1. In the figure, \( q_1 = 300 \mu C \), \( q_2 = 400 \mu C \) and \( q_3 = 500 \mu C \), \( r_{12} = 9 \text{ m} \), \( r_{13} = 12 \text{ m} \). Compute the magnitude of the total force on \( q_3 \).

The charges all have the same sign so the forces are repulsive.

For the force on \( q_3 \) due to \( q_2 \),

\[
F_{23} = k \frac{q_2 q_3}{r_{23}^2} = \left( 9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(400 \mu \text{C})(500 \mu \text{C})}{(15 \text{ m})^2} = 8.0 \text{ N}
\]

Now \( \cos \theta = \frac{x}{r} = \frac{12 \text{ m}}{15 \text{ m}} \) \& \( \sin \theta = \frac{9 \text{ m}}{15 \text{ m}} \)

And \( \vec{F}_{23} = 8.0 \text{ N} \left( \cos \theta \hat{i} - \sin \theta \hat{j} \right) \) (– sign from sketch)

So, \( \vec{F}_{23} = 8.0 \text{ N} \left( \frac{12}{15} \hat{i} - \frac{9}{15} \hat{j} \right) = 6.4 \hat{i} - 4.8 \hat{j} \)

For the force on \( q_3 \) due to \( q_1 \), which is entirely along x,

\[
F_{13} = k \frac{q_1 q_3}{r_{13}^2} = \left( 9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \frac{(300 \mu \text{C})(500 \mu \text{C})}{(12 \text{ m})^2} = 9.38 \text{ N}
\]

\( \vec{F}_{13} = 9.38 \hat{i} \)

Then

\[
\vec{F}_{\text{net}} = \vec{F}_{23} + \vec{F}_{13} = 6.4 \hat{i} - 4.8 \hat{j} + 9.38 \hat{i}
\]

\( \vec{F}_{\text{net}} = 15.78 \hat{i} - 4.8 \hat{j} \)

& \( F_{\text{net}} = \sqrt{(15.78 \text{ N})^2 + (4.8 \text{ N})^2} = 16.5 \text{ N} \)
2. An electron and a proton are separated by a distance of $d = 5.3 \times 10^{-11}$ m. What is the magnitude of the acceleration of the electron immediately after both particles are released from rest?

\[
\begin{align*}
(1) & \quad 9.0 \times 10^{22} \text{ m/s}^2 \\
(2) & \quad 8.2 \times 10^{-8} \text{ m/s}^2 \\
(3) & \quad 4.8 \times 10^{12} \text{ m/s}^2 \\
(4) & \quad 5.6 \times 10^{41} \text{ m/s}^2 \\
(5) & \quad 1.0 \times 10^{13} \text{ m/s}^2
\end{align*}
\]

\[F = m_e a = k \frac{q_e q_p}{r^2} = k \frac{e^2}{r^2} \]

\[a = k \frac{e^2}{m_e r^2} = \frac{(1.6 \times 10^{-19} \text{ C})^2}{(9.1 \times 10^{-31} \text{ kg})(5.3 \times 10^{-11} \text{ m})^2} = 9.0 \times 10^{22} \frac{\text{m}}{\text{s}^2}\]

3. Five charges are equally spaced along the x-axis. Each charge has the same magnitude, $e$, but some of the charges are $+e$ and some are $-e$. Four different configurations of charge are labelled A, B, C, D in the figure at right. Rank the magnitude of the force on the middle charge for the different configurations with largest first and smallest last.

\[
(1) \quad \text{C, D, A, B} \\
(2) \quad \text{C, A, D, B} \\
(3) \quad \text{D, C, A, B} \\
(4) \quad \text{A, C, D, B} \\
(5) \quad \text{C, B, D, A}
\]

