.1I_0 \), we have \( 0.1I_0 = 0.5I_0 \cos^2 \theta \), so that \( \theta = 63.4^\circ \).