Exam 1 Solution

1. Four charges are placed at the corners of a rectangle as shown in the figure. If $Q_1 = 1\, \mu C$, $Q_2 = -2\, \mu C$, $Q_3 = 1\, \mu C$, and $Q_4 = 2\, \mu C$, what is the magnitude of the net force on $Q_4$?

Answer: 11.4 N

Solution: Make a table showing the components of each of the forces and then add the components.

<table>
<thead>
<tr>
<th></th>
<th>$F_{\text{on 4 by 1}}$</th>
<th>$F_{\text{on 4 by 2}}$</th>
<th>$F_{\text{on 4 by 3}}$</th>
<th>$F_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>$-k(1, \mu C)(2, \mu C)/(3, \text{cm})^2$</td>
<td>$-k(1, \mu C)(2, \mu C)/(4, \text{cm})^2$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$+0.8k(2, \mu C)(2, \mu C)/(5, \text{cm})^2$</td>
<td>$+0.6k(2, \mu C)(2, \mu C)/(5, \text{cm})^2$</td>
<td>$-0.27$ N</td>
<td>$-11.36$ N</td>
</tr>
</tbody>
</table>

The directions were obtained using opposite charges attract and like charges repel. The factors of 0.8 and 0.6 are the trigonometric factors to determine the x- and y-components. The magnitude of the force is $\sqrt{F_x^2 + F_y^2}$.

2. Five charges of magnitude $e$ and sign indicated in the figure are placed on a line. The separation between the charges is $d$. What is the magnitude and direction of the force on the middle charge?

Answer: $1.5\, ke^2/d^2$, left

Solution: For this problem again use opposite charges attract and like charges repel. Label the charges 1 to 5 from left to right. As for the previous problem it is helpful to make a table:

<table>
<thead>
<tr>
<th></th>
<th>$F_{\text{on 3 by 1}}$</th>
<th>$F_{\text{on 3 by 2}}$</th>
<th>$F_{\text{on 3 by 4}}$</th>
<th>$F_{\text{on 3 by 5}}$</th>
<th>$F_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$ke^2/(2d)^2$ to the right</td>
<td>$ke^2/(d)^2$ to the left</td>
<td>$ke^2/(d)^2$ to the left</td>
<td>$ke^2/(2d)^2$ to the right</td>
<td>$1.5, ke^2/d^2$ to the left</td>
</tr>
</tbody>
</table>

3. The following tests are made on an object $X$ of unknown charge and material.
   (i) A positively charged object is attracted to $X$.
   (ii) A negatively charged object is attracted to $X$.
Which of the following are possible explanations of this behavior?

Answer: $X$ is a neutral conductor.

Solution: As demonstrated in class and also in the book (see Fig. 21-4), when a charged object is brought next to a neutral conductor, there is an induced charge near the charged object. For example, a positive charge brought near a neutral conductor will induce a negative charge near the surface of the conductor. The metal will be attracted to the positive charge. The metal will also be attracted to a negative charge because induced charge in this case will be positive.

4. How many excess electrons are in $-2\mu C$ charge?

Answer: $10^{13}$

Solution: The charge of an electron is $-1.6 \times 10^{-19} C$. $(−2 \times 10^{-6} C)/(−1.6 \times 10^{-19} C) \approx 10^{13}$. 
5. A and B are identical conducting spheres. Sphere A is initially charged to 1 C, and sphere B has no charge initially. The following five steps are done.
(i) Sphere A is touched to sphere B.
(ii) The spheres are separated and sphere B is grounded.
(iii) Sphere A is again touched to sphere B.
(iv) The spheres are separated and sphere B is grounded.
(v) The spheres are touched together one last time.
What is the charge of sphere A after these five steps?

Answer: 0.125 C

Solution: After step (i) each sphere has charge $1/2$ C. After step (iii) each sphere has charge $1/4$ C, and after step (v) each sphere has charge $1/8$ C.

6. In the figure four charges with the same magnitude and indicated charge are placed on the corners of a square. What is the direction of the resulting electric field at the center of the square?

Answer: $\hat{-j}$

Solution: The electric field goes away from positive charges and towards negative charges. Adding the electric field at the center due to each charge gives a net electric field downward ($\hat{-j}$).

7. A wire segment of length $L$ has constant linear charge density $\lambda > 0$. Which of the following expressions gives the magnitude of the electric field a distance $D$ from the center of the wire (see figure)?

