Exams 2 Solutions

Note that there are several variations of some problems, indicated by choices in parentheses.

**Problem 1:** Two light bulbs have resistances of $100 \Omega$ and $300 \Omega$. They are connected in parallel across a $120V$ line. What is the total power dissipated by the two bulbs (or the $100 \Omega$ or $300 \Omega$ bulbs)?

This problem was similar to Exercise 26-C done in class.

$$P = IE = E^2 R^{-1} = E^2 (R_1^{-1} + R_2^{-1}) = (120)^2 (\frac{1}{100} + \frac{1}{300})W = 192W;$$

or 144 W and 48 W individually.

**Problem 2:** In the circuit shown in the figure both batteries have insignificant internal resistance and the idealized ammeter reads $I_1 = 4.0A$ in the direction shown. Find the E.M.F. of the battery (a negative answer indicates that the E.M.F. polarity is opposite to what is shown).

This problem was similar to Exercise 26-D done in class.

$$75.0V = \mathcal{E} = R_1 I_1 + R_2 (I_1 - I_2); \quad -\mathcal{E} = R_1 I_2 + R_2 (I_2 - I_1); \quad I_1 = 4.0A; \quad R_1 = 12.0\Omega; \quad R_2 = 48.0\Omega; \quad R_3 = 15.0\Omega;$$

We note that the first equation in (1.2) implies $I_2 = \frac{1}{R_3}((R_1 + R_2)I_1 - \mathcal{E})$, which can be immediately put into the 2nd equation to yield $\mathcal{E}$, as,

$$\mathcal{E} = R_2 I_1 - (R_2 + R_3)I_2 = \frac{R_2 I_1}{R_2} - \frac{R_2 + R_3}{R_3}((R_1 + R_2)I_1 - \mathcal{E}) = -24.5625V;$$

**Problem 3:** A capacitor with an initial potential difference of $V(0)=150V$ is discharged through a resistor when a switch between them is closed at $t=0$. At $t=10.0$, the potential difference across the capacitor is $V_1 = 1.5V$. What is the potential difference across the capacitor at $t=20s$?

This is based on homework problem 27.64
Recalling that \( q(t) = CV(t) = q_0 + q_1 e^{-t/\tau} \) in which the discharging-conditions \( q(0) = CE = CV(0) \) and \( q(\infty) = 0 \) give \( q_0 = 0 \) and \( q_1 = CV(0) \), we have \( V(t) = V(0)e^{-t/\tau} \). The third condition \( V(10.0s) = 1.5V = (150V)e^{-(10.0s)/\tau} \) determines \( \tau \) to be \( \tau = 10.0s / \left(-\ln \frac{15}{150}\right) = 2.1715s \), which determines the voltage at all times, and so we calculate directly \( V(20.0s) = (150V)e^{-(20.0s)/(2.1715s)} = 0.015V \).

**Problem 4:** In the figure \( \mathcal{E} = 14V \), \( R_1 = R_3 = 1.00\Omega \), and \( R_2 = 2\Omega \). What is the potential difference \( V_A - V_B \)?

This is based on a Ch.27 homework problem (27.35), and Exercise 26-F done in class.

Let the current through the EMF be \( i \), that through \( R_1 \) be \( i_1 \), and the current through \( R_2 \) be \( i_2 \).

Junction rules:
\[
i = i_1 + i_2 \]
\[
i_1 = i_2 + i_3 \Rightarrow i_3 = i_1 - i_2
\]

Left loop rule, substituting \( R_1 = 1 \Omega \) and \( R_2 = 2 \Omega \):
\[
14 - i_1 - 2i_2 = 0
\]
\[
i_1 = 14 - 2i_2
\]

Zig-zag loop rule:
\[
14 - i_1 - i_3 - i_1 = 0
\]
\[
14 - 2i_1 -(i_1 - i_2) = 0
\]
\[
14 - 3i_1 + i_2 = 0
\]
\[
14 - 3(14 - 2i_2) + i_2 = 0
\]
\[
-28 + 7i_2 = 0 \Rightarrow i_2 = 4A
\]
\[
\Rightarrow i_1 = 14 - 2i_2 = 6A
\]
\[
\Rightarrow i = i_1 + i_2 = 10A
\]

Now \( V_A - V_B = i_1 R_1 = (6A)(1\Omega) = 6V \)
Problem 5: A proton travels through uniform magnetic and electric fields. The magnetic field is \( \mathbf{B} = -2.50 \hat{i} \) mT. At one instant the velocity of the proton is \( \mathbf{v} = 2000 \hat{j} \) m/s. At that instant what is the net force acting on the proton if the electric field is \( 4.00 \hat{k} \) V/m?

