Exam 1 Solutions

Note that there are several variations of some problems, indicated by choices in parentheses.

Problem 1

Three particles are fixed on the x axis. Particle 1 of charge $q_1$ is at $x = -a$, and particle 2 of charge $q_2$ is at $x = +a$. If their net electrostatic force on particle 3 of charge $+Q$ is to be zero, what must be the ratio $q_1/q_2$ when particle 3 is at $x = 1.5a$?

(1) -25  (2) -9  (3) -1  (4) +9  (5) +25

This is problem 21-14 in the text. If we factor out the irrelevant constants $(Q / (4\pi\epsilon_0 a^2))$, the condition for zero force on $Q$ is

$$\frac{q_1}{\left(\frac{5}{2}\right)^2} + \frac{q_2}{\left(\frac{1}{2}\right)^2} = 0$$

Solving for the ratio gives $q_1/q_2 = -25$.

Problem 2

Three identical conducting spheres initially have the following charges: sphere A, $+4Q$; sphere B, $-6Q$; and sphere C, 0. Sphere A and B are fixed in place with a center-to-center separation that is much larger than the spheres. Two experiments are conducted. In experiment 1, sphere C is touched to sphere A and then (separately) to sphere B, and then it is removed. In experiment 2, starting with the same initial states, the procedure is reversed: sphere C is touched to sphere B and then (separately) to sphere A, and then it is removed. What is the ratio of the electrostatic force between A and B at the end of experiment 2 to that at the end of experiment 1?

(1) 3/8  (2) 8/3  (3) 1/1  (4) 4/1  (5) 1/4

This is problem 21-8 from the text.

After each contact the charge distributes itself equally on each sphere. So the result of experiment 1 is

A: $+4Q \rightarrow +2Q \rightarrow +2Q$
B: $-6Q \rightarrow -6Q \rightarrow -2Q$
C: $0 \rightarrow +2Q \rightarrow -2Q$

Hence the force between A and B (at separation $R$) is

$$F_i = -4Q^2 / (4\pi\epsilon_0 R^2).$$
The result of experiment 2 is 
A: $+4Q \rightarrow +4Q \rightarrow +Q/2$
B: $-6Q \rightarrow -3Q \rightarrow -3Q$
C: $0 \rightarrow -3Q \rightarrow +Q/2$

Hence the force between A and B (at separation $R$) is

$$F_2 = -\frac{3}{2} \frac{Q^2}{(4\pi\varepsilon_0R^2)}$$

The ratio is $F_2 / F_1 = \frac{3}{8}$

**Problem 3**

Four particles form a square as shown. The charges are $q_1 = q_4 = Q$ and $q_2 = q_3 = q$. What is $Q/q$ if the net electrostatic force on particles 1 and 4 is zero?

(1) $-\sqrt{8}$  (2) $+\sqrt{8}$  (3) $-2$  (4) $+2/2$  (5) $-1$

This is problem 21-10 in the text, which was solved in class on Aug. 26.

Note first that charges 1 and 4 must repel one another, with the force directed along the diagonal between them. So the only chance for zero force on 1 is if 2 and 3 have the same charge, which is opposite to that of 1. Because these two forces are equal in magnitude and perpendicular to one another, the magnitude of their vector sum is $\sqrt{2}$ times the magnitude of either one. If we remove the common overall factor of $Q / (4\pi\varepsilon_0a^2)$ then the condition for zero force on 1 is

$$\frac{Q}{2} + \frac{\sqrt{2}q}{1} = 0$$

Solving for the ratio gives $Q/q = -2\sqrt{2} = -\sqrt{8}$. 
Problem 4

The figure shows a plastic ring of radius \( R = 50.0 \text{ cm} \). Two small charged beads are on the ring. Bead 1 of charge +2.00 \( \mu \text{C} \) is fixed in place at the left side; bead 2 of charge +6.00 \( \mu \text{C} \) can be moved along the ring. The two beads produce a net electric field of magnitude \( E = 2.00 \times 10^5 \text{ N/C} \) at the center of the ring. What is the angle \( \theta \) in degrees?

