Exam 2 Solutions

1. The positive terminals of two batteries with emf's of $\varepsilon_1$ and $\varepsilon_2$, respectively, are connected together. Here $\varepsilon_2 > \varepsilon_1$. The circuit is completed by connecting the negative terminals. If each battery has an internal resistance of $r$, the rate in watts at which electrical energy is converted to chemical energy in the smaller battery is:

Answer: $\left(\varepsilon_2 - \varepsilon_1\right)\varepsilon_1 / 2r$

Solution: The energy conversion in the smaller battery is $\varepsilon_1 i$. However, $i = \left(\varepsilon_2 - \varepsilon_1\right)/2r$ because of the arrangement of the batteries, so the total power is $\left(\varepsilon_2 - \varepsilon_1\right)\varepsilon_1 / 2r$.

2. The current in the 12.0 $\Omega$ resistor in the circuit shown is:

Answer: 0.5 A

Solution: From the arrangement in the circuit, the voltage across the top branch is 12 V (same as battery). The resistance of the top branch is easily computed to be 8 $\Omega$ so the current in the branch is 12 / 8 = 1.5 A. The voltage across the 12 $\Omega$ resistor is 12 - 4 x 1.5 = 6 V, thus the current running through it is 6 V / 12$\Omega$ = 0.5 A.

3. In the diagrams, all light bulbs are identical and all emf devices are identical. In which circuit will each of the bulbs glow with the same brightness as in circuit X?

Answer:

Solution: Let the voltage be $\varepsilon$ and the resistance of the bulb be $R$. Then the current in X is $\varepsilon / R$. The only circuit that has current = $\varepsilon / R$ passing through each bulb is the one shown.

4. A certain capacitor, in series with a 720 $\Omega$ resistor, is being charged. At the end of 10 ms, its charge is half the final value. The capacitance is about:

Answer: 20 $\mu$F

Solution: We have to solve the charging equation $1 - e^{-t/RC} = 1/2$ for $C$, where $t = 0.010$ s. Solving yields $C = 20 \mu$F.
5. If $R_1 = 6 \, \Omega$, $R_2 = 8 \, \Omega$, $R_3 = 2 \, \Omega$, $\mathcal{E}_1 = 8 \, \text{V}$, and $\mathcal{E}_2 = 28 \, \text{V}$, what is the current in $R_2$?

**Answer:** 2 A

**Solution:** This is a 2-loop problem. Let the currents be as shown in the figure with $i_2 = i_1 + i_3$.

Then we can write the loop equations for the left and right branches, using counterclockwise and clockwise loops, respectively, as

\[-8i_2 - 6i_1 - 8 = 0\]
\[-8i_2 - 2i_3 + 28 = 0\]

Solving yields $i_2 = +2.0$, $i_1 = -4.0$ and $i_3 = +6.0$. Thus we have 2 A flowing upwards through $R_2$. Note that we picked $i_1$ to be in the wrong direction; it is actually flowing opposite to the arrow shown. This is no problem, the solution tells us the direction.

6. An electron is moving at $3 \times 10^5 \, \text{m/s}$ in the positive $x$ direction. A magnetic field of 0.8 T is in the positive $z$ direction. The magnetic force on the electron is:

**Answer:** $4 \times 10^{-14} \, \text{N}$ in the positive $y$ direction

**Solution:** The total force is $F = evB = 4.0 \times 10^{-14}$. The right hand rule for a positive charge gives a direction of $-y$, so for a negative charge the direction is $+y$.

7. An electron is launched with velocity $\mathbf{v}$ in a uniform magnetic field $\mathbf{B}$. The angle $\theta$ between $\mathbf{v}$ and $\mathbf{B}$ is between 0 and 90°. As a result, the electron follows a helix, its velocity vector $\mathbf{v}$ returning to its initial value in a time interval of:

**Answer:** $2\pi m / eB$

**Solution:** The centripetal force balance condition is $mv_\perp^2 / r = ev_\perp B$, where $v_\perp = v \sin \theta$, which yields $r = mv_\perp / eB$. The total orbital time is $T = 2\pi r / v_\perp = 2\pi m / eB$. 
8. In a certain mass spectrograph, an ion beam passes through a velocity filter consisting of mutually perpendicular fields $E$ and $B$. The beam then enters a region of another magnetic field $B'$ perpendicular to the beam. The radius of curvature of the resulting ion beam is proportional to:

**Answer:** $E / BB'$

**Solution:** The velocity selector balances electric and magnetic forces so $qE = qvB$ or $v = E / B$. The ions then enter the second magnetic field where the centripetal force condition is $mv^2 / R = qvB'$. The radius $R$ is then $R = mv / qB' = mE / qB' \propto E / BB'$.