Configuration C has all the + charge, repelling $Q$, on one side and all – charge, attracting $Q$, on the other so it will have the greatest net force on $Q$. Config. B has the same sign charges equidistant on either side of $Q$ resulting in forces that cancel each other (net force on $Q$ is zero) so it will be the smallest. In config. D the two outermost like charges cancel each other leaving only the net force from the two oppositely signed nearer charges. Config. A has the same situation for the nearer charges giving the same net force magnitude as in D from these but it has opposite sign outer charges oriented to oppose the force from the inner charges making the magnitude of the force in config. D greater than that in A so: C, D, A, B.
4. Charges $+3Q$ and $-9Q$ are held in place at positions $x = 0\text{ m}$ and $x = 2\text{ m}$, respectively. At what position in $x$ (in m) should a third charge be placed so that it is in equilibrium?

\begin{align*}
(1) \quad & -2.73 \\
(2) \quad & +1.33 \\
(3) \quad & +3.73 \\
(4) \quad & -1.33 \\
(5) \quad & -1.86
\end{align*}

The sketch shows approximately where the charge must lie.

For equilibrium,

\begin{align*}
\vec{F}_{3,1} + \vec{F}_{3,2} &= 0 \\
k \frac{|q_3||q_1|}{r_{3,1}^2} - k \frac{|q_3||q_2|}{r_{3,2}^2} &= 0 \\
\frac{|q_1|}{r_{3,1}^2} - \frac{|q_2|}{r_{3,2}^2} &= 0 \\
\frac{|q_1|^2}{r_{3,1}^2} &= \frac{|q_2|^2}{r_{3,2}^2} \\
\frac{r_{3,2}}{r_{3,1}} &= \sqrt{\frac{|q_2|}{|q_1|}} = \sqrt{\frac{-9Q}{3Q}} = \sqrt{3} \\
r_{3,2} &= \sqrt{3}r_{3,1} \\
x_2 - x_3 &= \sqrt{3} (x_1 - x_3) \\
2m - x_3 &= \sqrt{3} (0m - x_3) \\
2m - x_3 &= -\sqrt{3}x_3 \\
2m &= x_3 - \sqrt{3}x_3 = (1 - \sqrt{3})x_3 \\
\frac{2m}{(1 - \sqrt{3})} &= x_3 \\
x_3 &= -2.73\text{ m}
\end{align*}
5. Which of the four-charge configurations has the strongest electric field at its center? The plus symbol indicates a positive charge +q, and the minus symbol indicates a negative charge −q.

(1) (2) (3) (4) (5) Insufficient information

Draw equal length arrows representing the force on a unit positive test charge (by definition the electric field) at the center point, due to the charges at the corners for each case. For clarity the arrows are color coded to go with the charge that produced it. The net field at the center is zero in cases (3) & (4). In case (1) the x components of the individual forces cancel while their y components sum to make the magnitude proportional to $4\cos(45^\circ)=2.83$. In case (2) the magnitude is proportional to 2, which is smaller.
6. A ball of mass $m = 0.75$ grams is suspended from a thread of negligible mass. The ball carries a charge $q = +32 \times 10^{-6}$ C, and is placed in a uniform electric field of magnitude $E$ which points to the left. If the angle $\theta = 28^\circ$, what is the magnitude of the electric field? (Recall $g = 9.80$ m/s$^2$.)

(1) 120 N/C  
(2) 60 N/C  
(3) 30 N/C  
(4) 20 N/C  
(5) 10 N/C

Equilibrium requires that,

\[ \sum F_x = 0 \rightarrow qE = T \sin \theta \]
\[ \sum F_y = 0 \rightarrow mg = T \cos \theta \]

Divide the first eqn by the second,

\[ \frac{qE}{mg} = \frac{T \sin \theta}{T \cos \theta} = \tan \theta \]

Solve for $E$:

\[ E = \frac{mg}{q} \tan \theta = \frac{(0.00075 \text{kg})(9.8 \text{ m/s}^2)}{32 \times 10^{-6} \text{ C}} \tan(28^\circ) = 122 \frac{\text{N}}{\text{C}} \]
7. An electron gun sends electrons through a region with a constant electric field of $1.5 \times 10^4 \text{ N/C}$ over a distance of 2.5 cm. If the electrons start from rest, how long does it take for the electrons to traverse the gun?