![Diagram of a plastic ring with two charged beads](image)

(1) 67.8 (2) 22.2 (3) 20.7 (4) 69.3 (5) 53.1

This is problem 22-16 in the text, which was discussed in class on Aug. 31.

Note that \( q_2 = 3 q_1 \), so each of the electric fields will contain a common factor of \( E_i = q_i / (4\pi\varepsilon_0 R^2) \approx 7.193 \times 10^4 \text{ N/C} \approx 0.3597 E \). In this notation the two electric field vectors are,

\[
\vec{E}_1 = +E_i \hat{i}, \quad \vec{E}_2 = 3E_i \left( -\cos(\theta) \hat{i} - \sin(\theta) \hat{j} \right)
\]

The vector sum is

\[
\vec{E}_{\text{tot}} = E_i \left( [1 - 3\cos(\theta)] \hat{i} - 3\sin(\theta) \hat{j} \right)
\]

The magnitude of this field is

\[
E_{\text{tot}} = E_i \sqrt{[1 - 3\cos(\theta)]^2 + 9\sin^2(\theta)} = E_i \sqrt{10 - 6\cos(\theta)}
\]

We now set this equal to \( E \), divide by \( E_i \) and square both sides

\[
10 - 6\cos(\theta) = \frac{E^2}{E_i^2} \approx 7.730
\]

Solving first for \( \cos(\theta) \) and then for \( \theta \) gives

\[
\theta \approx \cos^{-1}(0.3783) \approx 67.8^\circ
\]
Problem 5

The figure shows three circular arcs centered on the origin. On each arc, the uniformly distributed charge is given in terms of $Q = 2.00 \, \mu\text{C}$. The radii are given in terms of $R = 10.0 \, \text{cm}$. What is the value of the x component of the electric field (in $10^6 \, \text{N/C}$) at the origin?

![Figure showing three circular arcs centered on the origin.]

\[ (1) \ +1.14 \quad (2) -1.14 \quad (3) \ 0 \quad (4) \ +1.80 \quad (5) -1.80 \]

This is problem 22-25 in the text.

Note first that the outer arc of $+9Q$ at $3R$ cancels the effect (at the origin) of the middle arc of $-4Q$ at $2R$. So the net field comes from the inner arc of $+Q$ at $R$. We can express this as an integral over the angle $\theta$ measured from the positive x axis,

\[ E = \frac{2}{\pi} \int_{\frac{\pi}{2}}^{\pi} d\theta \frac{Q}{4\pi\varepsilon_0 R^2} (-\cos(\theta)\hat{i} - \sin(\theta)\hat{j}) \]

\[ = \frac{Q}{2\pi\varepsilon_0 R^2} (-\sin(\theta)\hat{i} + \cos(\theta)\hat{j}) \bigg|_{\frac{\pi}{2}}^{\pi} = \frac{Q(\hat{i} - \hat{j})}{2\pi\varepsilon_0 R^2} \]

Hence the x component is $E_x = \frac{Q}{2\pi^2\varepsilon_0 R^2} \approx 1.14 \times 10^6 \, \text{N/C}$.

Problem 6

A nonconducting rod of length $L = 8.15 \, \text{cm}$ has a charge $-q = 4.23 \, \text{fC}$ uniformly distributed along its length. What is the magnitude of the electric field (in $10^{-3} \, \text{N/C}$) produced at point $P$ a distance $a = 12.0 \, \text{cm}$ from the rod?

![Rod with charge and point P labeled.]

\[ (1) \ 1.57 \quad (2) \ 0.94 \quad (3) \ 2.64 \quad (4) \ 8.23 \quad (5) \ 2.64 \]
This is problem 22-31 from the text.

