1. In the multiloop circuit shown, what is the current in the left hand loop (abcd), in units of Amperes?
   a. $\frac{3}{55}$ b. $\frac{3}{11}$ c. $\frac{12}{55}$ d. 0 e. $\frac{9}{55}$

**Solution:** Use Kirchhoff's rules (sum of currents at a point = 0 and sum of EMF's around a loop = 0) to obtain: (direction of currents chosen - remember these can be $180^\circ$ wrong, which will give a minus sign in the solution - are shown in red)

- eq. #1: $i_1 + i_2 = i_3$ (sum of currents at point b)
- eq. #2: $-5i_1 - 10i_3 + 3V = 0$ (sum of EMF's around left hand loop starting at point a and going clockwise)
- eq. #3: $+5i_2 - 6V + 10i_2 + 10i_3 = 0$ (sum of EMF's around right hand loop starting at point b and going clockwise)

Solve these 3 eqns for $i_1$:

- $\#3 \Rightarrow 15i_2 + 10i_3 = 6$
- $\#2 \Rightarrow 10i_3 = -5i_1 + 3$ put this in eq. above
- $15i_2 + (-5i_1 + 3) = 6$ also, put eq. above as $i_3 = (1/10)(-5i_1 + 3)$ in eq. #1 to get
- $i_1 + i_2 = (1/10)(-5i_1 + 3)$ use $* \ to express \ i_2 = (6 - 3 + 5i_1)/15 \ and \ put \ in \ ** \ to \ get$

$i_1 + 1/5 + (1/3)i_1 = - (1/2)i_1 + 3/10$ solve for $i_1$, get $i_1 = 3/55$ Amp

However, the wording of the problem was found to be ambiguous, as the current through the middle branch (10 ohm resistor) is not the same as the left current (through the 3V battery). Both were marked correct, so if one chose $i_3 = 3/11$ that is also correct.
2. A capacitor starts fully charged with charge $q_0$. It is then, through the closing of a switch, discharged through a series resistor, see circuit in Figure. If $C=10 \mu F$ and $R=100 \Omega$, how long in units of milliseconds (1 millisecond = $10^{-3}$ s) does it take the charge on the capacitor to reach 0.5 $q_0$? $(1 \mu F = 10^{-6} F)$

   a. 0.69  b. 0.63  c. 0.37  d. 0.31  e. 0.5

Solution: $q = q_0 e^{-t/RC}$ if want $q/q_0 = 0.5$, then $\ln(0.5) = -t/RC$, or $-0.6931 = -t/(100*10^{-5})$ or $t = 0.69 \times 10^{-3}$ s, so answer (in units of milliseconds) is 0.69.

3. An electron is accelerated from rest by a potential difference of 20,000V. It then enters a uniform magnetic field of magnitude 0.02 T with its velocity perpendicular to the direction of the field. Calculate the radius in meters of its circular path in the magnetic field.

   (a) 0.024  (b) $5.7 \times 10^{-4}$  (c) 40.0  (d) 0.017  (e) 0.0034

Solution: The radius of curvature is given by $r = p/qB$, where we can find the momentum $p$ by equating the potential energy of the electron before acceleration to kinetic energy afterwards:

\[
\Delta U = e\Delta V = E_k = \frac{1}{2} m_e v^2
\]
\[
\Rightarrow p = m_e v = \sqrt{2m_e e \Delta V}
\]
\[
\Rightarrow r = \frac{p}{eB} = \frac{2m_e \Delta V}{eB^2} = 0.024 \text{ m}
\]

4. The circular coil shown in the figure carries a current $i = 2.0 A$ in the direction indicated, is parallel to the $x$-$z$ plane, has 3 turns, and has a radius of $r = 2 \text{ cm}$. A uniform magnetic field is present pointing in the $x$-direction: $B = 0.025 \text{ T}$. What is the magnitude of the magnetic torque on the coil (in N-m)?