(1) 4.4 ns    (2) 1.1 $\mu$s    (3) 2.2 ns    (4) 2.2 $\mu$s    (5) 1.1 ns

\[ F = qE = ma \]
\[ a = \frac{qE}{m} \]
\[ x = x_o + v_o t + \frac{1}{2} at^2 \]

0 (starts from rest)

\[ \frac{1}{2} at^2 = x - x_o \]

\[ t^2 = \frac{2(x - x_o)}{a} \]

\[ t = \sqrt{\frac{2(x - x_o)}{a}} = \sqrt{\frac{2m(x - x_o)}{qE}} = \sqrt{\frac{2(9.1 \times 10^{-31} \text{ kg})(0.025 \text{ m} - 0)}{(1.6 \times 10^{-19} \text{ C})(1.5 \times 10^4 \text{ N/C})}} = 4.4 \times 10^{-9} \text{ s} \]
8. The figure shows a closed Gaussian surface in the shape of a cube of edge length 1 m, with one corner at the origin of a coordinate system. The cube lies in a region where the electric field vector is given by $\vec{E} = -2.0 \hat{i} + 2.0 \hat{j}$ N/C, with $x$ and $y$ in meters. What is the net charge, in pC, contained by the cube?

(1) 17.7 (2) −8.9 (3) 0 (4) 8.9 (5) 17.7

$q_{\text{enc}} = \varepsilon_0 \int \vec{E} \cdot d\vec{A}$

Since there is no electric field along the $z$ direction there is no electric flux through the faces perpendicular to the $z$ axis. There is electric flux passing through the faces perpendicular to the $y$ axis but the field is in the same direction through each face, the areas are the same but their normal vectors are in opposite directions so they contribute to the integral with equal magnitudes but opposite sign and so they vanish. This leaves only the integral over the faces perpendicular to the $x$ axis. For the one at $x = 0$ the electric field is zero leaving only,

$$q_{\text{enc}} = \varepsilon_0 \int E_{x=1m} \cdot d\vec{A} = \varepsilon_0 \int E_{x=1m} dA = \varepsilon_0 E_{x=1m} \int dA = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} (-2 \frac{N}{C})(1m^2) = 17.7 \times 10^{-12} C$$
9. A uniformly charged thin non-conducting shell (hollow sphere) of radius $R$ with the total positive charge $Q$ is placed at a distance $d$ away from an infinite non-conducting sheet carrying a uniformly distributed positive charge with a surface density $\sigma$. The distance $d$ is measured from shell’s center (point O). What is the magnitude of the total electric field at the center of the shell? (Hint: Consider electric field at point O made by the shell alone and by the sheet alone and then, use the principle of superposition of fields.)

\[
\begin{align*}
(1) \quad & \frac{\sigma}{2\varepsilon_0} \\
(2) \quad & \frac{Q}{4\pi\varepsilon_0 R^2} + \frac{\sigma}{2\varepsilon_0} \\
(3) \quad & \frac{Q}{4\pi\varepsilon_0 R^2} + \frac{\sigma}{\varepsilon_0} \\
(4) \quad & \frac{Q}{4\pi\varepsilon_0 R^2} \\
(5) \quad & \frac{Q}{4\pi\varepsilon_0 R} + \frac{\sigma}{\varepsilon_0}
\end{align*}
\]

The electric field inside a uniform shell of charge *due to the charge on the shell* is zero (draw a Gaussian surface just inside the shell. It encloses zero charge, making the field inside, *from the shell*, zero). But there is field there due to the infinite sheet of charge *given by*,

\[E = \frac{\sigma}{2\varepsilon_0}\]
(see section 23-5 of your textbook for the proof)
10. A point charge $+Q$ is placed at the center of a thick conductive shell carrying charge $-2Q$. What is the charge on the inner surface of the shell?