9. A square loop of wire measured 2 cm $\times$ 2 cm carries a current of 1.25 A. The loop's magnetic dipole moment initially is aligned with a 0.5-T magnetic field. To rotate the loop so its dipole moment is perpendicular to the field and hold it in that orientation, you must do work of:

**Answer:** $2.5 \times 10^{-4}$ J

**Solution:** The potential energy of a current loop (which is a magnetic dipole) is $U_B = -\mu \cdot B$ where $\mu = iA$ is the magnitude of the magnetic moment of the current loop. Since the loop is already in this minimum energy position, the energy required by an external agent clearly has to be positive. More quantitatively, the work done by the external agent moving the loop from its initial position ($0^\circ$) to its final position ($90^\circ$) is the negative of the work done by the magnetic field or $W = +\Delta U = -\mu B \cos 90^\circ + \mu B \cos 0^\circ = \mu B = 2.5 \times 10^{-4}$ J.

10. A long wire is bent into the form of three-quarter concentric loops as shown in the figure. The loops have radii 4 cm and 8 cm and carry a common current of 2.0 A. What is the magnitude of the magnetic field at the center of the loops?

**Answer:** 11.8 $\mu$T

**Solution:** For a partial loop of radius $r$ and angle $\phi$, the magnetic field at the center of the loop is $B = \mu_0 i \phi / 4\pi r$. In the loop shown, the B fields of the two loops are opposite one another (the straight sections do not contribute anything) and $\phi = 3\pi / 2$. Putting it all together, with $r_1 = 0.04$ m and $r_2 = 0.08$ m, yields $B = 11.8$ $\mu$T.
12. In the figure, four long parallel wires of equal length are carrying identical currents in the directions shown. If the force per unit length on wire B is $F$ what is the force per unit length on B if the currents in wires B and C reverse their directions?

**Answer:** $F / 3$

**Solution:** Let the currents be $i$ and the distance between the wires be $d$. The force per unit length on wire B is $\left(\mu_0 i^2 / d\right) \left(2 - \frac{1}{2}\right)$ (wires A and C contribute in the same direction and D is opposite but farther away. If the current directions in B and C are reversed then the total force/length is $\left(\mu_0 i^2 / d\right) \frac{1}{2}$ (A and C cancel, leaving only D) which is 1/3 of the original.

12. A toroid is like a solenoid that has been bent around in a circle until its ends meet, as shown in the figure (inner radius 4 cm, outer radius 8 cm). What is the magnitude of the magnetic field inside a toroid of 1000 turns carrying a current 0.5 A at a distance 6 cm from the center of the toroid?

**Answer:** 1.67 mT

**Solution:** We solve using Ampere’s law: $\oint B \cdot ds = \mu_0 i_{enc}$ where the integral is along a closed path and $i_{enc}$ is the enclosed current within the path. For the path we use a circular loop at $r = 6$ cm where the B field is constant, obtaining $B(2\pi r) = N\mu_0 i$, where $N = 1000$ is the number of turns. Solving yields $B = 1.67$ mT.

13. As shown in the figure, a long wire carries a current of 12 A in the direction shown. What is the net force acting on the rectangular wire loop having dimensions 4 cm × 8 cm, current 2 A, and whose closest distance to the wire is 4 cm (in μN)?

**Answer:** 4.8

**Solution:** The B field at a distance $r$ due to the current in the long wire is $B = \mu_0 i_1 / 2r$ and the magnetic force acting on a parallel length $L$ of the rectangular loop is $F = i_2 LB$. The forces for the near and far loop components are in the opposite direction while the forces on the other two sides exactly cancel. The net force is $F = \mu_0 i_1 i_2 L / 2\pi \left(1/r_1 - 1/r_2\right) = 4.8 \times 10^{-6}$ N.
14. A 14.0 g conducting rod of length 1.30 m and resistance 8.0 Ω slides freely downward between two vertical conducting rails without friction. The entire apparatus is placed in a 0.43 T uniform magnetic field. Ignore the electrical resistance of the rails. What is the terminal velocity of the rod? ($g = 9.8 \text{ m/s}^2$)

**Answer:** 3.51 m/s

**Solution:** The motional emf is $\mathcal{E} = BLv$ which generates a loop current of $i = \mathcal{E} / R = BLv / R$. At terminal velocity, the magnetic force on this current equals the force due to gravity or $mg = BiL = B^2L^2v / R$. Solving for the velocity yields $v = 3.51 \text{ m/s}$.

15. The number of turns is tripled for an ideal solenoid, and its length is doubled while holding its cross-sectional area constant. If the old inductance is $L$, what is the new inductance?

