Exam 3 Solutions

1. The hexagonal array of charge as shown in the figure has the length of the side=1 m, and therefore (since all the triangles shown are equilateral, with 60 degree angles) the distances from all the q’s to the center point P are also 1 m. If the charges at the corners of the hexagon are \( q_1 = q_2 = q_3 = +10^{-9} \text{ C} \), and \( q_4 = q_5 = q_6 = -10^{-9} \text{ C} \), what is the magnitude, in N/C, of the x-component of the force from the sum of all the charges at point P in the center? (Hint – it’s in the negative direction.)

\[
\begin{align*}
\text{y-axis} & \quad \uparrow \\
\rightarrow & \quad \rightarrow \\
\text{x-axis} & \quad \rightarrow \rightarrow \rightarrow
\end{align*}
\]

a.) 18   b.) 0   c.) 9   d.) 27   e.) 36

Solution: From symmetry, the field from the pair \( q_1 q_4 \) (which is directed from \( q_1 \) along the dotted line towards \( q_4 \)) and the field from the pair \( q_2 q_5 \) (which is directed from \( q_2 \) along the dotted line towards \( q_5 \)) sum to give a zero component in the x-direction. Thus, the x-component of the net force at point P is just

\[
E = E_{\text{from } q_3} + E_{\text{from } q_6} = \frac{1}{4\pi \varepsilon_0} \left( \frac{10^{-9} \text{ C}/1\text{ m}^2}{1\text{ m}})(-e_x) + \frac{10^{-9} \text{ C}/1\text{ m}^2}{1\text{ m}})(-e_x) \right) \] where \( e_x \) is a unit vector in the x-direction and the two minus signs indicate that both \( E_{\text{from } q_3} \) and \( E_{\text{from } q_6} \) point to the left, i. e. in the negative x-direction.

Thus, the magnitude of \( E \) at point P from all the charges is just

\[8.99 \times 10^9 \text{ N m}^2/\text{C}^2](2 \times 10^{-9} \text{ C/m}^2) = 17.98 \text{ N/C}\]

2. In an oscillating LC circuit (no resistance), the maximum current in the circuit is 0.1 A, \( L=2 \text{ mH} \), and \( C=3 \mu \text{F} \). What is the maximum charge on the capacitor, in units of \( \mu \text{C} \)?

a. 7.7   b. 6.0   c. 0.13   d. 0.17   e. 60

This is like a homework problem in chapter 31. Eq. 31-14 tells us that the amplitude of the current in an oscillating LC circuit is \( \omega Q \), where \( \omega \), the frequency, is just \( 1/(LC)^{1/2} \) and \( Q \) is the maximum charge on the capacitor.

\[Q = \frac{I_{\text{max}}}{\omega} = I_{\text{max}}(LC)^{1/2} = 0.1(2 \times 10^{-3} \times 3 \times 10^{-6})^{1/2} = 7.7 \mu \text{C}\]
3. A non-uniform electric field is given by \( E = 2.0x \hat{i} \), where \( E \) is a vector in the \( x \)-direction, \( \hat{i} \) is a unit vector (length=1) vector in the \( x \)-direction, and the ‘\( x \)’ in the equation is a variable. Thus, the strength of the electric field is a function of ‘\( x \)’ and is, for example, twice as large at \( x=4.0 \) m than at \( x=2.0 \) m. What is the charge inclosed in the Gaussian surface shown, in units of \( 10^{-12} \) Coulombs?

\[ \begin{align*}
\text{a. } & 142 & \text{b. } & 8.85 & \text{c. } & 17.7 & \text{d. } & 283 & \text{e. } 0
\end{align*} \]

Solution: Use Gauss’ law (eq. 23-7 in the text). (This problem is similar to sample problem 23-2)

\[ \varepsilon_0 \int E \cdot dA = q_{enc} \]

where the integral is over the Gaussian surface, and \( E \cdot dA \) is the vector dot product between the electric field vector and the \( dA \) over each surface of the cube shown. Since \( E \) is entirely in the \( x \)-direction, any surface on the cube whose \( dA \) vector is perpendicular to \( x \) will give a zero dot product. Thus, only the left face (at \( x=1.0 \) m and parallel to the \( yz \) plane) and the right face (at \( x=3.0 \) m and also parallel to the \( yz \) plane) will have non-zero contributions to the integral.

