1. A current $i$ travels in the direction of the arrows shown in the figure in three wire segments. The straight segments (upper left and upper right in the figure) both are lined up with the center point, $P$, which is the center of the circular arc segment wire shown which subtends (takes up) an angle of $120^\circ$. The straight wires are both of length $R$; the radius of the circular arc is also $R$. What is the correct expression for the magnitude of the total field produced by the current in all three wire segments at point $P$?

a. $\mu_0i/(6R)$  
b. $\mu_0i/(3R)$  
c. $\mu_0i/(2R)$  
d. $\mu_0i/(4R)$  
e. $\mu_0i/(12R)$

Solution: The current in the straight wires that would go thru center point $P$ if extended contributes zero to the field (as discussed in lecture for chap. 29). The field at the center of a circular arc of wire (eq. 29-9) is $B=\mu_0i\phi/(4\pi R)$, where $\phi$ is the angle taken up by the circular arc. $120^\circ$ is $1/3$ of $360^\circ$, where $360^\circ$ is $2\pi$ radians, so $120^\circ=(1/3) 2\pi$ radians, so $B=\mu_0i\phi/(4\pi R)=\mu_0i(2/3)\pi/(4\pi R)=\mu_0i/(6R)$.

2. Consider the toroid shown in the figure. Wire is wrapped around the core in a doughnut geometry, current $i=10$ A in the wire. As shown, the wire carries current out of the page (this direction of the wire is shown as small 'o'), around the toroid, and back into
the page (denoted as 'x'). There are 40 windings or turns on this toroid. Three circular Amperian loops – the smallest one (#1), the middle-sized one between the inner and outer windings (#2), and the largest one outside the toroid entirely (#3), with radii $R_1 < R_2 < R_3$, are shown. What is the relationship between the magnitudes of the magnetic fields at each radius?

a. $B_1=B_3=0$ and $B_2=\mu_0(10 \text{ A})40/(2 \pi R_2)$
b. $B_1=B_2=B_3=0$
c. $B_1=0$, $B_2=\mu_0(10 \text{ A})40/(2 \pi R_2)$, $B_3=\mu_0(10 \text{ A})40/(2 \pi R_3)$
d. $B_1=0$, $B_2=\mu_0(10 \text{ A})40/(2 \pi R_2)$, $B_3=\mu_0(20 \text{ A})40/(2 \pi R_3)$
e. $B_1=0$, $B_2=0$, $B_3=\mu_0(20 \text{ A})40/(2 \pi R_3)$

Solution: The B field at $R_2$ is $\mu_0(10 \text{ A})40/(2 \pi R_2)$ (eq. 29-24 in the text), where $i=10 \text{ A}$, and $N$, the total number of turns, is 40. This field comes from the current included inside the radius $R_2$, which is all in the same direction (40 turns carry wire out of the page inside $R_2$.) Since radius $R_1$ has no current included inside of it, B at this radius is 0. Similarly, since the sign of the included current is important, at radius $R_3$ there are included both 40 turns carrying current out of the page (call this direction '+') AND 40 turns carrying current into the page (call this direction '-'). Thus, $B_3$ is also zero, just as discussed in class and in the text.

3. The axial field in the 5 cm diameter center, or bore, of a 100 turn coil is being changed from +10T to 0 in 20 ms (where the '+' sign indicates the direction of the field.) What is the magnitude (answers given are in units of V) of the induced voltage in the coil during this ramp down? Also, which direction does the field produced by the current in the coil caused by this induced voltage point? '+' denotes the same direction as the 10 T starting field, '-' denotes the opposite direction.

a. 98, + b. 980, + c. 9.8, + d. 0.98, + e. 0.98, -

Solution: induced electro-motive force $= -Nd\Phi_B/dt = -N\Delta\Phi_B/\Delta t$, where $N=$ # of turns in coil=100, $\Phi_B=BA$, where $A$ is the area of the coil, thus at the beginning $\Phi_B=10 \text{ T} \times \pi r^2$, where the radius of the coil is $2.5 \text{ cm} = 2.5 \times 10^{-2} \text{ m}^2$, so $\Phi_B=0.0196 \text{ T} \text{ m}^2$, so $-N\Delta\Phi_B/\Delta t= -100*(-0.0196 \text{ T} \text{ m}^2)/0.02 \text{ s}$, or 98 V. (The sign comes out + because of the '-' in front of 'N' and the fact that $\Phi_B$ is decreasing with time. But the magnitude was asked anyway.) The direction of the field is in the direction of the original 10 T (+) since we know from Lenz's law that the induced electro-motive force causes a current that produces a field that opposes the change in the field.
4. In the electrical circuit shown, the inductor \( L = 1 \) Henry, the resistor \( R = 5 \Omega \), and the battery voltage \( V = 12 \) V. What is the time constant, \( \tau \), in units of seconds, of this circuit?

a. 0.2  b. 0.5  c. 1.0  d. 2  e. 5

Solution: the time constant for a circuit containing a resistance and an inductance is \( \tau = L/R \), so that the time constant for this problem is 0.2 s.