   (a) $6\pi \times 10^{-5}$  (b) $2\pi \times 10^{-5}$  (c) $\pi \times 10^{-5}$  (d) $3\pi \times 10^{-5}$  (e) 0
Solution: the torque on a current loop is \( \tau = \mu \times B \) where the magnetic dipole moment is \( \mu = N i A \) where \( N \) is the number of turns and \( A \) is the area vector of the loop. The direction of \( \mu \) is given by the current and points in the \(-\hat{y}\) direction. Thus the torque is about the \( z \)-axis. The magnitude is \( \tau = N i r^2 B = 6\pi \times 10^{-5} \text{ N-m} \)

5. A semi-infinite wire of 1 mm diameter (wire X in diagram) carries a current \( I \) to the right. At the end of wire A, the current flows into a perpendicular wire (wire Y in diagram) which has a length of 1 cm. The current leaves this short wire, flows into another semi-infinite wire of 1 mm diameter (wire Z in diagram) that carries the current \( I \) back to the left.

What current \( I \) (in units of Amperes accurate to 3 sig figs) has to flow in the wires to create a force of 1 N on wire Y? (Hint 1: do the integral \( F = \int dF \) for the force on the current in wire Y from the (spatially dependent) field due to the current in the semi-infinite wire X. Then double this force due to the same force on the current in wire Y from the field due to the current in semi-infinite wire Z.) (Hint 2: for the limits on the integral, use 0.001 m for the lower limit – i.e. treat as a negligible correction the field along wire Y generated from the current inside wires X and Z – and use 0.01 m (the length of wire Y) as the upper limit.)

Solution: \( F = \int \int \int I (dL \text{ along wire Y}) \times B \text{(function of distance from wire X)} \) for the force on the perpendicular wire (wire Y). Need to do integral from 1 mm to the end of the wire, 10 mm. Inside the semi-infinite wire, the integral would have involved \( B \propto r \) (but we’re told to ignore this contribution), while outside the semi-infinite wire \( B = \mu_0 i/(4\pi r) \). Once we get this integral, then double it because wire Z makes the same contribution in force.

\[
F = \int_{0.001}^{0.01} I dx \mu_0 i/(4\pi r)
\]

where \( dL \) along wire Y is called \( dx \) and the distance from the semi-infinite wire, \( r \), is called \( x \) to get the variables consistent. We have two factors of the current, \( I \), and we want \( 2 \times F \) (the contribution from both semi-infinite wires) to be 1 N.
1 \ N = 2 \ * \ I^2 \ * \ \mu_0/(4 \ \pi) \ \ln x \ \text{evaluated between 0.001 and 0.01} \\
1 \ N = 2 \ * \ I^2 \ * \ \mu_0/(4 \ \pi) \ \{\ln(0.01) - \ln(0.001)\} = 2 \ 10^{-7} \ \text{(Tm/A)} \ I^2 \ 2.303 \ (-2 \ -{-3}) \\
so \ I^2 = 1 \ N/(4.606 \ 10^{-7} \ \text{Tm/A}) \\
remember (p. 738, chap. 28, that 1 \ \text{Tesla} = 1 \ \text{N/(coulomb} \ * \ \text{m/s)} and a \ \text{coulomb/s is an Ampere, so} \ I^2 = N/[46.06 \ 10^{-8} \ \text{(N/Am)(m/A)}] \ \text{so} \ \text{the units are correct.} \\
I=1470 \ \text{A (a lot!)}

6. Using Ampere’s law, what is the magnetic field (in units of T) in the tangential direction shown in the diagram 2 mm from the center of a long wire (diameter = 1 mm) carrying 1000 A?

\[ B = \text{?} \]

\[
\begin{array}{c}
\text{wire} \\
\leftarrow 2 \ \text{mm} \rightarrow \\
\text{O} \\
\end{array}
\]

Solution: \[ \oint B \ ds = \mu_0 i_{\text{enc}} = 4 \pi \ 10^{-7} \ 10^3 \ \text{(Tm/A)(A)} \ \text{(units are right!)} \]
left side is just \( B \ 2\pi \ r \) where \( r=0.002 \ \text{m} \) so \( B=(4\pi/2\pi)10^{-4} \ \text{T}/0.002 = 0.1 \ \text{T} \)

