Exam 2 Solutions

1. What is the magnitude and direction of the magnetic field at the origin of the shown arrangement of current-carrying wires. Each wire carries a current of $i$ and points either into or out of the page as indicated, and each wire is a distance $d$ from the origin.

Answer: $\frac{\mu_0 i}{\pi d} (\hat{i} + \hat{j})$

Solution: Each wire produces a magnetic field at the origin of magnitude $\mu_0 i/(2\pi d)$. The two wires on the x-axis each produce magnetic fields in the $\hat{j}$ direction. The two wires on the y-axis each produce magnetic fields in the $\hat{i}$ direction. Sum the four magnetic fields to get the net magnetic field at the origin.

2. A wire is bent into the shown planar shape and carries a current $i = 15$ mA along the indicated directions. What is the magnitude of the magnetic field at the center of the loop, $C$ if the radius of the loop is $R = 5$ cm?

Answer: $1.3 \times 10^{-7}$ T

Solution: The wire produces a magnetic field coming out of the page at $C$, and the loop produces a magnetic field going into the page at $C$. The net field into the page at $C$ is

$$B_C = \frac{\mu_0 i}{2R} - \frac{\mu_0 i}{2\pi R}$$

3. A solenoid that is 50 cm long with a radius of 1 cm is to be constructed such that it has a field strength of 0.04 T within its volume. If a current of 3A can be provided, how many windings of a wire in total are required for the entire solenoid to achieve this field?

Answer: 5300

Solution: The magnetic field inside a long solenoid is $B = \mu_0 iN/L$, where $N$ is the number of turns to be determined and $L = 0.5$ m.

4. Two straight parallel power lines are separated by a distance of 2m. Each carries a current of 5000A, but in opposite directions. What is the magnitude of the force per unit length on the wires, and is the force attractive or repulsive?

Answer: 2.5 N/m, repulsive

Solution: The force is repulsive because the currents are in opposite directions, and the force per unit length is $\mu_0 i^2/(2\pi r)$. 
5. A solid cylindrical wire of radius 2mm carries a current of 10A distributed uniformly throughout its cross section. What is the magnitude of the magnetic field inside the wire at a radius of 1mm from the central axis?

**Answer:** \(5 \times 10^{-4}\) T

**Solution:** Ampere’s law states for a loop of radius \(r = 1\) mm inside the wire states that

\[
\oint \mathbf{B} \cdot d\mathbf{s} = 2\pi r = \mu_0 i_{\text{enc}} = \mu_0 (10 A) \frac{\pi (1\text{mm})^2}{\pi (2\text{mm})^2}.
\]

Only a fraction of the current, \(1/2^2 = 1/4\), is enclosed in the 1mm path of integration.

6. An air-filled parallel-plate capacitor has a capacitance of 1 pF. The plate separation is then doubled and a wax dielectric is inserted, completely filling the space between the plates. As a result, the capacitance becomes 2 pF. The dielectric constant of the wax is:

**Answer:** 4.0

**Solution:** The capacitance of a parallel plate capacitor with a dielectric is \(C = \epsilon_0 \kappa A/d\). If the plate separation is doubled, the capacitance goes down by a factor of two. Inserting the dielectric multiplies the capacitance by \(\kappa\). Since overall the capacitance changes by two in going from 1 pF to 2 pF, \(2 = \kappa/2 \rightarrow \kappa = 4\).

7. To store a total of 0.040 J of energy in the two identical capacitors shown, each should have a capacitance of:

**Answer:** 1.0 \(\mu\)F

**Solution:** The energy stored in a capacitor is \((1/2)CV^2\). There are two capacitors so the total energy stored is \(CV^2\) with \(V = 200\) volts.

8. Two parallel-plate capacitors with different capacitance but the same plate separation are connected in series to a battery. Both capacitors are filled with air. The quantity that is the same for both capacitors when they are fully charged is:

**Answer:** charge on the positive plate

**Solution:** Capacitors in series have the same charge.

9. The diagram shows four 6\(\mu\)F capacitors. The capacitance between points a and b is:

**Answer:** 6 \(\mu\)F

**Solution:** Two 6\(\mu\)F capacitors in series have capacitance of 3\(\mu\)F. The two sets of capacitors in series are in parallel so the net capacitance is 3\(\mu\)F + 3\(\mu\)F = 6\(\mu\)F.

10. A 10-ohm resistor has a constant current. If 1200C of charge flows through it in 4 minutes what is the value of the current?

**Answer:** 5 A

**Solution:** Current is charge per unit time: \(i = (1200C)/(4 \times 60s) = 5A\).
11. A cylindrical copper rod has resistance \( R \). It is reformed to twice its original length with no change of volume. Its new resistance is:

Answer: \( 4R \)

Solution: The resistance of a wire is \( R = \rho L/A \). The volume of a wire is \( L A \). If \( L \) increases by 2, then \( A \) must decrease by 2 in order for the volume to remain constant. If \( L \) increases by 2 and \( A \) decreases by 2, then \( L/A \) increases by 4.

12. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1m. Conductor B is a hollow tube of inside diameter 1m and outside diameter 2m. The ratio of their resistance, \( R_A/R_B \), is:

Answer: 3

Solution: The resistance of a wire is \( R = \rho L/A \). Since the wires are made of the same material, the resistivity, \( \rho \), is the same. The length is also the same. Thus, the only difference is the cross-sectional area, \( A \): \( R_A/R_B = (\pi 2^2 - \pi 1^2)/(\pi 1^2) \).

13. A current of 0.3 A is passed through a lamp for 2 minutes using a 6-V power supply. The energy dissipated by this lamp during the 2 minutes is

Answer: 216 J

Solution: The energy dissipated is the power used by the lamp multiplied by the time: \( E = (0.3A)(6V)(120s) = 216 J \).

14. Resistors of 2.0\( \Omega \), 4.0\( \Omega \), and 6.0\( \Omega \) are connected in series across a 24-V emf. The potential difference across the 2\( \Omega \) resistor is:

Answer: 4V

Solution: The net resistance of the three resistors in series is 12\( \Omega \). Consequently, the current in the circuit is 24V/12\( \Omega \) = 2A. The voltage across the 2\( \Omega \) resistor is \( iR = 4V \).

15. In the circuit shown, the capacitor is initially uncharged. At time \( t = 0 \), switch S is closed. If \( \tau \) denotes the time constant, the approximate current through the 3\( \Omega \) resistor when \( t = \tau/10 \) is:

Answer: 1.0A

Solution: The charge across the capacitor as a function of time is

\[ q(t) = CV(1 - e^{-t/\tau}) , \]

where \( \tau = RC \) and \( R = 9\Omega \). The current at time \( t \) is

\[ i(t) = \frac{dq}{dt} = \frac{CV}{\tau} e^{-t/\tau} = \frac{V}{R} e^{-t/\tau} . \]
16. At one instant an electron is moving in the x-y plane. Its velocity is \( \vec{v} = (5 \times 10^5 \hat{i} + 3 \times 10^5 \hat{j}) \text{m/s} \). In that region of space there is an electric field of 200,000V/m in the \(-z\) direction and a magnetic field of 0.8T in the \(+x\) direction. At that instant the magnitude of the net force on the electron is:

**Answer:** \( 7.0 \times 10^{-14} \text{ N} \)

**Solution:** The force due to the magnetic field is \((-e) \vec{v} \times \vec{B} \). Since \( \hat{i} \times \hat{i} = 0 \) and \( \hat{j} \times \hat{i} = -\hat{k} \), the magnetic force is \( 3.84 \times 10^{-14} N \hat{k} \). The electric force is \((-e) \vec{E} = 3.2 \times 10^{-14} N \hat{k} \). Add these two forces together.

17. An electron is accelerated from rest through a potential difference \( V \) after which it enters a region having a magnetic field of magnitude \( B \) and direction perpendicular to the electron’s velocity. The radius of the electron orbit in this magnetic field is given by:

**Answer:** \( \frac{1}{B} \sqrt{\frac{2m_eV}{e}} \)

**Solution:** The kinetic energy of the electron when it enters the magnetic field is \((1/2)m \nu^2 = eV\). For circular motion in a magnetic field \( m(\nu^2/r) = evB \). Solve for the radius.

18. A single circular loop of wire for which the wire length is 80 cm carries a current of 1.0A. At one instant the plane in which the loop lies is at an angle of 45° to a magnetic field of magnitude 2.0T. The torque magnitude acting on the loop at that instant is:

**Answer:** 0.072 N·m

**Solution:** The circumference of the loop is \( 2\pi r = 0.8 \text{ m} \) so the radius is 0.127 m. The magnetic moment of the wire is \( \mu = N i A = 1 \times (\pi r^2) \times 1 \text{A} \). The torque on the loop is \( \mu B \sin(\theta) \), where \( \theta \) is the angle between the magnetic moment and the magnetic field, which is 45°.

19. The dashed horizontal line in the Figure shows the direction of a uniform magnetic field. The short arrowed lines represent the orientation of the magnetic moment of a current carrying loop in a magnetic field. Rank the potential energy of the loop associated with each letter from greatest to least:

**Answer:** e, c = d, a, b

**Solution:** The potential energy of a magnetic moment in a magnetic field is \( U = -\vec{\mu} \cdot \vec{B} - \mu B \cos(\theta) \), where \( \theta \) is the angle between the magnetic moment and the magnetic field. The potential energy is greatest when the moment is in the opposite direction of the field, and smallest when the moment is in the same direction as the field.

20. The absolute value of the potential difference between the points labeled a and b in the Figure is:

**Answer:** 14V

**Solution:** Let \( i \) be the current going clockwise in the loop. Applying Kirchhoff’s loop rule going clockwise around the loop starting at a,

\[
0 = 12 - 1000i + 12 - 4 - 1000i,
\]

we find that \( i = 10mA \). Consequently, \( V_b - V_a = 12 - 1000i + 12 = 14V \).