To avoid sign confusion, let’s define $Q = +4.23 \, \text{rm fC}$. The $x$-component of the electric field is

$$E_x = \int_0^L \frac{dxQ}{4\pi\varepsilon_0 (x-a-L)^2}$$

$$= \frac{Q}{4\pi\varepsilon_0 L} \left( \frac{1}{a} - \frac{1}{a+L} \right) = \frac{Q}{4\pi\varepsilon_0 a(a+L)} \approx 1.57 \, \text{N/C}$$

**Problem 7**

Two very large, nonconducting plastic sheets, each 10 cm thick, carry uniform charge densities $\sigma_1$, $\sigma_2$, $\sigma_3$, and $\sigma_4$ on their surfaces as shown in the figure. These surface charge densities have the values (in μC/m$^2$): $\sigma_1 = -6$, $\sigma_2 = 4$, $\sigma_3 = 2$, and $\sigma_4 = -1$. Find the magnitude of the electric field at point A (or B), far from the edges of these sheets.

![Diagram of two plastic sheets with charge densities](image)

(1) $5.6 \times 10^4 \, \text{N/C}$
(2) $1.7 \times 10^5 \, \text{N/C}$
(3) $2.8 \times 10^5 \, \text{N/C}$
(4) $7.3 \times 10^5 \, \text{N/C}$
(5) $0 \, \text{N/C}$

This is example “23-D” covered in lecture. Also similar to homework problem 23-35.

The magnitude of the electric field from a sheet of charge is given by: $E_i = \sigma_i / 2\varepsilon_0$

The field at point A is given by:

$$E_A = \left( E_1 - E_2 - E_3 + E_4 \right) \hat{i} = (6-4-2+1) \frac{10^{-6}}{2\varepsilon_0} = 5.6 \times 10^4 \, \text{N/C}$$

The field at point B is given by:

$$E_A = \left( -E_1 + E_2 - E_3 + E_4 \right) \hat{i} = (-6+4-2+1) \frac{10^{-6}}{2\varepsilon_0} = 1.7 \times 10^5 \, \text{N/C}$$
Problem 8

A very small object with mass $8 \times 10^{-9}$ kg and positive charge $6 \times 10^{-9}$ C is projected directly toward a very large insulating sheet of positive charge that has a uniform surface charge density of $5 \times 10^{-8}$ C/m$^2$. The object is initially 0.5 m from the sheet, and its initial speed is 40 m/s. What is the distance of closest approach to the sheet?

(1) 0.12 m  (2) 0.21 m  (3) 0.31 m  (4) 0.38 m  (5) 0 m

This is similar to example “23-E” covered in lecture, and also homework problem 23-41.

Since the electric field from a large sheet of charge is uniform ($E = \sigma / 2\varepsilon_0$), the acceleration will be constant. At the turn-around point the speed, $v$, will be zero.

Therefore:

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$v_0^2 = 2 \frac{q \sigma}{m 2\varepsilon_0}(x_0 - x)$$

$$\Rightarrow x = x_0 - \frac{mv_0^2\varepsilon_0}{q\sigma} = 0.12 \text{ m}$$

Problem 9

A hollow, conducting sphere with an outer radius of 0.5 m and an inner radius of 0.25 m has a uniform surface charge density of $+3 \times 10^{-6}$ C/m$^2$. A charge of $-3 \mu$C is now introduced into the center of the cavity inside the sphere. Calculate the magnitude of the electric field just outside the sphere.

(1) $2.3 \times 10^5$ N/C  (2) $4.5 \times 10^5$ N/C  (3) $3.4 \times 10^5$ N/C  (4) $1.1 \times 10^5$ N/C  (5) 0 N/C
This is example “23-H” covered in lecture.

Since the shell is conducting the electric field must be zero inside it. So the inner surface needs a charge of $+3\mu C$. This is borrowed from the shell, so the outer surface has an additional $+3\times 10^{-6}$ C/m². Since a spherical shell is symmetric, the field outside has exactly the same form of a point charge located at the center of the shell. So we just need to find the total charge on the shell to use as input to the field calculation:

$$q_{\text{tot}} = q + \sigma (4\pi r^2) = -3\times 10^{-6} + (3\times 10^{-6}) (4\pi \times 0.5^2) = 6.4 \times 10^{-6} \text{ C}$$

The field is given by:

$$E = \frac{q_{\text{tot}}}{4\pi \varepsilon_0 r^2} = 2.3 \times 10^5 \text{ N/C}$$

**Problem 10**

What is the net enclosed charge in the shown cube if the electric field is given by $E = 4 \hat{1} + (y-2) \hat{j}$ (or $E = 4 \hat{1} + 2(y-2) \hat{j}$) and the cube has a side length of 4?

