1. A thin straight glass rod of length \( L \) is mounted on the \( x \)-axis with one end at the origin. See sketch. The rod is charged NON-uniformly with a linear charge density \( \lambda = ax \). Here \( a \) is a constant with dimensions charge/length. Calculate the electric potential at the point \( P \) a distance \( D \) away, setting potential at infinity to be zero.

\[
\begin{align*}
(1) & \quad ka (L - D \ln (1 + L/D)) \\
(2) & \quad ka (L + D \ln (1 + L/D)) \\
(3) & \quad -ka (L - D \ln (1 + L/D)) \\
(4) & \quad ka (D - L \ln (1 + L/D)) \\
(5) & \quad -ka (D - L \ln (1 + L/D))
\end{align*}
\]

**SOLUTION:**

Line element \( dx \) has charge \( dq = \lambda dx = axdx \) and is a distance \( x + D \) from point \( P \). Therefore

\[
V = k \int_{0}^{L} \frac{dq}{x + D} = k \int_{0}^{L} \frac{ax}{x + D} dx
\]

Change variables using \( u = x + D \) and \( du = dx \)

\[
V = ka \int_{D}^{D+L} \frac{u-D}{u} du = ka \int_{D}^{D+L} \left( 1 - \frac{D}{u} \right) du
\]

\[
= ka(u - D \ln u)|_{D}^{D+L}
\]

\[
= ka(L - D \ln(1 + L/D))
\]

**NOTE:** One could eliminate the wrong answers by applying common sense only:

- a) At \( L = 0 \), \( V \) must be zero (no charge left). This eliminates (4) and (5).
- b) \( V \) must be positive. This eliminates (3): e.g., try \( L = D \)
- c) At \( D = \infty \), \( V = 0 \). This eliminates (2): \( ka \left( L + D \ln (1 + \frac{L}{D}) \right) \approx ka \left( L + D \frac{1}{D} \right) \approx 2kaL \neq 0 \).

2. A parallel plate capacitor (plate area \( A \), plate separation \( d \)) of capacitance \( C \) is charged to a potential difference \( V \), creating an electrostatic field \( E \) the between plates. The stored energy is \( U \). The charging battery is disconnected and a slab of dielectric (dielectric constant \( k \)) is now inserted between the plates without touching them. The electrostatic field between the plates and the stored energy become, respectively:

\[
(1) \ E/k; \ U/k \quad (2) \ kE; \ kU \quad (3) \ kE; \ U/k \quad (4) \ E/k; \ U/2k \quad (5) \ E/k^2; \ U/k^2
\]

**SOLUTION:**

- a) The electrostatic field in material of dielectric constant \( k \) is \( k \) times smaller than it would be in vacuum. Therefore \( E_k = \frac{E}{k} \)
- b) With dielectric inserted \( C_k = kC, \ U_k = \frac{1}{2} \frac{E^2}{k} = \frac{1}{2} \frac{E^2}{k} = \frac{U}{k} \)
3. An automobile battery jumper cable is composed of 9 identical strands of copper wire, twisted together. A length \( L \) of this cable having resistance \( R \) has the wires untwisted, laid end-to-end, and welded together making a single-strand conductor of length \( 9L \). Neglecting the effects of welding, what is the resistance of the ‘new’ wire?

(1) \( 81R \)  
(2) \( 9R \)  
(3) \( R \)  
(4) \( 18R \)  
(5) \( R/9 \)

**SOLUTION:**
A parallel combination of 9 strands with resistance \( R \) implies that each strand has resistance \( r = \frac{9R}{9} \). Therefore, 9 strands laid end-to-end should have resistance of \( 9r = 81R \).

4. How many time constants \( \tau \) must elapse for an initially uncharged capacitor in an RC series circuit to be charged to 99% of its equilibrium charge?

(1) \( 4.6\tau \)  
(2) \( 0.99\tau \)  
(3) \( 0.03\tau \)  
(4) \( 1.7\tau \)  
(5) \( 2.3\tau \)

**SOLUTION:**
Charge \( q \) across the capacitor is \( q = q_0(1 - e^{-t/\tau}) \) Solve \( 0.99 = 1 - e^{-t/\tau} \) to find \( t/\tau = 4.6 \), or \( t = 4.6\tau \).

5. Find the current in the 5.0Ω resistor in the circuit shown.

(1) 1.5A  
(2) 0.42A  
(3) 3.0A  
(4) 2.4A  
(5) 0.67A

**SOLUTION:**
Resistance of upper branch is the parallel combination of 6.0Ω and 12Ω in series with 4Ω. Calculate \( R_{\text{upper}} = \frac{6 \times 12}{6+12} + 4 = 8\Omega \). Resistance of lower branch is the series combination 3Ω + 5Ω = 8Ω. The two branches in parallel have a resistance of 4Ω. The current through the combined resistors is \( \frac{12V}{4\Omega} = 3 \text{ A} \). This current splits evenly between the two branches (they have equal resistance); hence 1.5 A per branch or 1.5 A through the 5.0Ω resistor.
6. Four charges are fixed at the corners of a rectangle, as shown. Assume that \( a = 4.0 \text{ m} \), \( b = 3.0 \text{ m} \), and that \( q_1 = +q \), \( q_2 = -2q \), \( q_3 = 3q \), \( q_4 = -4q \), in which \( q = 1.0 \mu \text{C} \). Find the electric potential energy of this system of charges.