5. A square wire loop of side length 10.0 cm carries a current of 10A. It is placed so that the normal to its plane makes an angle of 30º with respect to the direction of a uniform magnetic field of 4.0T. What is the magnitude of the torque acting on the loop in N-m?

(1) 0.20  (2) 0.63  (3) 4.0  (4) 0.35  (5) 10.0

Solution:
\[
\tau = \mu \times B \\
|\tau| = NiAB\sin\theta = (1)(10)(0.1^2)(4)\sin 30^\circ \\
|\tau| = 0.2
\]

6. A proton of mass \( 1.67 \times 10^{-27} \) kg and velocity \( 10^8 \) m/s enters a uniform magnetic field with its velocity perpendicular to the field direction. If the radius of its path in the magnetic field is 50 cm, what is the strength of the magnetic field in T? (The charge of the proton is \( q = e \))

(1) 2  (2) 0.5  (3) \( 10^{-8} \)  (4) 1  (5) 4

Solution:
\[
r = \frac{p}{qB} \implies B = \frac{p}{qR} = \frac{mv}{qR} \\
B = \frac{(1.67\times10^{-27})(10^8)}{(1.6\times10^{-19})(0.5)} = 2
\]
7. In the circuit shown, what is the magnitude of the current flowing through the battery with potential difference $\varepsilon_1$?

\begin{align*}
(1) \quad & \frac{\varepsilon_1}{R_2} + \frac{\varepsilon_2}{R_2} \\
(2) \quad & \frac{\varepsilon_2}{R_2} \\
(3) \quad & \frac{\varepsilon_1}{R_1} + \frac{\varepsilon_2}{R_2} \\
(4) \quad & \frac{\varepsilon_1 + \varepsilon_2}{R_1 + R_2} \\
(5) \quad & \frac{\varepsilon_1}{R_1 + R_2}
\end{align*}

Solution:
Use Kirchoff's junction and loop rules after you define current directions:

Junction: $i_1 = i_2 + i_3$

Outer loop counterclockwise:

$\varepsilon_1 + \varepsilon_2 - i_1 R_2 = 0$

$\Rightarrow i_1 = \frac{\varepsilon_1 + \varepsilon_2}{R_2}$

8. A DC power supply delivers 18 W of power at a constant EMF of 6 V to a purely resistive load. What is the resistance of the load?

\begin{align*}
(1) \quad & 2 \Omega \\
(2) \quad & 3 \Omega \\
(3) \quad & 0.33 \Omega \\
(4) \quad & 0.5 \Omega \\
(5) \quad & 6 \Omega
\end{align*}

Solution:
The power delivered must equal the power dissipated:

$P = \frac{\varepsilon^2}{R}$

$\Rightarrow R = \frac{6^2}{18} = 2\Omega$
9. An electron with velocity $\mathbf{v} = v_0 \hat{x}$ enters a region where there is a uniform electric field $\mathbf{E} = E_0 \hat{y}$. If the electron is to continue undeflected from its initial trajectory, what magnetic field is necessary?

\begin{align*}
(1) \quad &\frac{E_0}{v_0} \hat{z} \\
(2) \quad &-\frac{E_0}{v_0} \hat{z} \\
(3) \quad &-\frac{E_0}{v_0} \hat{y} \\
(4) \quad &E_0 v_0 \hat{z} \\
(3) \quad &-E_0 v_0 \hat{y}
\end{align*}

Solution:
The forces must balance:

$$
\mathbf{F} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0
$$

$$
\mathbf{E} = -\mathbf{v} \times \mathbf{B} \quad \text{where} \quad \mathbf{E} = E_0 \hat{y} \quad \text{and} \quad \mathbf{v} = v_0 \hat{x}
$$

$$
\Rightarrow \mathbf{B} = \frac{E_0}{v_0} \hat{z}
$$

10. An initially uncharged capacitor in an RC circuit reaches a potential difference of 63 V in a time of 2 s after the circuit is connected to an EMF source of 100 V. If the resistance in the circuit is 1000 Ω, what is the capacitance of the capacitor?

\begin{align*}
(1) \quad &0.002 \text{ F} \\
(2) \quad &0.001 \text{ F} \\
(3) \quad &2 \text{ F} \\
(4) \quad &0.004 \text{ F} \\
(5) \quad &0.0005 \text{ F}
\end{align*}

Solution:

$$
V_C = \frac{q}{C} = \varepsilon \left(1 - e^{-t/RC}\right)
$$

$$
\Rightarrow 63 = 100 \left(1 - e^{-t/RC}\right)
$$

$$
\Rightarrow e^{-t/RC} = 1 - 0.63 = 0.37
$$

$$
\frac{t}{RC} = -\ln 0.37 = 1
$$

$$
\Rightarrow C = \frac{t}{R} = \frac{2}{1000} = 0.002
$$