At the left face, \( \int E \cdot dA = 2.0 \int (-1)x dA \) where the 2.0 is the constant in \( E=2.0x \hat{i} \), the minus sign is because \( dA \) for the left face points in the minus \( x \)-direction and \( E \), the electric field vector, points in the +\( x \)-direction. All over the left face, \( x=1.0 \) m, and the integral of \( dA \) is just the area of the face, 4 \( m^2 \). Thus, the integral at the left face is \(-8 \text{ Nm}^2/\text{C}\). Similarly, for the right face, \( x=3.0 \) m, \( dA \) points to the right in the same direction as the \( E \) field so the dot product is positive and the integral at the right face is \(+2.0(3.0)(+4)m^2 = +24 \text{ Nm}^2/\text{C}\). So we have

\[ \varepsilon_0 \int E \cdot dA = q_{enc} = \varepsilon_0 ( -8 + 24) \text{Nm}^2/\text{C} = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 (16 \text{ Nm}^2/\text{C})=142 \times 10^{-12} \text{ C} \]

4. Green light of wavelength 520 nm is traveling in vacuum until it reaches a glass lens with index of refraction 1.4. In units of \( 10^8 \) m/s, what is the speed of the green light in the glass? (2.998 \times 10^8 \) m/s is the speed of light in vacuum.)

\[ \begin{align*}
\text{a. } & 2.14 & \text{b. } & 2.998 & \text{c. } & 4.2 & \text{d. } & 0.58 & \text{e. } 0.52
\end{align*} \]

Solution: The frequency of light in matter remains the same, however the wavelength is reduced from the freespace value of the wavelength by \( n \), the index of refraction, i.e. \( \lambda_{\text{in glass}} = \lambda_{\text{vacuum}}/n = 520 \) nm/1.4. The speed of the propagation of the light in the glass is \( \lambda_{\text{in glass}} \times \text{freq} \). Since the frequency is unchanged, the speed in glass is just reduced by the same factor that \( \lambda \) is, i.e. speed = 2.998 \times 10^8 m/s / 1.4 = 2.14 \times 10^8 \) m/s.
5. Which of the following equations is not a law of electromagnetism?

\begin{align*}
(1) & \quad \oint \vec{E} \cdot d\vec{A} = d\Phi_B/dt \\
(2) & \quad \oint \vec{B} \cdot d\vec{A} = 0 \\
(3) & \quad \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q \\
(4) & \quad \oint \vec{E} \cdot d\vec{s} = -d\Phi_B/dt \\
(5) & \quad \oint \vec{B} \cdot d\vec{s} = \mu_0 i + \varepsilon_0 \mu_0 d\Phi_E/dt
\end{align*}

Solution:
Law (1) is not a law of electromagnetism.
Law (2) is Gauss’ Law for magnetic fields.
Law (3) is Gauss’ Law for electric fields.
Law (4) is Faraday’s Law of induction.
Law (5) is Ampere’s Law with Maxwell’s Law of Induction.

6. A beam of unpolarized light is sent into a system of two polarizing sheets with polarizing directions at angles $\theta_1=20^\circ$ and $\theta_2=60^\circ$ with respect to the y-axis. What is the intensity of the light transmitted by the system relative to the initial intensity?