(a) $64 \varepsilon_0$ (b) $0$ (c) $4 \varepsilon_0$ (d) $128 \varepsilon_0$ (e) $32 \varepsilon_0$

This is like homework problem 23-11, but with a different functional dependence.

Because of the direction of the electric field, the flux could only be non-zero for the top-bottom and left-right faces. However, the left-right faces have a constant field pointing perpendicular to their surfaces, and so no net flux out of the cube (as much flux going in as out). The relevant component of the field on the bottom face is $E = -2 \hat{j}$ (or $E = -4 \hat{j}$), and the field on the top face is $E = 2 \hat{j}$ (or $E = 4 \hat{j}$). In other words, the field points out from the cube in both cases and is considered positive. So the net charge enclosed is:

$$q_{\text{enc}} = \varepsilon_0 \Phi = \varepsilon_0 \Phi_{\text{bottom}} + \varepsilon_0 \Phi_{\text{top}} = \varepsilon_0 E_{\text{bottom}} \cdot A_{\text{top}} + \varepsilon_0 E_{\text{bottom}} \cdot A_{\text{top}}$$

$$= \varepsilon_0 2 (2)(4)^2 = 64 \varepsilon_0 \quad \text{or} \quad = \varepsilon_0 2 (2)(4)^2 = 128 \varepsilon_0$$
Problem 11

Two identical raindrops, each with a radius of 1 mm and a charge of $-2.8 \times 10^{-12}$ C, (or $-1.4 \times 10^{-12}$ C) collide and merge into one larger and spherical raindrop. What is the potential at the surface of this merged raindrop if its charge is distributed over its volume?

(1) $-40$ V  (2) $-25$ V  (3) $-50$ V  (4) $-20$ V  (5) $-12.5$ V

This is example “24-D” covered in class.

The merged raindrop will have twice the volume of a single raindrop. Therefore its radius can be determined from equating the volume:

$$V' = 2V$$

$$\frac{4}{3} \pi R'^3 = 2 \frac{4}{3} \pi R^3$$

$$\Rightarrow R' = (2)^{1/3} R$$

The charge will be twice the charge of one drop: $q' = 2q$

The potential outside (or on the surface) of a sphere is the same as that of a point charge at the center of the sphere from symmetry, where the charge is the total charge enclosed up to that radius. So:

$$V = \frac{q'}{4 \pi \varepsilon_0 R'} = \frac{2}{2^{1/3}} \frac{q}{4 \pi \varepsilon_0 R} = -40V \quad \text{or} \quad -20V$$

Problem 12

In a certain region of space, the electric potential is $V(x, y, z) = Ax^2 - Bxy + Cz$, where $A, B, \text{and} C$ are positive constants. At which of the following points is the electric field equal to zero?

(1) $x = 2AC/B^2$, $y = C/B$, $z = 0$
(2) $x = 0$, $y = 0$, $z = 0$
(3) $x = A/(B - C)$, $y = 1$, $z = 0$
(4) $x = -2BC/A^2$, $y = -C/A$, $z = 0$
(5) $x = -2AC/B^2$, $y = -C/B$, $z = 0$
This is similar to example “24-I” covered in class.

The electric field can be determined from the potential by:

\[ E_x = \frac{\partial V}{\partial x} = By - C \]
\[ E_y = \frac{\partial V}{\partial y} = -2Ay + Bx \]
\[ E_z = \frac{\partial V}{\partial z} = 0 \]

These are zero when:

\[ By - C = 0 \Rightarrow y = C / B \]
\[ -2Ay + Bx = 0 \Rightarrow x = 2Ay / B = 2AC / B^2 \]
\[ z = \text{anything} \]

**Problem 13**

How much work is required (as measured in eV) to assemble three electrons into an equilateral triangle with a side length of $10^{-9}$ m? The electrons are initially separated infinitely far from one another.