7. What is the Lenz’s law force in units of N on the loop shown being pulled at constant \( v=1 \ \text{m/s} \) out of a field of 0.1 T. The width of the loop perpendicular to the motion is 10 cm, the resistance \( R \) of the loop is 1 \ \Omega. (The x’s mark the uniform magnetic field directed into the page.)

| x x x x x x x x x | \uparrow \\
| x x x x x x x | \uparrow \downarrow \\
| x x x x x x x | \rightarrow v=1 \ \text{m/s} \\

Solution: \[ F = B^2 L^2 v/R = 0.1^2 \ 0.1^2 \ 1/1 \ \text{(T^2 m^2 \ {m/s}/\Omega)} \]
\( T=\text{N/(Am)} \ \Omega=\text{V/A} \)
So units are \{N^2/(A^2 m^2)\} m^2 m/s A/V
The m^2 factors cancel, one of the A in the denominator cancels
Have N^2 m/AsV
a VA is a W, a Ws is a J; a J/m is a N, so have N (units are OK!)
\( F=10^{-7} \ \text{N} \)
8. A cylindrical solenoid that is 13m long has a radius of 3m. There are 2070 turns of wire carrying a current of 20,000A. Find the total energy stored in Joules in the magnetic field inside the solenoid (neglect end effects).

(a) $2.3 \times 10^9$  (b) $6.4 \times 10^9$  (c) $4.0 \times 10^{11}$  (d) $1.1 \times 10^9$  (e) $2.0 \times 10^8$

Solution: The energy density of a solenoid is given by: $u = \frac{B^2}{2\mu_0}$. The magnetic field of a solenoid is given by: $B = \mu_0 ni$, where $n$ is the number of turns per unit length.

So multiplying the density by the volume and plugging in that $n = \frac{2070}{13} m$:

$$U = \frac{(\mu_0 ni)^2}{2\mu_0} \pi r^2 L = 2.3 \times 10^9 \text{ J}$$

9. A charged capacitor ($C = 40\mu F$) is connected across an inductor ($L = 0.2H$) to form an LC circuit. When the charge on the capacitor is $6.3 \times 10^{-3} C$ the current is 2A. What will be the maximum current in amps through the circuit?

(a) 3.0  (b) 2.2  (c) 9.0  (d) 350  (e) 4.2

Solution: In an LC circuit without resistance, the total energy in the circuit is conserved. So we can compute the energy in the circuit given the parameters of the problem, and then set that equal when the current is a maximum (and the charge=0):

$$U = \frac{q^2}{2C} + \frac{1}{2} Li^2 = \frac{(6.3 \times 10^{-3})^2}{2 \times 40 \times 10^{-6}} + \frac{1}{2} (0.2)(2)^2 = 0.9 \text{ J}$$

$$U = \frac{1}{2} L i_{\text{max}}^2 \Rightarrow i_{\text{max}} = \sqrt{\frac{2U}{L}} = 3.0 \text{ A}$$

10. An RLC series circuit is driven by a sinusoidal EMF with angular frequency $\omega_d$. If $\omega_d$ is increased without changing the amplitude of the EMF the current amplitude increases. If $L$ is the inductance, $C$ is the capacitance, and $R$ is the resistance, this means that:

(a) $\omega_d L < 1/(\omega_d C)$  (b) $\omega_d L > 1/(\omega_d C)$  (c) $\omega_d L = 1/(\omega_d C)$  (d) $\omega_d L > R$  (e) $\omega_d L < R$

Solution: if the current is increasing with $\omega_d$, then we are on the low frequency side of the resonance peak of the RLC circuit. Thus, $\omega_d L < 1/(\omega_d C)$ since at the peak itself $\omega_d L = 1/(\omega_d C)$. The circuit is thus capacitive.