(1) $-Q$  (2) $+Q$  (3) $-2Q$  (4) $+2Q$  (5) 0

To make the field inside the metal of the conducting shell zero (as it must be) the charge on the inner surface of the conducting shell must have the same magnitude but opposite sign as the charge inside the shell (to make the net charge inside a Gaussian surface drawn entirely within the conductor, see sketch, zero).
11. A plastic rod has been bent into a circle of radius \( R = 11.8 \) cm. It has a charge \( Q_1 = +8.58 \) pC uniformly distributed along one-quarter of its circumference and a charge \( Q_2 = -6Q_1 \) uniformly distributed along the rest of the circumference (see figure). With \( V = 0 \) at infinity, what is the electric potential in volts at point \( O \), which is on the center of the circle?

\[
V = \frac{1}{4\pi\varepsilon_0} \left( \frac{Q_1}{r} + \frac{Q_2}{r} \right) = \frac{1}{4\pi\varepsilon_0} \left( Q_1 + Q_2 \right) = \frac{1}{4\pi\varepsilon_0} \left( Q_1 - 6Q_1 \right) = \frac{-5Q_1}{4\pi\varepsilon_0 r}
\]

\[
V = \frac{-5Q_1}{4\pi\varepsilon_0 r} = \frac{-5(8.58 \times 10^{-12} \text{C})}{4\pi \left( 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2 \right) (0.118 \text{ m})} = -3.27 \text{ V}
\]
The integral of $f(x)\,dx$ is the signed area between the function and $f(x)=0$ between the limits of the integral. So,

$$\int_{V_1}^{V_2} E(x)\,dx$$

12. A graph of the $x$ component of the electric field as a function of $x$ in a region of space is shown in figure. The scale of the vertical axis is set by $E_{x,0} = 20.0 \text{ N/C}$. The $y$ and $z$ components of the electric field are zero in this region. If the electric potential at the origin is 30 V, what is the electric potential (in V) at $x = 4.0 \text{ m}$?

\[ V_f - V_i = -\int E \cdot d\vec{s} \]

\[ V_f = V_i - \int E \cdot d\vec{s} \]

\[ V_f = 30\text{V} - \int_0^4 E(x)\,dx \]

The integral of $f(x)\,dx$ is the *signed area* between the function and $f(x)=0$ between the limits of the integral. So,

\[ V_f = 30\text{V} - \frac{1}{2}(4\text{m})(20 \frac{\text{N}}{\text{C}}) = 30\text{V} - 40\text{V} = -10\text{V} \]
13. An infinite non-conducting thin plate has negative uniform charge density $-\sigma$. What is the difference of potentials between points $P$ and $O$: $V_P - V_O$?

\[
\begin{align*}
(1) & \quad \frac{\sigma}{2\epsilon_0} \cdot d \\
(2) & \quad -\frac{\sigma}{2\epsilon_0} \cdot d \\
(3) & \quad \frac{\sigma}{2\epsilon_0} \cdot \sqrt{2d} \\
(4) & \quad -\frac{\sigma}{2\epsilon_0} \cdot \sqrt{2d} \\
(5) & \quad -\frac{\sigma}{\epsilon_0} \cdot d
\end{align*}
\]

$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$ Where the conservative nature of the electric force let's us go from $O$ to $P$ by any path to get the same value. In that case we chose the path shown below. The electric field for the negative sheet of charge is uniform and points in, towards the sheet with magnitude (everywhere) given by $E = \frac{\sigma}{2\epsilon_0}$

Then

$V_P - V_O = -\int_O^P \vec{E} \cdot d\vec{s} = -\int_O^T \vec{E} \cdot d\vec{s} - \int_T^P \vec{E} \cdot d\vec{s}$