This is problem 28.10, which was assigned in homework

\[
\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = q(E\hat{k} + v\hat{j} \times B(-\hat{i})) = \frac{q(E + vB)\hat{k}}{1.602 \times 10^{-19} (4.00 + (2000)(2.50 \times 10^{-3})) \hat{k} N} = 1.442 \times 10^{-18} \text{ N}\hat{k}
\]

(1.5)

Problem 6: In the figure a charged particle moves into a region of uniform magnetic field \( \mathbf{B} \), goes through half a circle, and then exits that region. The particle is either a proton or an electron (you must decide which). It spends 130 ns in the region. What is the magnitude of \( \mathbf{B} \)?

This is problem 28.26, which was assigned in homework

\[
\mathbf{F} = F_x \hat{i} = q(v(-\hat{j})) \times \hat{B} = qvB(-\hat{i}) \rightarrow qvB > 0 \rightarrow \begin{cases} q = e = 1.602 \times 10^{-19} \text{ C}; \\ m = m_p = 1.672 \times 10^{-27} \text{ kg}; \end{cases}
\]

(1.7)

We immediately notice that the particle velocity \( \mathbf{v} = v(-\hat{j}) \) is deflected in the \( -\hat{i} \) direction by the magnetic field \( \mathbf{B} = B\hat{k} \) pointing out of the page. Looking at the vector-value of the forces,

\[
\mathbf{F}_x = F_x(-\hat{i}) = q\mathbf{v} \times \mathbf{B} = q(v(-\hat{j})) \times (B\hat{k}) = qvB(-\hat{i}) \rightarrow qvB > 0 \rightarrow \begin{cases} q = e = 1.602 \times 10^{-19} \text{ C}; \\ m = m_p = 1.672 \times 10^{-27} \text{ kg}; \end{cases}
\]

While in the circular region, the charged particle has constant speed \( v = |\mathbf{v}| \), and maintains this velocity. This is because the magnetic force \( \mathbf{F}_x \) (due to magnetic field \( \mathbf{B} = B\hat{k} \) and velocity \( \mathbf{v} = v \cdot d\theta = R\omega \cdot d\theta \) ) is perpendicular to the displacement, and thus does no work. Letting the half-circle have radius \( R \), and letting the particle have charge \( q \), we have,

\[
dK = dW = \mathbf{F}_x \bullet d\mathbf{s} = (q\mathbf{v} \times \mathbf{B}) \bullet Rd\theta = qvBR(\hat{r} \cdot d\hat{\theta}) = qvBR(0) = 0 \rightarrow dK = dW = 0 \rightarrow K_i = K_f = K_a
\]

(1.8)

Hence, no kinetic energy is added or subtracted to the particle of mass \( m \). The velocity \( \frac{\mathbf{v}}{v} \) is of constant magnitude. In the circular trajectory, Newton’s 2nd Law then is,

\[
\sum F = m_p a = m_p \frac{-v^2}{R} = -\mathbf{F}_x \rightarrow m_p \frac{-v^2}{R} = -q\mathbf{v} \times \mathbf{B} \rightarrow m_p \frac{-v^2}{R} = -qv\hat{i} \times \hat{B} \rightarrow m_p \frac{-v^2}{R} = qvB \sin\theta \rightarrow B = \frac{m_p v}{Rq}
\]

(1.9)

In (1.9), the velocity is given by \( v = \frac{2\pi R}{T} = \frac{\pi R}{(1/2)t} \), so,
In the last steps of (1.10), we illustrate the units\(^1\) of magnetic field.

**Problem 7:** In a certain cyclotron a proton moves in a circle of radius 0.5 m. The magnitude of the magnetic field is 1.2 T. What is the kinetic energy of the proton in million electron-volts (MeV)?