(1) 0.29  (2) 0.59  (3) 0.13  (4) 0.22  (5) 0.38

Solution:
Unpolarized light passed through the first filter will be reduced to $\frac{1}{2}$ of the original intensity. The intensity after the second filter is thus given by:
\[
\frac{I_f}{I_0} = \frac{1}{2} \cos^2 (\theta_1 - \theta_2) = \frac{1}{2} \cos^2 (60 - 20) = \frac{1}{2} \cos^2 (40^\circ) \approx 0.29
\]

7. An object is placed 40 cm in front of a concave spherical mirror with a radius of curvature of 60 cm. Where is the image?

(1) 120 cm in front of the mirror
(2) 60 cm behind the mirror
(3) 120 cm behind the mirror
(4) 17 cm behind the mirror
(5) 40 cm in front of the mirror
Solution:
The focal length of the mirror is \( f = \frac{R}{2} = 30 \text{ cm} \).
The image is found by solving:
\[
\frac{1}{p} + \frac{1}{i} = \frac{1}{f}
\]
\[
\frac{1}{i} = \frac{1}{f} \frac{1}{p} = \frac{1}{30} \frac{1}{40} = \frac{4}{120}
\]
\( \Rightarrow i = 120 \text{ cm} \)

Image distance is positive, so image is real and is thus in front of mirror.

8. An electron with velocity \( \mathbf{v} = v_0 \hat{x} \) enters a region where there is a uniform electric field \( \mathbf{E} = -E_0 \hat{y} \). If the electron is to continue undeflected from its initial trajectory, what magnetic field is necessary?

\[ (1) -\frac{E_0}{v_0} \hat{z} \quad (2) \frac{E_0}{v_0} \hat{z} \quad (3) -\frac{E_0}{v_0} \hat{y} \quad (4) E_0v_0 \hat{x} \quad (5) -E_0v_0 \hat{y} \]

Solution:
The forces must balance:
\[
\mathbf{F} = q \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right) = 0
\]
\[
\mathbf{E} = -\mathbf{v} \times \mathbf{B} \quad \text{where} \quad \mathbf{E} = -E_0 \hat{y} \quad \text{and} \quad \mathbf{v} = v_0 \hat{x}
\]
\( \Rightarrow \mathbf{B} = -\frac{E_0}{v_0} \hat{z} \)

9. In the circuit shown, what is the magnitude of the current flowing through the battery with potential difference \( \varepsilon_1 \)?

\[
(1) \frac{\varepsilon_1}{R_1} - \frac{\varepsilon_2}{R_1} \quad (2) \frac{\varepsilon_2}{R_1} \quad (3) \frac{\varepsilon_2}{R_1} + \frac{\varepsilon_2}{R_2} - \frac{\varepsilon_1}{R_1} \quad (4) \frac{\varepsilon_2 - \varepsilon_1}{R_1 + R_2} \quad (5) \frac{\varepsilon_1}{R_1 + R_2}
\]

Solution:
Use Kirchoff’s junction and loop rules after you define current directions:
Junction: \( i_1 = i_2 + i_3 \)
Outer loop counterclockwise:
\[ \varepsilon_i - i R_1 - \varepsilon_2 = 0 \]
\[ \Rightarrow i = \frac{\varepsilon_i - \varepsilon_2}{R_1} \]

10. A 75pF capacitor is charged to a potential difference of 100V, and the charging battery is disconnected. The capacitor is then connected in parallel with a second (initially uncharged) capacitor. If the potential difference across the first capacitor drops to 75V, what is the capacitance of this second capacitor?

(1) 25pF  (2) 225pF  (3) 75pF  (4) 100pF  (5) 300pF

Solution:
The total charge is conserved, as it cannot be removed. The capacitance and potential difference does change, however:

\[ q = C_1 \Delta V_1 = (C_1 + C_2) \Delta V_2 \]
\[ \Rightarrow \left( \frac{\Delta V_1}{\Delta V_2} - 1 \right) C_1 = C_2 \]
\[ \left( \frac{100}{75} - 1 \right) 75 = 25 \]
\[ \Rightarrow C_2 = 25 \text{pF} \]