From $O$ to $T$ $\vec{E} \cdot d\vec{s} = Eds\cos(90^\circ) = 0$ while from $T$ to $P$ $\vec{E} \cdot d\vec{s} = Eds\cos(180^\circ) = -Eds$ So,

$V_P - V_O = 0 - \int_T^P -Eds = E\int_T^P ds = Ed = \frac{\sigma}{2\epsilon_0}d$
14. The potential (in volts) in a given region is given by \( V(x, y, z) = xyz \), where \( x, y, \) and \( z \) are expressed in meters. What is the magnitude of the electric field in V/m at the point (1,1,1)?

(1) 1.7 \hspace{1cm} (2) 1.0 \hspace{1cm} (3) 3.0 \hspace{1cm} (4) 0 \hspace{1cm} (5) 1.4

\[ E_x = -\frac{\partial V}{\partial x} = -\frac{\partial (xyz)}{\partial x} = -yz = -1 \frac{\text{V}}{\text{m}} \text{ (at 1,1,1)} \]

\[ E_y = -\frac{\partial V}{\partial y} = -\frac{\partial (xyz)}{\partial y} = -xz = -1 \frac{\text{V}}{\text{m}} \text{ (at 1,1,1)} \]

\[ E_z = -\frac{\partial V}{\partial z} = -\frac{\partial (xyz)}{\partial z} = -xy = -1 \frac{\text{V}}{\text{m}} \text{ (at 1,1,1)} \]

The magnitude is then: \( E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{1 + 1 + 1} = \sqrt{3} = 1.7 \)
15. The rectangle shown in the figure has sides 5 cm and 8 cm. The charges are $q_1 = 6 \mu C$ and $q_2 = 9 \mu C$. How much work done by an external force is required to move a charge $q_3 = 4.0 \mu C$ from point A to point B?

\[
V_A = k \sum_i \frac{q_i}{r_i} = k \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) = k \left( \frac{6 \mu C}{0.05m} + \frac{9 \mu C}{0.08m} \right) = k \left( \frac{48 \mu C}{0.40m} + \frac{45 \mu C}{0.40m} \right) = k \frac{93 \mu C}{0.40m} = 2.093 \times 10^6 V
\]

\[
V_B = k \sum_i \frac{q_i}{r_i} = k \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) = k \left( \frac{6 \mu C}{0.08m} + \frac{9 \mu C}{0.05m} \right) = k \left( \frac{30 \mu C}{0.40m} + \frac{72 \mu C}{0.40m} \right) = k \frac{102 \mu C}{0.40m} = 2.295 \times 10^6 V
\]

\[
W_{app} = \Delta U = q\Delta V = q(V_f - V_i) = q(V_B - V_A)
\]

\[
W_{app} = q(V_B - V_A) = (4 \times 10^{-6}C)(2.295 \times 10^6 V - 2.093 \times 10^6 V) = 0.81J
\]
16. Consider two square parallel conducting plates of side length 2 cm, separated by a 3 mm slab of dielectric with \( \kappa = 1.5 \), that are charged to ±4 nC and then disconnected from the power supply. Considering only electric forces (i.e., ignoring adhesion, friction, gravity...), how much work (in \( \mu J \)) must be done by someone to withdraw the dielectric slab?

\[
W = \Delta U = U_f - U_i = U_o - \frac{U_o}{\kappa} = U_o \left( 1 - \frac{1}{\kappa} \right) = \frac{q^2}{2C_o} \left( 1 - \frac{1}{\kappa} \right)
\]

Where

\[
C_o = \frac{\varepsilon_o A}{d}
\]

\[
W = \frac{q^2 d}{2 \varepsilon_o A} \left( 1 - \frac{1}{\kappa} \right) = \frac{(4 \times 10^{-9} \text{C})^2 (0.003 \text{m})}{2 \left( 8.85 \times 10^{-12} \frac{C^2}{\text{N-m}^2} \right) (0.02 \text{m})^2} \left( 1 - \frac{1}{1.5} \right) = 2.26 \times 10^{-6} \text{J}
\]
17. What is the equivalent capacitance of the combination shown?