This is problem 28-38, which was assigned in homework.

\[
K = \frac{1}{2}mv^2 = \frac{1}{2}m(\omega_e R)^2 = \frac{1}{2}m\left(\frac{eB}{m} R\right)^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{kg})\pi \left(\frac{1.602 \times 10^{-19} \text{C}}{130 \times 10^{-6} \text{s}}\right)^2 = 0.252 \frac{\text{kg}}{\text{C} \cdot \text{s}} = 0.252 \frac{N}{\pi \cdot \text{C}} = 0.252 \text{T} \text{MeV}; \quad (1.11)
\]

**Problem 8:** The figure shows a wood cylinder of mass \(m = 0.250 \text{ kg}\) and length \(L = 0.100 \text{ m}\), with \(N = 10\) turns of wire wrapped around it longitudinally, so that the plane of the wire contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle \(\theta\) to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.500 T, what is the least current \(i\) through the coil that keeps the cylinder from rolling down the plane?

This is problem 28-51, which was assigned in homework.

Let the wood-cylinder be of mass \(m\), and have moment of inertia \(I\). Newton’s 2\(^{\text{nd}}\) Law for translational equilibrium between the force of static\(^3\) friction \(f\) and magnetic force \(F_B\) and rotational equilibrium between the torque of static friction \(Rf\) and magnetic torque \(\tau_B\) is,

\[
ma = m \cdot 0 = 0 = \sum F = f - mg \sin \theta; \quad I\alpha = 0 = \sum \tau = \sum_{i=1}^{4} R F_i \sin \theta_i - fR = \sum_{i=1}^{4} R F_i \sin 90^\circ - fR = \sum_{i=1}^{4} R F_i - Rf; \quad (1.13)
\]

\(^1\) A tesla is a unit of force per unit velocity per unit charge; essentially the units of electric field divided by velocity.

\(^2\) The radius of the wooden cylinder is \(R\), but the wooden-material may be inhomogeneous, so assume \(I \neq \frac{1}{2} mR^2\).

\(^3\) CAUTION: The force of static friction is a reaction force, and its magnitude is unknown. The maximum value the force of friction could take on, if we knew the static-friction-coefficient \(\mu_s\), is \(\max f = \mu_s N\), where \(N\) is a reaction force which has a known contribution \(mg \cos \theta\) from gravity, but an unknown contribution from the net magnetic force.
PHY2049 Fall 2015 – Acosta, Woodard

The problem is to find \( F_i, F_2, F_3, F_4 \): the magnetic forces upon the four sections of the square-loop shown in the Figure. The magnitude of the force upon a wire of length \( l \) making angle \( \theta \) with a magnetic field \( B \) carrying current \( I = \frac{dq}{dt} \) is given by \( F = I l B \sin \theta \). There are \( N \) such wires producing identical and superimposing forces. Thus, let the wood-cylinder be of radius \( R \). Sections 1 and 3 are of length \( 2R \), while Sections 2 and 4 are of length \( L \). Then,

\[
\begin{align*}
R_1 F_1 &= R_1 l B \sin \theta N = R(1)(2R)B \sin(90^\circ - \theta)N = +2i R^2 B \cos \theta N; \\
R_2 F_2 &= R_2 l B \sin \theta N = R(1)(2R)B \sin(270^\circ - \theta)N = -2i R^2 B \cos \theta N = -R_1 F_1; \\
R_3 F_3 &= R_3 l B \sin \theta N = R(1)(2R)B \sin(270^\circ - \theta)N = -2i R^2 B \cos \theta N = -R_1 F_1; \\
R_4 F_4 &= R_4 l B \sin \theta N = R(1)(2R)B \sin(90^\circ - \theta)N = +2i R^2 B \cos \theta N = R_1 F_1; \\
\end{align*}
\]

(1.14)

Combining (1.13) and the explicit forces (1.14), and noting the simplification \( R_1 F_1 = -R_2 F_2 \), we have,

\[
\frac{f}{mg \sin \theta} = \sum_{i=1}^4 R_i F_i = R_1 F_1 + R_2 F_2 + R_3 F_3 + R_4 F_4 = RiLB \sin \theta N + RiLB \sin \theta N + 0 = 2RiLB \sin \theta N; \quad (1.15)
\]

Solving (1.15) for \( i \), we have,

\[
Rmg \sin \theta = 2RiLB \sin \theta N \quad \text{solve for } i = \frac{mg}{2LBN} = \frac{(0.250)kg(9.81 m/s^2)}{2(0.100m)(0.500 \frac{N}{m/s/C})(10.0)} = 2.453 \frac{C}{s}; \quad (1.16)
\]