**Answer:** $9L / 2$

**Solution:** The inductance is proportional to the cross sectional area, the solenoid length and the square of $n = \text{number of turns/length}$: $L = \mu_0 n^2 A \ell$. Then, if $n \to 3n / 2$ (triple turns over twice the length) and $l \to 2l$ (length doubled), the new inductance will be $L \times (3 / 2)^2 \times 2 = (9 / 2) L$.

16. In the circuit shown in the figure, the switch is in position 1 for a long time. It is then moved to position 2. How long will it take for the current in the inductor to reach 0.25 A after the switch is moved to its new position? The values are $L = 12 \text{ mH}$, $R_1 = 48 \Omega$ and $R_2 = 16 \Omega$. The battery has a voltage of 96 V.

**Answer:** 1.56 ms

**Solution:** The current in the inductor while the switch is in position one reaches 96/48 = 2.0 A after a long time. When the switch is thrown to position 2, the current falls exponentially to 1/8 of its former value or $e^{-tR_2/L} = \frac{1}{8}$. Solving yields $t = 1.56 \text{ ms}$.

17. What resistance $R$ should be connected in series with an inductance $L = 500 \text{ mH}$ and capacitance $C = 120 \mu \text{F}$ for the maximum charge on the capacitor to decay to 25% of its initial value in 25 cycles? (Assume $\omega' = \omega_0$.)

**Answer:** 1.14 Ω

**Solution:** The maximum capacitor charge falls like $e^{-tR/2L}$, so we want to solve $e^{-tR/2L} = \frac{1}{4}$. Here $t = 25 \times 2 \pi / \omega'$, where $\omega' = \omega_0 = 1 / \sqrt{LC} = 129 \text{ rad/sec}$ is the natural angular frequency of the circuit. We find $t = 1.22 \text{ sec}$ and $R = 1.14 \Omega$. 


18. A hollow cylindrical conductor (inner radius = \(a\), outer radius = \(b\)) carries a current \(i\) uniformly spread over its cross section. Which graph below correctly gives \(B\) as a function of the distance \(r\) from the center of the cylinder?

**Answer:**

![Graph of B vs. r](image)

**Solution:** We use Ampere’s law \(\oint B \cdot ds = \mu_0 i_{\text{enc}}\), where the integral is around a circular path of radius \(r\) (constant \(B\)) and \(i_{\text{enc}}\) is the total enclosed current. The total enclosed current, using constant current density and integration over the cross sectional area using \(dA = 2\pi rd\), is

\[
i_{\text{enc}} = \int_a^b \pi (2\pi r) dr = \pi \int_a^b \left( r^2 - a^2 \right) dr \quad \text{for} \quad a \leq r \leq b \quad \text{and} \quad i_{\text{enc}} = \pi \int_b^\infty \left( b^2 - a^2 \right) dr \quad \text{for} \quad r > b.
\]

The RHS is \(B(2\pi r)\). Thus \(B \propto \left( r^2 - a^2 \right) / r \) for \(a \leq r \leq b\) and \(B \propto 1 / r \) for \(r > b\). The above curve is the only one that replicates this behavior.

19. An LC circuit has a capacitance of 30 \(\mu\)F and inductance of 15 mH. At time \(t = 0\) the charge on the capacitor is 10 \(\mu\)C and the current is 20 mA. The maximum charge on the capacitor is:

**Answer:** 17 \(\mu\)C

**Solution:** The total energy of an LC circuit is \(\frac{1}{2} Li^2 + \frac{1}{2} q^2 / C\). When \(q = q_{\text{max}}\), \(i = 0\), so

\[
\frac{1}{2} Li^2 + \frac{1}{2} q^2 / C = \frac{1}{2} q_{\text{max}}^2 / C.
\]

Solving yields \(q_{\text{max}} = \sqrt{LCi^2 + q^2} = 16.7 \mu\)C.

20. An ac generator producing 10 V (rms) at 200 rad/s is connected in series with a 50 \(\Omega\) resistor, a 400 mH inductor, and a 200 \(\mu\)F capacitor. The rms voltage (in volts) across the inductor is:

**Answer:** 10.8

**Solution:** The reactances are \(X_L = 200 \times 0.4 = 80 \Omega\) and \(X_C = 1 / (200 \times 0.0002) = 25 \Omega\). The total impedance is \(Z = \sqrt{50^2 + (80 - 25)^2} = 74.3 \Omega\) which gives for the rms current

\(i_{\text{rms}} = \epsilon_{\text{rms}} / Z = 10 / 74.3 = 0.135 \) A. The rms voltage across the inductor is therefore

\(V_L = i_{\text{rms}} X_L = 10.8 \) V.