\[
\begin{align*}
q_1 & \quad q_2 \\
q_3 & \quad q_4
\end{align*}
\]

\[\text{(1) } -42 \text{ mJ} \quad \text{(2) } -740 \text{ mJ} \quad \text{(3) } -3.0 \text{ mJ} \quad \text{(4) } +740 \text{ mJ} \quad \text{(5) } +42 \text{ mJ}\]

**SOLUTION:**

Four charges make six different pairs. The total energy is:

\[
U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}
\]

\[
= \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}
\]

\[
= kq^2 \left[ -\frac{2}{4} + \frac{3}{5} - \frac{4}{3} - \frac{6}{5} + \frac{8}{3} - \frac{12}{4} \right]
\]

\[
= 4.6kq^2 = -42 \text{ mJ}
\]

7. Two previously uncharged capacitors are connected in series and then charged with a 12-V source. One capacitor is 30\( \mu \text{F} \) and the other is unknown. If the voltage across the 30\( \mu \text{F} \) capacitor is 8 V, find the capacitance of the unknown capacitor.

\[
\begin{align*}
\text{C} & \quad \text{12 V} \\
30 \mu \text{F} & \quad \text{8 V}
\end{align*}
\]

\[
\text{(1) } 60 \mu \text{F} \quad \text{(2) } 120 \mu \text{F} \quad \text{(3) } 240 \mu \text{F} \quad \text{(4) } 4 \mu \text{F} \quad \text{(5) } 20 \mu \text{F}
\]

**SOLUTION:**

Charge on 30\( \mu \text{F} \) capacitor \( q = CV = (30 \mu \text{F}) (8 \text{V}) = 240 \mu \text{C} \). Voltage across unknown capacitor is 12 V – 8 V = 4 V. Same charge on both capacitors, therefore

\[
C = \frac{q}{V} = \frac{240 \mu \text{C}}{4 \text{V}} = 60 \mu \text{F}
\]
8. Two small spheres of mass \( m = 1 \text{ kg} \) are charged with \( q = 1 \text{C} \) each and placed at a distance of 1 m from each other. Since they repel each other, they start flying apart. Find the velocity of each of the spheres when they are separated by a distance of 2 m.

(1) 67 km/s  (2) 47 km/s  (3) 134 km/s  (4) 190 km/s  (5) 268 km/s

**SOLUTION:**

Initial energy = \( E_i = KE_i + PE_i = 0 + k\frac{q^2}{r_i} \)

Final energy = \( E_f = 2\left(\frac{1}{2}mv^2\right) + k\frac{q^2}{r_f} \)

Set \( E_i = E_f \) to get

\[ v = \sqrt{\frac{k}{m} \left( \frac{1}{r_i} - \frac{1}{r_f} \right)} = 67 \text{ km/s} \]

9. A cylindrical resistor of radius 5.0 mm and length 2.0 cm is made of material that has a resistivity of \( 3.5 \times 10^{-5} \Omega \cdot \text{m} \). What is the current density when the energy dissipation rate is 1.0 W?

(1) 1.35 E5 Am\(^{-2}\)  (2) 2.65 E3 A  (3) 6.58 E10 Am\(^{-1}\)  (4) 2.41 E6 Am\(^{-2}\)  (5) 1.19 E2 Am\(^{-2}\)

**SOLUTION:**

Power \( P = \dot{v}^2 R = (JA)^2 \frac{\rho L}{A} = \rho LJ^2 A \). Solve for \( J = \left(\frac{P}{\rho LA}\right) = 1.35 \times 10^5 \frac{A}{m^2} \).

Using \( L = 2.0 \text{cm} = 0.02m, \rho = 3.5 \times 10^{-5} \Omega \cdot \text{m}, \ P = 1 \text{ Watt} \) and \( A = \pi r^2 \) with \( r = 5.0 \text{mm} = 5 \times 10^{-3} \text{m} \).

10. What is the current through \( R_2 \) in the figure?

[Diagram of the circuit with labeled resistances and currents]

\( I_1 = 100 \ \Omega \)
\( R_2 = R_3 = 50 \ \Omega \)
\( R_4 = 75 \ \Omega \)
\( E = 6.0 \ \text{V} \)

(1) 0.02 A  (2) 1 A  (3) 0.05 A  (4) 0.015 A  (5) 0.25 A

**SOLUTION:**

\( R_2, R_3, R_4 \) are parallel. Therefore \( R_{eq} = R_1 + \frac{R_2 R_3 R_4}{R_2 R_3 + R_2 R_4 + R_3 R_4} = 100\Omega + 18.75\Omega = 118.75\Omega \). Voltage across \( R_3 \) is

\( I_1 R_1 = 5.05 \text{ V} \) and voltage across the parallel combination is therefore \( 6 \text{ V} - 5.05 \text{ V} = 0.95 \text{ V} \). Current \( I_2 = \frac{0.95 \text{ V}}{R_2} = 0.019 \text{ A} \) is

\( A = 0.02 \text{ A} \).