Afterword: We note that the net torque \( \tau_B \) due to the four branches of the loop has the property,

\[
\tau_B = \sum_{i=1}^4 R_i F_i = 2RiLB \sin \theta = \left| B \right| 2R \left| L \cdot N \right| \sin \theta = \left| B \right| A \cdot l \sin \theta = \left| B \right| \left| \mu \right| \sin \theta = \left| B \times \mu \right|; \quad (1.17)
\]

We introduced the magnetic dipole moment vector, \( \mu = i \hat{A} = iN \hat{n} = iNa \hat{n} \), where \( \hat{n} = \hat{k} \cos \theta + \hat{j} \sin \theta \) is the plane-normal defining the vector-area \( A = N \hat{a} = Na \hat{n} = N2LR \hat{n} \). Recall, also, that we encountered vector area in our study of the flux that naturally occurred in Gauss’s law.

**Problem 9:** The figure shows, in cross section, two long straight wires held against a plastic cylinder of radius \( R = 20cm \). Wire 1 carries current \( i_1 = 60mA \) out of the page and is fixed in place at the left side of the cylinder. Wire 2 carries current \( i_2 = 40mA \) out of the page and can be moved around the cylinder. At what (positive) angle \( \theta \) should wire 2 be positioned such that, at the origin, the net magnetic field due to the two currents has magnitude \( B = 80nT \)?

This is problem 29-34, which was worked in class on Oct.14
The two magnetic fields decompose as \( \hat{B}_1 = B_1 \hat{j} \) and \( \hat{B}_2 = B_2 (\sin \theta_2 \hat{i} - \cos \theta_2 \hat{j}) \), so the resultant of this, using Ampère’s law to say \( B_1 = \frac{\mu_0 i_1}{2\pi R} \) and \( B_2 = \frac{\mu_0 i_2}{2\pi R} \) (in which we clearly have \( i_1 = \frac{3}{2} i_2 \)), is,

\[
B = \sqrt{(B_2 \sin \theta_2)^2 + (B_1 - B_2 \cos \theta_2)^2} = \frac{\mu_0 i_1}{2\pi R} \sqrt{\sin^2 \theta_2 + (\frac{3}{2} - \cos \theta_2)^2} = \frac{\mu_0 i_2}{2\pi R} \sqrt{\sin^2 \theta_2 + (\frac{3}{2})^2 + \cos^2 \theta_2 - 2 \frac{3}{2} \cos \theta_2}
\]

\[
= \frac{\mu_0 i_2}{2\pi R} \sqrt{1 + \frac{9}{4} - 3 \cos \theta_2} \iff \theta_2 = \cos^{-1} \left( \frac{3}{4} \right) = \cos^{-1} \left( \frac{13}{4} - \left( \frac{2\pi R}{\mu_0 i_2} \right)^2 \right) = \cos^{-1} \left( \frac{13}{4} - \left( \frac{0.2(0.80 \times 10^{-7})}{2(40 \times 10^{-7})} \right)^2 \right) = 104.4775^\circ.
\]

**Problem 10:** In the figure a long straight wire carries a current \( i_1 = 30.0\,\text{A} \) and a rectangular loop carries current \( i_2 = 20.0\,\text{A} \). Take the dimensions to be \( a = 1.00\,\text{cm} \), \( b = 8.00\,\text{cm} \), and \( L = 30.0\,\text{cm} \). In unit vector notation, what is the force on the loop due to \( i_1 \)?

This is problem 29-41, which was assigned in homework.

**Problem 11:** In the figure, a long straight wire carries a current \( i_1 = 30.0\,\text{A} \) and a rectangular loop carries current \( i_2 = 20.0\,\text{A} \). Take the dimensions to be \( a = 1.00\,\text{cm} \), \( b = 8.00\,\text{cm} \), and \( L = 30.0\,\text{cm} \). In unit vector notation, what is the force on the loop due to \( i_1 \)?

This is problem 29-41, which was assigned in homework.