Answer: $\int_{-L/2}^{L/2} \frac{kD\lambda}{(x^2 + D^2)^{3/2}}dx$

Solution: A small element of charge is $dq = \lambda dx$. The distance from $dq$ to the point in question is $r = \sqrt{x^2 + D^2}$. By the symmetry of the problem the electric field is pointing away from the wire. To get this component of the electric field we need a geometrical factor of $D/r$.

$$E = \int \frac{kD}{r^2} \frac{D}{r} dq = \int_{-L/2}^{L/2} \frac{k}{(\sqrt{x^2 + D^2})^2 \sqrt{x^2 + D^2}} \frac{D}{\sqrt{x^2 + D^2}} \lambda dx$$

8. An electron with initial velocity of 500 km/s is traveling in the direction of a uniform electric field with magnitude 2 V/m. How long will it take the electron to return to its initial position?

Answer: 2.8 micro-sec.

Solution: Let the direction of the electron’s initial velocity be in the $\hat{i}$ direction. The force on the electron is $-eE$, and its acceleration is $-e|E|/m_e\hat{i}$. This acceleration is in the opposite direction of the initial velocity so the electron will slow down and eventually reverse direction. The time it takes the electron to reach an instantaneous zero velocity is obtained from $v_x = v_i - at = 0$ or $t = v_i/a$. The time it takes to return to it’s initial position is twice as long: $2v_i/a = 2v_i/m_e|E|$. 
9. The field lines for two point charges is shown at right. Based on the field lines, what are the signs of the charges?

**Answer:** positive charge on left, negative charge on right

**Solution:** Electric field lines go from positive charges to negative charges. Since the electric field lines leave the left hand side and go to the right hand side, there must be a positive charge on the left and a negative charge on the right.

10. A segment of a circular arc has charge per unit length 5\text{nC/m}. As shown in the figure, the segment covers one quarter of a circle of radius 1 cm. What is the magnitude of the electric field at the center of the circle?

**Answer:** 6.4 kV/m

**Solution:** By the symmetry of the problem the x and y components of the electric field at the origin are equal and both negative.

\[ E_x = -\int \frac{k}{r^2} \cos(\theta) dq = -\int_{\pi/2}^{\pi/2} \frac{k}{r^2} \cos(\theta) \lambda r d\theta = -\frac{k\lambda}{r}. \]

The magnitude of the electric field is \( |E| = \sqrt{E_x^2 + E_y^2} = \sqrt{2}|E_x| = \sqrt{2}k\lambda/r. \)

11. A cube of side 3 m has one corner at the origin as shown in the figure. If the electric field is equal to \( \vec{E} = (20 - 10x)\hat{i} \) with \( x \) measured in meters and the electric field measured in N/C, what is the net charge enclosed in the cube?

**Answer:** -2.4 nC

**Solution:** The electric field only has non-zero perpendicular components, \( \vec{E} \cdot \hat{n} \), at the \( x = 3 \) m (\( \hat{n} = \hat{i} \)) surface and the \( x = 0 \) m (\( \hat{n} = -\hat{i} \)). At the \( x = 3 \) m surface the perpendicular component is \( \vec{E} \cdot \hat{n} = -10N/C \). At the \( x = 0 \) m surface the perpendicular component is \( \vec{E} \cdot \hat{n} = -20N/C \), where the minus sign for this surface comes because \( \hat{n} = -\hat{i} \). The net flux through the cube is thus \( \Phi = (-10N/C - 20N/C)(3m)^2 = Q_{enc}/\varepsilon_o \). Solve for \( Q_{enc} \).

12. A very long insulating cylinder has radius of 4 mm. The cylinder is uniformly charged with charge density \( \rho = 5nC/m^3 \). What is the magnitude of the electric field at a distance of 2 mm from the center of the cylinder?

**Answer:** 0.56 N/C

**Solution:** Consider a Gaussian surface in the shape of a cylinder with length, \( L \), and radius \( r = 2 \) mm. The volume enclosed is this surface is \( \pi r^2L \), and the charge enclosed is \( \rho \pi r^2L \). Applying Gauss’s law, \( 2\pi rL \vec{E} \cdot \hat{n} = \rho \pi r^2L/\varepsilon_o \), yields \( \vec{E} \cdot \hat{n} = \rho r/(2\varepsilon_o) \).
13. A narrow charged solid cylinder is coaxial with a larger charged cylindrical shell (see figure). Both are nonconducting and thin and have uniform surface charge densities on their outer surfaces. The graph shows the radial component E of the electric field versus radial distance r from the common axis. If $E_s = 7 \times 10^3 \text{N/C}$, what is the charge per unit length on the charged solid cylinder (not the shell)?

**Answer:** $4.5 \times 10^{-9} \text{C/m}$

**Solution:** A cylinder of radius just under 3.5 cm encloses only the solid cylinder and has an electric field of $E_s/3$ at its surface. Apply Gauss’s law: $2\pi r L (E_s/3) = \lambda L/\varepsilon_0$, where L is the length of the Gaussian cylindrical surface. The L’s on both sides cancel, and one can solve for the charge per unit length, $\lambda$.

