Exam 1 Solutions

1. A particle with charge 3µC is placed at the origin, an identical particle, with the same charge, is placed 2m from the origin on the x axis, and a third identical particle, with the same charge, is placed 2m from the origin on the y axis. The magnitude of the force on the particle at the origin is:

   a. $2.9 \times 10^{-2}$ N   b. $1.4 \times 10^{-2}$ N   c. $2.0 \times 10^{-2}$ N   d. $4.0 \times 10^{-2}$ N   e. $8.1 \times 10^{-2}$ N

   As all charges are the same, the force is repulsive between the first and second particle (directed in $\hat{x}$ direction), and first and third particle (directed in $-\hat{y}$ direction). The magnitude of the force between each pair of particles is the same since the distances are the same:

   $$ F_{12} = k \frac{q_1 q_2}{d^2} = k \frac{q^2}{d^2} = \left(9 \times 10^9 \right) \left(3 \times 10^{-6} \text{ C} \right)^2 / (2 \text{ m})^2 = 0.02 \text{ N} $$

   But when the forces from each pair of particles are added vectorially, the overall magnitude is $\sqrt{2}F_{12} = 0.029 \text{ N}$

2. An electron and a proton are separated by a distance of $d=5.3 \times 10^{-11} \text{ m}$. What is the magnitude of the acceleration of the electron immediately after both particles are released from rest?

   a. $9.0 \times 10^{22} \text{ m/s}^2$  b. $8.2 \times 10^{-8} \text{ m/s}^2$  c. $4.8 \times 10^{12} \text{ m/s}^2$  d. $5.6 \times 10^{41} \text{ m/s}^2$  e. $1.0 \times 10^{13} \text{ m/s}^2$

   The proton has an equal and opposite electric charge to the electron. So the magnitude of the force between them is:

   $$ F = \left(9 \times 10^9 \right) \left(\frac{1.6 \times 10^{-19} \text{ C}}{5.3 \times 10^{-11} \text{ m}} \right)^2 = 8.2 \times 10^{-8} \text{ N} $$

   $$ a = \frac{F}{m_e} = \left(\frac{8.2 \times 10^{-8} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} \right) = 9 \times 10^{22} \text{ m/s}^2 $$
3. Arrangement of point charges on a square lattice, length of each side=d. ‘+’ marks the four corners of the square. The point marked by 'O' is in the center of the square. The positive charge at each corner is shown in parentheses after the '+' marking the corners; ‘n’ is some integer.

What is the component of the electric field in the horizontal, x direction, at point 'O' in the center of the square?

a. 0  b. \(\frac{2q}{(4\pi\varepsilon_0 d^2/2)}\)  c. \((\sqrt{2}/2)*\frac{2q}{(4\pi\varepsilon_0 d^2/2)}\)  d. \(\sqrt{2}\frac{2q}{(4\pi\varepsilon_0 d^2/2)}\)

e. \(-(\sqrt{2}/2)*\frac{2q}{(4\pi\varepsilon_0 d^2/2)}\)

One possible method to sum the electric field from all four corners is to take the charges in pairs so as to benefit from symmetry. Take the upper left and lower right corners together, and this gives a net electric field at 'O' pointed along the diagonal directly towards the upper left corner (i.e. at 45 degrees to the left of the vertical) of size:

\(2q_0/(4\pi\varepsilon_0 [d/\sqrt{2}]^2)\) where \(q_0\) is the elementary charge \(1.6\times10^{-19}\) C and the distance \(d/\sqrt{2}\) is half the length of the diagonal of the square as discussed in class. Taking the other pair of charges (upper right and lower left), this pair gives an E field at 'O' of the same size, but directed at 45 degrees to the right of the vertical. Thus, the sum of the x-components of the electric fields from all four corners at 'O' is zero.

4. What is the magnitude, in units of N/C, of the electric field from a dipole aligned along the z-axis at a point \(10^{-9}\) m from the center of the dipole and on the z-axis, with the positive charge in the dipole (=\(1.6\times10^{-19}\) C) positioned one Angstrom (=\(10^{-10}\) m) away from the negative charge (=\(-1.6\times10^{-19}\)C)?

a. \(2.88 \times 10^8\)  b. \(1.44 \times 10^8\)  c. \(5.76 \times 10^8\)  d. \(5.76 \times 10^{-20}\)  e. \(5.76 \times 10^8\)

Use the approximation for \(z>>d\), \(E=\left(\frac{1}{2\pi\varepsilon_0}\right)q*d/z^3\) = \(2*9*10^9 (N m^2/C^2) *1.6*10^{-19} C*10^{-10} m/10^{-27} m^3=18*1.6*10^7\) N/C = \(2.88*10^8\) N/C

5. Three concentric conducting shells have radii \(R_1 < R_2 < R_3\) and charges -3Q, +Q and +4Q, respectively, where Q is some positive number. What is the charge on the inner surface of the outermost shell?

a. +2Q  b. -2Q  c. +4Q  d. -Q  e. 0

The electric field must be zero inside an isolated conductor. So a spherical Gaussian surface within each conductive shell must have zero flux since \(E=0\), which implies that by Gauss’ Law the enclosed charge must be zero. Thus, the inside surface of each shell must have an electric charge that exactly balances the charge from the inner conductors. Therefore, the inside surface of the outermost shell must balance a charge of \((-3Q+Q)\), which is thus +2Q.
6. At each point on the surface of the cube shown, the electric field is parallel to the z-axis. The length of each edge of the cube is $a=2.5\text{m}$. On the top face of the cube $E=10\hat{z}\text{ N/C}$, and on the bottom face of the cube $E=-30\hat{z}\text{ N/C}$. Determine the net charge contained within the cube in nC ($1\text{ nC}=10^{-9}\text{ C}$).