The horizontal wires: Consider two typical wires, \( a \) and \( b \), a distance \( d \) apart, and carrying respective currents \( i_a \) and \( i_b \). A differential element of force \( d\vec{F}_{ba} \) acts upon wire- \( b \) and is due to wire- \( a \),

\[
\vec{F}_{ba} = \mu_0 i_a \mu_0 i_b \frac{d}{2\pi R^2} \hat{r}.
\]
By the Lorentz force law, the differential element of force $\frac{d\vec{F}}{dx}$ per unit length $dx$ is due to magnetic field\(^4\)
\[ \frac{d\vec{F}}{dx} = i \left( \frac{d\vec{B}}{dx} \right) \]
so we calculate the force per unit length $\vec{F}_{ba} \equiv \frac{d\vec{F}}{dx}$ as,
\[
\vec{F}_{ba} = \frac{d\vec{F}}{dz} = \frac{dq_b \cdot \vec{v}_b \times \vec{B}_a}{dz} = \frac{dq_b \cdot \frac{d\vec{v}_b}{dz} \times \left( \frac{\mu_0 i_a}{2\pi a} (-\hat{j}) \right)}{dz} = \frac{\mu_0 i_a}{2\pi d} \hat{k} \times (-\hat{j}) = \frac{\mu_0 i_a b}{2\pi d} \hat{i}; \tag{1.22}
\]
Evidently, the force between wires $a$ and $b$ is in the $+\hat{i}$ direction, and thus is attractive. Using this result (1.22) upon the two horizontal wires in the Figure (numbered 1 and 3 and note the different coordinates!),
\[
\vec{F}_1 = \int d\vec{F}_1 = \int_0^L \frac{d\vec{F}_1}{dx} dx = \int_0^L \left( \frac{\mu_0 i_a L}{2\pi a} \hat{j} \right) dx = \frac{\mu_0 i_a L}{2\pi a} j; \quad \vec{F}_3 = \int d\vec{F}_3 = \int_0^L \left( \frac{\mu_0 i_a L}{2\pi (a+b)} (-\hat{j}) \right) dx = -\frac{\mu_0 i_a L}{2\pi (a+b)} j; \tag{1.23}
\]
The vertical wires: The total forces upon wires 2 and 4 due to wire 0 (of infinite length) are,
\[
\vec{F}_2 = \int d\vec{F}_2 = \int_{a+b}^{a+b} \frac{d\vec{F}_2}{dy} dy = \int_{a+b}^{a+b} dq_x \cdot \vec{v}_x \times \vec{B}_0 \frac{dx}{dy} dy; \quad \vec{F}_4 = \int d\vec{F}_4 = \int_{a+b}^{a+b} dq_x \cdot \vec{v}_x \times \vec{B}_0 \frac{dx}{dy} dy;
\]
The integrands in (1.24) (i.e., the forces per unit $y$-length) are,
\[
\frac{d\vec{F}_2}{dy} = \frac{dq_x \cdot \vec{v}_x \times \vec{B}_0}{dy} = \frac{i_x \cdot (-\hat{j} \cdot dy) \times \frac{\mu_0 i_a L}{2\pi y} (-\hat{k})}{dy} = \frac{\mu_0 i_a L}{2\pi y} \hat{i}; \quad \frac{d\vec{F}_4}{dy} = \frac{dq_x \cdot \vec{v}_x \times \vec{B}_0}{dy} = \frac{i_x \cdot (+\hat{j} \cdot dy) \times \frac{\mu_0 i_a L}{2\pi y} (-\hat{k})}{dy} = -\frac{\mu_0 i_a L}{2\pi y} \hat{i} \tag{1.25}
\]
Combining (1.24) and (1.25), we have,
\[
\frac{\vec{F}_2}{a+b} = \frac{\mu_0 i_a L}{2\pi y} \hat{i} \ln \frac{a+b}{a}; \quad \frac{\vec{F}_4}{a+b} = -\frac{\mu_0 i_a L}{2\pi y} \hat{i} \ln \frac{a+b}{a} \tag{1.26}
\]
Looking at (1.26), we see $\vec{F}_2 = -\vec{F}_4$, so the superposition of these two forces make no contribution. Hence,
\[
\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \frac{\mu_0 i_a L}{2\pi a} j - \frac{\mu_0 i_a L}{2\pi (a+b)} j = \mu_0 i_a L \left( \frac{1}{2\pi} j \left( \frac{1}{a} - \frac{1}{a+b} \right) \right) \tag{1.27}
\]
\[
= \left( \frac{4\pi \times 10^{-7} \text{ Tm/A}}{2\pi} \right) (30.0 \text{ A})(20.0 \text{ A})(30.0 \text{ cm}) j \left( \frac{1}{1.00 \text{ cm}} - \frac{1}{1.00 \text{ cm} + 8.00 \text{ cm}} \right) = 0.0032 \text{ N} \cdot \hat{j};
\]
\textbf{Problem 11:} The current density $\hat{J}$ inside a long, solid, cylindrical wire of radius $a = 3.1 \times 10^{-3} \text{ m}$ is in the direction of the central axis, and its magnitude varies linearly with radial distance $r$ from the axis according to $\hat{J} = \hat{J}(r) = J_0 (r / a) \hat{k}$, where $J_0 = 310 \text{ A/m}$. What is the magnitude of the magnetic field at $r = a = 1/2$? You may need the Jacobian term $r dr d\theta$ for integration in polar coordinates.
This is problem 29-47, which was assigned in homework.

