1. Suppose the rod in the figure is moving at a speed of 5.0 m/s in a direction perpendicular to a 0.80-T magnetic field. The magnetic field is directed into the paper. The rod has a length of 1.6 m and has negligible electrical resistance. The rails also have negligible resistance, and the light bulb has a resistance of 96Ω. Find the induced current in the circuit.

\[ \text{(1) 0.067 A (2) 26 A (3) 614 A (4) 0.43 A (5) 6.4 A} \]

\text{SOLUTION:}

The area enclosed by the loop is \( L \cdot x \), where \( L \) is the length of the rod, and \( x \) is the distance from the left side. Combine \( E = \frac{d\Phi}{dt} \) and \( \Phi = B(\text{Area}) = BLx \) to get:

\[ E = -\frac{d}{dt}(BLx) = -BL \frac{dx}{dt} = -BLv \]

Therefore, \( E = iR = BLv \), and we get \( i = \frac{BLv}{R} = 0.067A \)

2. A source injects an electron of speed \( v = 2 \times 10^2 \text{ m/s} \) into a region with a uniform magnetic field of magnitude \( B = 1 \times 10^{-3} \text{ T} \). The velocity of the electron makes an angle \( \theta = 20^\circ \) with the direction of \( \vec{B} \). Find the distance, \( d \), from the point of injection at which the electron next crosses the field line that passes through the injection point.

\[ \text{(1) 0.67 m (2) 2.34 m (3) 1.29 m (4) 0.11 m (5) 1.71 m} \]

\text{SOLUTION:}

There is a typo here—the electron’s speed should be \( 2 \times 10^7 \text{ m/s} \). The electron follows a helical path around the field line, making one complete revolution in \( T = \frac{2\pi m}{qB} \) (Eq. 29–17). During this time, the electron also has a velocity component parallel to \( \vec{B} \), given by \( v \cos \theta \). So the distance travelled along the field line in one revolution is:

\[ d = (v \cos \theta)T = \frac{2\pi m}{qB}v \cos \theta = 0.67m \]

3. An electric field of 1.5 kV/m and a magnetic field of 0.4 T act on a moving electron to produce no net force. Calculate the \textbf{minimum} speed of the electron.

\[ \text{(1) 3.75 \times 10^3 \text{ m/s (2) 0 (3) 4.1 \times 10^6 \text{ m/s (4) 1.34 \times 10^2 \text{ m/s (5) 5.5 \times 10^4 \text{ m/s}}} \]

\text{SOLUTION:}

We know the force \( \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \). Here, \( F = 0 \), so \( \vec{E} = -(\vec{v} \times \vec{B}) \) and \( |E| = |vB\sin\theta| \). Therefore, \( |v| = \left| \frac{E}{B\sin\theta} \right| \). For minimum speed, we need \( \theta = 90^\circ \), and so \( |v| = \left| \frac{E}{B} \right| = 3.75 \times 10^3 \text{ m/s} \).
4. A steady current of 2.0 A flowing through a 50-turn solenoidal inductor produces a magnetic flux of 15 microWb through each turn. If the current in the inductor changes at a rate of 25 A/s, what will be the induced emf in this device?

(1) 9.4 milliV  (2) 188 microV  (3) 375 microV  (4) 15 V  (5) none of these

**SOLUTION:**
Self-induced emf \( E = L \frac{di}{dt} = -\frac{N \Phi_B di}{i} \)  
Plug in values provided to get 
\[ 9.4 \times 10^{-3} \text{V} \]

5. A 10 H inductor is connected in series with a resistor and a 20 volt battery. After the current reaches its maximum value, the energy stored in the magnetic field of the inductor is 80 J. The resistance of the circuit in ohms is:

(1) 5.0  
(2) 10.0  
(3) 2.83  
(4) 2.25  
(5) none of these

**SOLUTION:**
Stored magnetic energy \( U = \frac{1}{2} Li^2 = 80 \text{ J} \). From this we get the current \( i = 4 \text{ A} \). This maximum current is reached by letting \( t \to \infty \) in equation 31–43, thus \( i_{max} = \frac{E}{R} \). Therefore \( R = \frac{E}{i} = 5.0 \text{ ohms} \).