(1) 4.3 eV  (2) 1.4 eV  (3) $4.3 \times 10^9$ eV  (4) $2.3 \times 10^{-19}$ eV  (5) 2.9 eV

This is like example “24-F” covered in lecture, and similar to problem 24-74 on the homework.

The work required equals the potential energy of the final configuration, which is the sum of the potential energy between every pairwise combination of electrons (there are three of them: 1-2, 1-3, and 2-3):

\[ W = U = \frac{e^2}{4\pi\varepsilon_0 s} + \frac{e^2}{4\pi\varepsilon_0 s} + \frac{e^2}{4\pi\varepsilon_0 s} = \frac{3e^2}{4\pi\varepsilon_0 s} = 6.9 \times 10^{-19} \text{ J} \]

To convert from J to eV recall that 1 eV = $1.6 \times 10^{-19}$ J (charge of electron times 1 V). So the answer is 4.3 eV
Problem 14

A charged glass rod is bent into the shape of a semicircle of radius $R$ and has a linear charge density of $\lambda$. What is the potential at point C at the center of the semicircle? (Take $V = 0$ infinitely far away.)

\[
(1) \frac{\lambda}{4\varepsilon_0} \quad (2) \frac{\lambda}{4\pi\varepsilon_0 R} \quad (3) -\frac{\lambda}{2\pi\varepsilon_0 R} \quad (4) \frac{\lambda}{2\varepsilon_0} \quad (5) 0
\]

This is similar to homework problem 24-25.

The potential at point C can be integrated from the charge distributed along the semicircle:

\[
V = \frac{dq}{4\pi\varepsilon_0 R} = \frac{\lambda ds}{4\pi\varepsilon_0 R} = \frac{\lambda R d\theta}{4\pi\varepsilon_0 R} = \frac{\lambda}{4\varepsilon_0}
\]

Problem 15

The line labelled 1 on figure (a) gives the charge $q$ that can be stored on capacitor 1 versus the electric potential $V$ set up across it. The vertical scale is set by $q_s = 16.0$ $\mu$C, and the horizontal scale is set by $V_s = 2.0$ V. The lines labelled 2 and 3 are similar plots for capacitors 2 and 3, respectively. Figure b shows a circuit with these three capacitors and a 6.0 V battery. What is the charge (in $\mu$C) stored on capacitor 2 in that circuit?

\[
(1) 12 \quad (2) 24 \quad (3) 16 \quad (4) 32 \quad (5) 8
\]
This is problem 25-16 in the text, which was solved in class on Sept. 18.

First use the graph to find the three capacitances

\[ C_1 = \frac{3}{4} \times \frac{q_s}{V_s} , \quad C_2 = \frac{1}{2} \times \frac{q_s}{V_s} , \quad C_3 = \frac{1}{4} \times \frac{q_s}{V_s} \]

Now compute the equivalent capacitance. Because \( C_2 \) and \( C_3 \) are in parallel, they are equivalent to

\[ C_{23} = \frac{3}{4} \times \frac{q_s}{V_s} . \]

Because \( C_1 \) has the same capacitance we can recognize the voltage across \( C_{23} \) as \( \frac{1}{2} V \).

Hence the charge on \( C_2 \) is

\[ Q_2 = C_2 \times \frac{1}{2} V = \frac{V}{4V_s} \times q_s = 12 \ \mu C \]

**Problem 16**

The parallel plates in a capacitor, with plate area of 8.50 cm\(^2\) and an air-filled separation of 3.00 mm, are charged by a 6.00 V battery. They are then disconnected from the battery and pulled apart (without discharge) to a separation of 8.00 mm. Neglecting fringing, what is the potential difference between the plates?

(1) 16 V (2) 6 V (3) 2 V (4) 12 V (5) 8 V

This is problem 25-37 in the text and was worked in class on Sept. 21.

First note that the charge on the capacitor remains the same before and after the plates are separated. On the other hand, the capacitance changes from \( C \) to \( C' = \frac{3}{8} C \).