\textbf{Proof:} from Ampère’s law: $\int \vec{B}_{wire} \cdot d\vec{l} = \mu_0 i_{\text{encl}} \rightarrow \vec{B}_{wire} \cdot \vec{r} = \int ds = \mu_0 i_{\text{encl}} \rightarrow B_{wire} \frac{1}{2\pi d} = \mu_0 i_{\text{encl}} \rightarrow \frac{\mu_0 i_{\text{encl}}}{2\pi d} \hat{r}$. In this problem, $i_{\text{encl}} = i_a$ and (by the right hand rule) $\hat{r} \perp = -\hat{j}$.
The current enclosed by an Ampérian-loop of radius \(0 \leq b \leq a\) is,
\[
i_{\text{encl}} = \int_{\text{wire \ x.s.}} \frac{di}{dA} dA = \int_0^b \int_0^{2\pi} J_0(r/a) \cdot r \cdot d\phi = \frac{2\pi J_0}{a} \int_0^b r^2 dr = \frac{2\pi J_0}{3a} \left( b^3 - 0^3 \right) = \frac{2\pi J_0 b^3}{3a}; (1.28)
\]

The magnetic field at a distance \(b\) from the center of the wire, then, is given from the enclosed current (1.28) via the law of Ampère,
\[
\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{\text{encl}}(b) \rightarrow \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \frac{2\pi J_0 b^3}{3a} \rightarrow B\hat{\phi} = \mu_0 \frac{2\pi J_0 b^3}{3a} \quad \text{solve for } B = \mu_0 \frac{J_0 b^3}{3a}; (1.29)
\]

From (1.29) follows,
\[
B_b = B(\frac{1}{2}a) = \frac{\mu_0 J_0 \cdot (\frac{1}{2}a)^2}{3a} = \left( \frac{1}{12} \mu_0 a J_0 \right) = \frac{1}{12} (4\pi \times 10^{-7} \frac{Tm}{A})(3.1 \times 10^{-3} m)(310 \frac{A}{m}) = 1.0064 \times 10^{-7} T; (1.30)
\]

**Problem 12:** A solenoid that is \(L = 95\text{cm}\) long has a radius \(R = 2.0\text{cm}\) and a winding of \(N = 1200\) turns; it carries a current of \(i = 3.6A\). What is the magnitude of the magnetic field inside the solenoid?

This is problem 29-50, which was assigned in homework.

The magnetic field \(\mathbf{B}\) at the center of a solenoid made of a wire carrying current \(i\) with \(n = N / L\) turns per meter is of magnitude \(B = \mu_0ni\). Therefore, \(B = \mu_0i (N/L) = B = \frac{\mu_0 (N/L)}{1.0064 \times 10^{-7} T} = 5.7144\text{mT}\).

**Problem 13:** A wire loop of lengths \(L = 40\text{cm}\) and \(W = 25\text{cm}\) lies in a magnetic field \(\mathbf{B} = (0.08 \frac{T}{m})\mathbf{y} \cdot \mathbf{k}\). What are the magnitude and direction of the induced E.M.F.?