14. A conducting sphere of radius 1 cm is surrounded by a conducting spherical shell of inner radius 3 cm and outer radius 4 cm. If the electric field at $r = 2$ cm is going outwards with magnitude 300 V/cm and at $r = 5$ cm is also going outwards with magnitude 300 V/cm, what is the net charge on conducting spherical shell?

**Answer:** 7 nC

**Solution:** A sphere of radius $r = 2$ cm encloses only the inner conducting sphere. Apply Gauss’s law: $Q_{sphere} = 4\pi r^2 (300 \text{V/m}) \varepsilon_0$. A sphere of radius $r = 5$ cm encloses both the inner conducting sphere and the outer conducting shell: $Q_{sphere} + Q_{shell} = 4\pi (5\text{cm})^2 (300 \text{V/m}) \varepsilon_0$. Subtract these two equations to find $Q_{shell}$.

15. Three large charged insulating sheets have charge per unit area of $\sigma_1 = +1\mu\text{C/m}^2$, $\sigma_2 = -2\mu\text{C/m}^2$, $\sigma_3$. What is the charge density of sheet 3, $\sigma_3$, in order for the electric field to be zero in the region between sheets 2 and 3.

**Answer:** $-1\mu\text{C/m}^2$

**Solution:** In the region between sheets 2 and 3, sheet 1 produces an electric field of $(1\mu\text{C/m}^2)/(2\varepsilon_0)$ to the right, and sheet 2 produces an electric field of $(2\mu\text{C/m}^2)/(2\varepsilon_0)$ to the left. The electric field due to sheets 1 and 2 combined is thus $(1\mu\text{C/m}^2)/(2\varepsilon_0)$ to the left. To cancel this sheet 3 must produce an electric field of $(1\mu\text{C/m}^2)/(2\varepsilon_0)$ to the right, which corresponds to a charge density of $\sigma_3 = -1\mu\text{C/m}^2$.

16. The electric field as a function of position in one dimension is shown in the figure. If the electrical potential at $x = 0$ m is 4 V, what is the electrical potential at $x = 2$ m?

**Answer:** 1 V

**Solution:** To evaluate $V(2)$ we must evaluate the integral $V(2) - V(1) = -\int_1^2 E(x)dx$, but $\int_1^2 E(x)dx$ is just the area under the curve, which is 3 V. Consequently, $V(2) = 4V - 3V = 1V$. 

17. Four charges are placed at the corner of a square of side 5 cm. If the charges have values $Q_1 = 2nC$, $Q_2 = 3nC$, $Q_3 = -1nC$, and $Q_4 = -2nC$, what is the electrical potential at the center of the square?

**Answer:** 510 V

**Solution:** The electrical potential of a point charge is $V = kq/r$. For this problem all the point charges are the same distance from the center of the square, $r = 5cm/\sqrt{2}$. Consequently, we can just add the total charge in computing the potential: $V = k(2nC)/r$.

18. The electrical potential as a function of position is shown in the figure. At which of the labeled points is the x-component of the electric field its maximum positive value?

**Answer:** C

**Solution:** The electric field in the x-direction is $E_x = -\partial V/\partial x$. In other words it is the negative of the slope of this graph. To find the maximum of $E_x$, we must find where the slope is the most negative, which is at point C.

19. The electrical potential as a function of position is equal to $V = 2 - x^2$, where $V$ is measured in volts and the position, $x$, is measured in meters. A proton is released from rest at $x = 1$ m. What is the velocity of the proton when it reaches $x = 2$ m?

**Answer:** 24 km/s

**Solution:** The initial kinetic energy is zero, and the final kinetic energy is $K_f = (1/2)m_pv^2$. The initial potential energy is $U_i = eV(1) = e(1V)$, and the final potential energy is $U_f = eV(2) = e(-2V)$. Using conservation of energy, $K_i + U_i = K_f + U_f$, the final velocity satisfies $v^2 = 2e(3V)/m_p$.

20. An electron is initially $10^{-10}$ m from a proton. The electron is moved to a distance of $2 \times 10^{-10}$ m from the proton. What is the change in the potential energy of the electron, $U_f - U_i$?

**Answer:** $1 \times 10^{-18}$ J

**Solution:** The potential energy initially is $U_i = -ke^2/(10^{-10}m)$, and the potential energy in the end is $U_f = -ke^2/(2 \times 10^{-10}m)$ so the change is $U_f - U_i = ke^2/(2 \times 10^{-10}m) = 1.15 \times 10^{-18} J$. 