Hence the new voltage is

\[ V' = \frac{Q}{C'} = \frac{8}{3} \times \frac{Q}{C} = \frac{8}{3} V = 16 \ V . \]

**Problem 17**

The space between two concentric conducting spherical shells of radii \( b = 1.70 \) cm and \( a = 1.20 \) cm is filled with a substance of dielectric constant \( \kappa = 23.5 \). A potential difference of \( V = 73.0 \) V is applied across the inner and outer shells. What is the free charge (in nC) on the inner shell?

(1) 7.78 (2) 7.45 (3) 0.33 (4) 2.29 (5) 3.24
This is problem 25-55 in the text and was worked in class on Sept. 21.

First compute the capacitance
\[ C = \frac{4\pi \kappa \varepsilon_0}{a-b} \approx 106.63 \text{ pC} \]
Hence the charge is \( Q = CV \approx 106.63 \text{ pC} \times 73.0 \text{ V} \approx 7.784 \text{ nC} \).

**Problem 18**

A 9.00 V battery is connected to a resistive strip that consists of three sections with the same cross-sectional areas but different conductivities. The graph shows the electric potential \( V(x) \) versus position \( x \) along the strip. The horizontal scale is set by \( x_s = 8.00 \text{ mm} \). Section 3 has conductivity \( 3.00 \times 10^7 (\Omega \text{m})^{-1} \). What is the conductivity of section 1 in units of \( 10^7 (\Omega \text{m})^{-1} \)?

(1) 6 (2) 4 (3) 1 (4) 2 (5) \( \frac{1}{2} \)

This is problem 26-26 in the text and worked in class on Sept. 25.

First note that, because the current through each of the three portions is the same, the formula \( R = V/i \) implies that the ratios of the first two resistances to the third are
\[ \frac{R_1}{R_3} = \frac{V_1}{V_3} = \frac{2}{3}, \quad \frac{R_2}{R_3} = \frac{V_2}{V_3} = \frac{4}{3} \]

Now use the formula \( R = L / (\sigma A) \) to conclude
\[ \frac{R_1}{R_3} = \frac{L_1}{L_3} \times \frac{\sigma_3}{\sigma_1} = \frac{4 \sigma_3}{3 \sigma_1}, \quad \frac{R_2}{R_3} = \frac{L_2}{L_3} \times \frac{\sigma_3}{\sigma_2} = \frac{1 \sigma_3}{3 \sigma_2} \]
Now just compare the two different results for the ratio \( R_1/R_3 \) to conclude \( \sigma_1 = 2 \sigma_3 = 6 \times 10^7 (\Omega \text{m})^{-1} \).
Problem 19

A certain wire has resistance $R$. What is the resistance of a second wire, made of the same material, that is half as long and has half the diameter?

(1) $2R$ (2) $R$ (3) $R/2$ (4) $4R$ (5) $R/4$

This is problem 26-20 in the text and was worked in class on Sept. 23.

Note that the area of a circle is $A = \pi(d/2)^2$, so having half the diameter means having one fourth the area. Hence the second resistor has

$$R_2 = \frac{\rho L}{\frac{1}{4}A} = 2 \frac{\rho L}{A} = 2R$$

Problem 20

Wire C and D are made from different materials and have length $L_C = L_D = 1.0$ m. The resistivity and diameter of wire C are $2.0 \times 10^{-6}$ m and 1.00 mm, and those of wire D are $1.0 \times 10^{-6}$ m and 0.50 mm. The wires are joined as shown, and a current of 2.0 A is set up in them. What is the rate at which energy is dissipated in wire C?

![Diagram of wires C and D](image)

(1) 10 W (2) 20 W (3) 8 W (4) 32 W (5) 16 W

This is 26-51 in the text and was worked in class on Sept. 25.

The resistance of C is

$$R_C = \frac{\rho_C L_C}{\frac{\pi}{4}d_C^2} = \frac{8}{\pi} \Omega$$

Now compute the power dissipated in this wire

$$P_C = i^2 R_C = \frac{32}{\pi} J \approx 10.1 J$$