This is problem 30-12, which was worked in class on Oct.19

\[
\mathcal{E} = -\frac{d}{dt} \int_0^L dy \int_0^W b_0 \mathbf{y} \cdot \mathbf{v} \cdot dx = -b_0 \left( \frac{1}{2} W^2 - \frac{1}{2} 0^2 \right)(L - 0) = -\frac{1}{2} b_0 W^2 L = -1.00\text{mV}; \quad \text{clockwise;}
\]

**Problem 14:** The figure shows a rod of length \(L = 10\text{cm}\) that is forced to move at constant speed \(v = 5\text{m/s}\) along the horizontal rails. The rod, rails and connecting strip at the right form a conducting loop. The rod has a resistance \(R = 0.4\Omega\); the rest of the loop has negligible resistance. A current \(i = 100\text{A}\) through the long straight wire at a distance \(a = 10\text{mm}\) from the loop sets up a nonuniform magnetic field through the loop. At what rate (in \(\mu W\)) is thermal energy generated in the rod?

This is problem 30-33, which was worked in class on Oct.21
The magnetic field \( \mathbf{B} \) due to the long straight wire at a distance \( y \in [a, L + a] \) is \( \mathbf{B} = \frac{\mu_0 i}{2\pi y} \hat{k} \), and the differential vector area through which it fluxes is \( dA(y) = dy(\nu t + x_0) \hat{k} \), and so the E.M.F. generated is \( \mathcal{E} = -\frac{d}{dt} \int \mathbf{B} \cdot dA \), yielding the power \( P = I\mathcal{E} = \frac{\mathcal{E}}{R} \). Putting all this together,

\[
P = \frac{1}{R} \left( -\frac{d}{dt} \int_a^{L+a} (\nu t + x_0) \frac{\mu_0 i}{2\pi y} dy \right)^2 = \frac{1}{R} \left( \frac{\mu_0 iv}{2\pi} \right)^2 \left( \int_a^{L+a} \frac{dy}{y} \right)^2 = \frac{1}{R} \left( \frac{\mu_0 iv}{2\pi} \right)^2 \left( \ln \frac{L + a}{a} \right)^2
\]

(1.32)

**Problem 15**: The current \( i = i(t) \) through a \( L = 4.6H \) inductor varies with time \( t \) as shown in the graph, where the vertical axis scale is set by \( i_s = 8.0A \) and the horizontal axis scale is set by \( t_s = 6.0 \times 10^{-3} \) s. The inductor has a resistance of \( R = 12\Omega \). What is the magnitude of the induced E.M.F. during the time interval

\[2 \times 10^{-3} \text{s} < t \leq 5 \times 10^{-3} \text{s}?
\]

This is problem 30-46, which was assigned in homework.

Recall the definition of inductance: \( L \) is the E.M.F.-magnitude \( \mathcal{E} \) per unit rate-of-change of current \( \frac{di}{dt} \), or \( \Phi_B \) per unit \( i \), and specialize it to the case of \( \frac{di}{dt} = \frac{\Delta i}{\Delta t} \),

\[
L = \frac{\mathcal{E}}{\frac{di}{dt}} = \frac{\Phi_B}{i} ; \quad \frac{di}{dt} = \frac{\Delta i}{\Delta t} ;
\]

(1.33)

Within the time-interval \( 2 \times 10^{-3} \text{s} < t \leq 5 \times 10^{-3} \text{s} \), we have,

\[
\left( \frac{\Delta i}{\Delta t} \right) = \frac{(5.0 - 7.0)A}{(5.0 - 2.0) \times 10^{-7} s} \rightarrow \mathcal{E} = L \left( \frac{\Delta i}{\Delta t} \right) = \frac{(5.0 - 7.0)A}{(5.0 - 2.0) \times 10^{-3} s} = 3.1 \times 10^2 \text{V} ;
\]

(1.34)
**Problem 16:** The switch in the figure is closed on a at time \( t = 0 \). What fraction of the total voltage drop \( E \) occurs across the inductor at time \( t = \frac{2L}{R} \)?