(1) $10 \mu F$
(2) $29 \mu F$
(3) $40 \mu F$
(4) $12 \mu F$
(5) $25 \mu F$

$$C_{eq} = 20 \left| (12 \left| 24 \right) + 12 \right|$$ With all values in $\mu F$

$$C_{eq} = 20 \left| \left( \frac{12 \cdot 24}{12 + 24} \right) + 12 \right| = 20 \left| 8 + 12 \right| = 20 \cdot 20 = 10 \mu F$$
18. In the figure the battery has a potential difference of $V = 10.0 \, \text{V}$ and the five capacitors each have a capacitance of $10.0 \, \mu\text{F}$. What is the charge, in $\mu\text{C}$, on capacitor 2?

The equivalent capacitance for the right side of the circuit (all in $\mu\text{F}$) is

$$C_{eq} = (10 \, \mu\text{F}) + (10 \, \mu\text{F}) = 20 \, \mu\text{F}.$$  

That means the total charge on that network is

$$Q = C_{eq} \cdot V = 20 \, \mu\text{F} \cdot 10 \, \text{V} = 200 \, \mu\text{C}.$$  

That’s also the charge on the cap we’ve labeled $C_3$. The voltage drop across that is

$$V_3 = \frac{Q}{C_3} = \frac{60 \, \mu\text{C}}{10 \, \mu\text{F}} = 6 \, \text{V}.$$  

The voltage drop across the upper part of that network is then $4 \, \text{V}$. Since that splits evenly between the series combination of $C_2$ and the equal cap below it, $C_2$ has $2 \, \text{V}$ across it. Then, 

$$Q_2 = C_2 V_2 = 10 \, \mu\text{F} \cdot 2 \, \text{V} = 20 \, \mu\text{C}.$$  

The answer is **(4) 80**.
19. Three resistors A, B, and C are successively connected to the same 12 V power supply. While they are connected to the supply you feel each resistor in turn and determine that B was hottest while C was coolest. The resistance values of the three resistors must be such that:

(1) C > A > B  (2) C > B > A  (3) B > A > C  (4) B > C > A  (5) A > B > C

Since we had not yet covered resistors in series at the time of this exam the word successive here was meant to convey its temporal meaning i.e. that the resistors each got 12V across them, connecting them (successively in time) one at a time. In that case, the resistor with the smallest resistance has the greatest current flowing through it and using $P = IV$ with the voltages all the same (12V) we see that the smaller the resistor the greater power dissipated (as heat). In that case the resistor values are such that C > A > B.

Many of you, however, interpreted successive as the resistors in series. In that case the same current flows through each resistor (limited by the series resistance of the 3 series resistors). In that case using the form for the power, $P = I^2R$, since the same current flows through each resistor the power scales with the resistance and the order of the resistances would be, B > A > C.
Let \( f \) refer to the final case and \( i \) to the initial, then, with \( L_f = 4L_i \), for constant volume, we must have,

\[
L_f A_f = L_i A_i
\]

\[
4L_i A_f = L_i A_i
\]

\[
A_f = \frac{A_i}{4}
\]

Now, \( R_f = \rho \frac{L_f}{A_f} \) and \( R_i = \rho \frac{L_i}{A_i} \) divide the first equation by the 2nd,

\[
\frac{R_f}{R_i} = \frac{\rho \frac{L_f}{A_f}}{\rho \frac{L_i}{A_i}} = \frac{L_f A_i}{A_f L_i} = \frac{4L_i A_i}{\frac{A_i}{4} L_i} = 16
\]

\[
R_f = 16R_i
\]