a. 2.2  b. 1.1  c. -1.1  d. 0.55  e. -1.7

The electric field points along the z-axis, so the electric flux through a surface, defined by $\Phi = E \cdot A$, is non-zero only for the top and bottom faces. For both faces, the flux is positive (outward), since $E \cdot A > 0$. Thus, the flux adds and indicates a positive amount of charge enclosed. By Gauss’ Law, one can solve for the charge enclosed:

$$\Phi = \sum E_i \cdot A_i = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = \varepsilon_0 \left(10 + 30\right) \left(a^2\right) = \left(8.85 \times 10^{-12} \text{ F/m}\right) \left(40 \text{ N/C}\right) \left(2.5 \text{ m}\right)^3$$

$$q_{enc} = 2.2\text{nC}$$

7. What is the potential $V$ at point P (marked with an 'x' in the figure) at a distance $d$ from the center of a line of uniform charge of length $L$? The linear density of charge in the line is $\lambda$.

a. $(2\lambda/4\pi\varepsilon_0)\left[\ln(L/2 + \left(L^2/4 + d^2\right)^{1/2}) - \ln d\right]$  
b. $(\lambda/4\pi\varepsilon_0)\left[\ln(L/2 + \left(L^2/4 + d^2\right)^{1/2}) - \ln d\right]$  
c. $(2\lambda/4\pi\varepsilon_0)\left[\ln(L + \left(L^2 + d^2\right)^{1/2}) - \ln d\right]$  
d. $(\lambda/4\pi\varepsilon_0)\left[\ln(L + \left(L^2 + d^2\right)^{1/2}) - \ln d\right]$  
e. 0

Do the integral of $dV=(1/4\pi\varepsilon_0)dq/r$ where $r$ is the distance between an element of charge $dq(=\lambda dx)$ and point $P$, so $r=(x^2+d^2)^{1/2}$, between $-L/2$ and $+L/2$.

$$V = \int_{-L/2}^{L/2} \left(1/4\pi\varepsilon_0\right) \lambda dx/\left(x^2+d^2\right)^{1/2} \quad \text{this integral is symmetrical around 0, so}$$

$$V = 2\int_0^{L/2} \left(1/4\pi\varepsilon_0\right) \lambda dx/\left(x^2+d^2\right)^{1/2} = (2\lambda/4\pi\varepsilon_0)\left[\ln(L/2 + \left(L^2/4 + d^2\right)^{1/2}) - \ln d\right]$$

This integral is the same one discussed in class and worked in the text, p. 639, with the change that point $P$ is in the center, not over one end, and the limits on the integral are $-L/2$ to $+L/2$ instead of $0$ to $L$. 


8. In the figure, initially the switch is open and charge \( q_0 \) is stored on capacitor \( C_1 \) via a 12 V battery (not shown). The battery is then removed. When the switch is closed, some of charge \( q_0 \) flows onto capacitor \( C_2 \). If \( C_1= X \ \mu F \) and \( C_2= Y \ \mu F \), what voltage is equally on \( C_1 \) and \( C_2 \) after charge flow has stopped after the switch is closed?

a. 4 V  
b. 6 V  
c. 8 V  
d. 12 V  
e. 2 V

\( q_0 = \) the sum of the charge after the switch is closed on \( C_1 \) and \( C_2=q_1+q_2 \)

\( q_1=C_1V \) and \( q_2=C_2V \) since, after the switch is closed, the voltage on both capacitors (as stated in the problem) is the same.

\[ q_0 = q_1 + q_2 \]
\[ C_1V (\text{time}=0 \text{ before switch is closed}) = C_1V(\text{after switch is closed}) + C_2V(\text{after switch is closed}) \]
\[ C_1*12 V = C_1V_{after} + C_2V_{after} \]
so \( V_{after}=12 V \ C_1/(C_1+C_2) = 12V*X \mu F / (X \mu F + Y \mu F) \)
If, for example, \( X=5 \) and \( Y=10 \), then \( V_{after}=4 V \)

9. An air filled parallel plate capacitor has a capacitance of \( X \ \mu F \). If the separation between the plates is decreased by a factor of \( N \) and a material with a dielectric constant of 12 (like Silicon) is inserted between the parallel plates, what is the new capacitance, in units of \( \mu F \)?

a. 96  
b. 48  
c. 4  
d. 24  
e. 0.67

\( C=\varepsilon_0 A/d \) for a parallel plate capacitor with air in between the plates, with \( A=\text{area} \) and \( d=\text{separation of the plates} \). With a dielectric with dielectric constant \( \kappa \), \( C=\varepsilon_0 \kappa A/d \).

Thus, the new \( C \) is \( N \kappa \) larger, or \( XN \kappa \ \mu F \). If, for example, \( X=4 \) and \( N=2 \), then the new \( C \) would be 96 \( \mu F \).

10. A certain amount of copper is formed into a cylindrical rod of length \( L \) and has a resistance measured between the two ends of \( R \). If the rod is compressed to half of its original length while maintaining its original volume, what is the new resistance of the rod?

a. \( R/4 \)  
b. \( R/2 \)  
c. 4\( R \)  
d. 2\( R \)  
e. \( R \)

Resistance is given by the expression:

\[ R = \rho \frac{L}{A} = \rho \frac{L^2}{V} \]

where \( V = LA = \) volume and \( \rho = \) resistivity of copper
The key point is that the volume remains the same, so the cross sectional area must change.

Thus

\[
\frac{R_2}{R_1} = \frac{\rho \left( \frac{L_2^2}{V} \right)}{\rho \left( \frac{L_1^2}{V} \right)} = \frac{L_2^2}{L_1^2} = \frac{1}{4}
\]

So \(R_2 = R/4\)