6. A small bar magnet falls through a cylindrical metal tube as shown in the figures. Which of the five figures correctly represents the induced currents in

**SOLUTION:**
Lenz’s law. The falling magnet increases the magnetic flux downward through the bottom face and so the opposing \( B \)-field will point up, indicating a counterclockwise current (as viewed from above). The flux downward through the top face is decreasing, thus the induced current must produce a downward field. So the top current is clockwise (as viewed from above).
7. A long straight cylindrical conductor of radius \( a \) carries a uniformly distributed current \( i \). Calculate the magnitude of the magnetic field inside the cylinder for any radial distance \( r \) from the center of the cylinder.

\[
\text{(1) } \frac{\mu_0 i}{2\pi a^2} \quad \text{(2) } \frac{\mu_0 i}{2\pi r} \quad \text{(3) } \frac{\mu_0 i}{2\pi r^2} \quad \text{(4) } \frac{\mu_0 i}{2\pi r} \ln(a/r) \quad \text{(5) } \frac{\mu_0 i}{2\pi r} \ln(r/a)
\]

**SOLUTION:**
Apply Ampere’s Law: \( \oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{enc}} \). Since the current is uniform, the current density is \( J = \frac{i}{\pi a^2} \). Draw circular Amperian loop at \( r (r < a) \). The current enclosed by this loop will be \( i_{\text{enc}} = J\pi r^2 = \frac{r^2}{a^2} \). The integral on the LHS is \( \oint \vec{B} \cdot d\vec{l} = B2\pi r \). Setting this equal value to \( \mu_0 i_{\text{enc}} \):

\[ B = \frac{\mu_0 i r}{2\pi a^2} \]

8. Two long, straight, parallel wires are perpendicular to the page and separated by 1.75 cm. The upper wire (see sketch) carries a current of 13 A out of the page. What must be the current (magnitude in Amperes and direction) in the lower wire if the magnetic field at point P (2.25 cm below the lower wire) is zero?

\[
\begin{align*}
\text{(1) } & 7.3, \text{ in} \\
\text{(2) } & 7.3, \text{ out} \\
\text{(3) } & 5.7, \text{ in} \\
\text{(4) } & 5.7, \text{ out} \\
\text{(5) } & 10.1, \text{ in}
\end{align*}
\]

**SOLUTION:**
At the point P, the top wire (wire 1) produces a field to the right with magnitude \( B_1 = \frac{\mu_0 i_1}{2\pi (d_1 + d_2)} \). The bottom wire (wire 2) produces a field \( B_2 = \frac{\mu_0 i_2}{2\pi d_2} \) at point P, their magnitudes must be equal and their directions opposite. Therefore: \( |B_1| = |B_2| \) leads to \( i_2 = i_1 \left( \frac{d_2}{d_1 + d_2} \right) = 7.3 \text{ A, into the page.} \)

9. A wire of length \( L \) carries a current \( i \). If the wire is bent into a circular coil of \( N \) turns and placed into a uniform magnetic field \( B \), what is the magnitude of the maximum possible torque on the loop?

\[
\begin{align*}
\text{(1) } & \frac{L^2 i B}{4\pi N} \\
\text{(2) } & \frac{Li B}{2N} \\
\text{(3) } & \frac{2L^2 i B}{\pi^2 N^2} \\
\text{(4) } & \frac{3Li B}{\pi N} \\
\text{(5) } & \frac{L^2 \pi i B}{4N^2}
\end{align*}
\]

**SOLUTION:**
If the length \( L \) is coiled into \( N \) turns, the circumference of each turn will be \( 2\pi r = \frac{L}{N} \). Torque is \( \vec{\tau} = \vec{\mu} \times \vec{B} = NiAB \sin \theta \).

You should immediately set \( \theta = 90^\circ \) for maximum torque. \( A = \text{area} = \pi r^2 \) where \( r = \frac{L}{2\pi N} \) as found above. Therefore:

\[ \tau_{\text{max}} = Ni\pi r^2 B = \frac{L^2 i B}{4\pi N} \]
10. At a certain position outside Gainesville, the magnetic field of the Earth is 39\mu T, horizontal to the surface, and directed due North. If the magnitude of the total field is exactly zero 8 cm above a long straight, horizontal wire that carries a constant current \( i \), what is the magnitude and direction of the current?

(1) 16 A, west to east  (2) 8 A, north to south  (3) 16 A, east to west  (4) 4.2 A, west to east  (5) 8 A, south to north

**SOLUTION:**
A long, straight wire produces a field of magnitude

\[ B = \frac{\mu_0 i}{2\pi r}. \]

At \( r = 0.08 \) m above the wire, the magnitude of this field equals that of the Earth, so setting \( B = 39\mu T \), we get \( i = 16\text{A} \). Since Earth’s field points North, that of the wire must point South. A current running West to East will produce field lines pointing South at points directly above the wire.