![Inductor circuit diagram](image)

This is problem 30-52, which was worked in class on Oct.23

Recall that an LR-circuit has a current \( i(t) = i_0 + i_1 e^{-t\tau} \), where \( \tau = L / R \). The conditions \( i(0) = 0 \) and \( i(\infty) = E / R \) respectively yield \( i_0 = -i_1 \) and \( i_0 = E / R \), yielding \( i(t) = (E / R)(1 - e^{-t\tau}) \), meaning that at a time of \( t = \frac{2L}{R} = 2\tau \) the inductor’s voltage in units of \( E \) is,

\[
\frac{v_i}{E} = \frac{1}{E} \frac{L}{d} \frac{d}{dt} \int \frac{E}{R} d(t) = \frac{L}{R} \int \left( 0 - \frac{1}{\tau} e^{-t\tau} \right) = \frac{L}{R} \frac{1}{\tau} e^{-\frac{t}{2\tau}} = E \left( 1.353 \right)
\]

**Problem 17:**
An LC circuit has a capacitance of \( 20\mu F \) and an inductance of \( 10 \text{ mH} \). At time \( t = 0 \) the charge on the capacitor is \( 27\mu C \) and the current is \( 80 \text{ mA} \). What is the maximum possible charge in \( \mu \text{C} \) (or what is the maximum possible current)?

\[
U_C = \int_0^q V(C) \cdot dQ = \int_0^q \frac{Q}{C} \cdot dQ = \frac{q^2}{2C}; \quad U_L = \int P \cdot dt = \int \mathbb{E} \cdot d\mathbb{F} = \int L \frac{dI}{dt} dI = \int (LI \cdot dI) = \frac{1}{2} Li^2
\]

Thus, the total energy in the circuit, by energy-conservation, is \( U = U_C + U_L = \frac{1}{2C} q^2 + \frac{1}{2} Li^2 \), yielding a maximum charge on the capacitor of \( \max q_C = U = \frac{1}{2C} q_{\max}^2 \leftrightarrow q_{\max} = \sqrt{2CU} \), which, explicitly, is,

\[
q_{\max} = \sqrt{2CU} = \sqrt{2C \left( \frac{1}{2C} q^2 + \frac{1}{2} Li^2 \right)} = \sqrt{q^2 + LCi^2} = \sqrt{(27\mu C)^2 + (10\text{mH})(20\mu F)(80\text{mA})^2} = 44.822\mu \text{C}
\]

\[
i_{\max} = \sqrt{2U / L} = 100 \text{ mA}
\]

**Problem 18:** A sinusoidally varying source of E.M.F. with an amplitude of \( 10V \) and a cyclic frequency of \( 5\text{GHz} \) is applied across a \( 100\mu \text{H} \) inductor. What is the current amplitude through the inductor?

This is based on a HITT clicker question given in class Oct.30

\[
I_{\max} = V_{\max} \frac{X_L}{\omega_d L} = \frac{V_{\max}}{2\pi f_d L} = \frac{10V}{2\pi(5\text{GHz})(100\mu H)} = \frac{1}{2\pi(5)(10)} \times 10^{-3} = 3.18 \times 10^{-6} \text{ A}
\]
Problem 19: A 218Ω resistor, a \( L = 0.775H \) inductor, and a 6.50\( \mu F \) capacitor are connected in series across a sinusoidally varying source of E.M.F. that has voltage amplitude 31.0\( V \) and a cyclic frequency of 37.5Hz. What is the magnitude of the phase difference between the current in the resistor and the E.M.F.?

This is similar to Example 31-E done in class.

\[
|\phi| = \left| \tan^{-1} \frac{X_L - X_C}{R} \right| = \left| \tan^{-1} \frac{\omega L - (\omega C)^{-1}}{R} \right| = \tan^{-1} \left( \frac{(235 \frac{rad}{s})(0.775H) - ((235 \frac{rad}{s})(6.50 \mu F))^{-1}}{218\Omega} \right) = \left[ -65.234^\circ \right];
\]

Problem 20: A transformer connected to a \( V_p^{RMS} = 120V \) AC line is to supply \( V_s^{RMS} = 12,000V \) for a neon sign. To reduce shock hazard, a fuse is to be inserted in the primary circuit; the fuse is to blow when the R.M.S. current in the secondary circuit exceeds \( i_s^{RMS} = 3.0mA \). What current rating should the fuse in the primary circuit have?

The power delivered into the primary is the same as that on the secondary (or less for realistic transformers)

\[
i_p^{RMS} = \frac{N_s}{N_p} i_s^{RMS} = \frac{V_s^{RMS}}{V_p^{RMS}} i_s^{RMS} = \frac{1.2 \times 10^4V}{120V} (3.0mA) = 300mA;
\]  \hfill (1.40)