So why did you get your brother a gnome?
I said it's a metronome, he plays piano.
No, my brother...
The gnome plays piano?
No, the gnome plays your brother? That's even weirder.
How did he learn to play your brother if they didn't know each other already?
This is insane.
You're the one babbling about brother-playing gnomes.

Get Fuzzy: Jan 20, 2009
HITT Quiz

Two identical conducting spheres A and B carry equal charge. They are separated by a distance much larger than their diameters. A third identical conducting sphere C is uncharged. Sphere C is first touched to A, then to B, and finally removed. As a result, the electrostatic force between A and B, which was originally F, becomes:

A. F/2  B. F/4  C. 3F/8  D. F/16  E. 0
A question

Two protons (p1 and p2) are on the axis, as shown here. The directions of the electric Field at points 1, 2, and 3, respectively, are:

A. →, ←, →
B. ←, →, ←
C. ←, →, →
D. ←, ←, ←
E. ←, ←, →
Another Question

Positive charge $+Q$ is uniformly distributed on the upper half of a rod and negative charge $-Q$ is uniformly distributed on the lower half. What is the direction of the electric field at point $P$, on the perpendicular bisector of the rod?
Electric fields due to a point charge (Coulomb’s), a wire and a plane.
Charge shells don’t act inside.
In an insulator with uniformly distributed charge, only charge enclosed inside contributes to the field outside.
Gauss’ theorem
The electric field has been calculated along a line passing through the center of the ring.

It is zero at the center of the ring, increases up to \( z \sim R \) and then decreases. It decreases as \( 1/R^3 \) for large \( z \). Looks like a spring. Problem 76
Wires, Rings etc..

Inside a uniformly charged ring, \( E = 0 \).
Inside a uniformly charged spherical shell, \( E = 0 \).
Electric field lines emerge from a positive charge. They end at a negative charge. Electric field lines do not cross.
\[ E = \frac{\sigma}{2\epsilon_0} \]

Does not depend on the distance.

C/L^2 dimensionally OK
PHY 2049: Physics II

- $E = \sigma/\varepsilon_0$ inside
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- Flux and Gauss’ theorem
Gaussian (imaginary) surfaces

Flux = \( \Phi = \sum E \cdot dA \)

\( E \cdot dA \) for a cube is easy to visualize.

Let calculus do it for a sphere or any other shape.

\( \Phi_c = \frac{q_{\text{total}}}{\varepsilon_0} \)
- $S_1$: $\Phi_c = \frac{q}{\varepsilon_0}$
- $S_2$: $\Phi_c = -\frac{q}{\varepsilon_0}$
- $S_3$: $\Phi_c = 0$
- $S_4$: $\Phi_c = 0$
• For a cylinder with axial field, no contribution to flux from the side walls.
• From the far top, $>0$
• Near (bottom), $<0$
No force/field from the charge outside

On inside circle
\[ E = 4\pi r^2 \frac{q}{\varepsilon_0} \] Coulomb’s law

Far outside, \[ E = k \frac{5q}{r^2} \]
The electric field in a metal is zero
Charge +q on inside surface.
Because it is neutral, outside surface must be -q.
- Electric field is radial.
- E.A = 0 for the top and bottom surfaces.
- Sideways:
  \[ E \cdot 2\pi rh = \lambda h / \varepsilon_0 \]
  \[ E = \lambda / 2\pi r \varepsilon_0 \]
- Plane
- $2EA = \sigma A/\varepsilon_0$

$E = \sigma/2\varepsilon_0$
Forces and torques exerted on electric dipoles by a uniform electric field

Consider the electric dipole shown in the figure in the presence of a uniform (constant magnitude and direction) electric field $\vec{E}$ along the $x$-axis.

The electric field exerts a force $F_+ = qE$ on the positive charge and a force $F_- = -qE$ on the negative charge. The net force on the dipole $F_{net} = qE - qE = 0$

The net torque generated by $F_+$ and $F_-$ about the dipole center is:

$$\tau = \tau_+ + \tau_- = -|F_+| \frac{d}{2} \sin \theta - |F_-| \frac{d}{2} \sin \theta = -qEd \sin \theta = -pE \sin \theta$$

In vector form: $\vec{\tau} = \vec{p} \times \vec{E}$

The electric dipole in a uniform electric field does not move but can rotate about its center

$$\vec{F}_{net} = 0 \quad \vec{\tau} = \vec{p} \times \vec{E}$$

(22-14)
Potential energy of an electric dipole in a uniform electric field

\[ U = -\int_{90^\circ}^{\theta} \tau \, d\theta' = -\int_{90^\circ}^{\theta} pE \sin \theta \, d\theta' \]

\[ U = -pE \int_{90^\circ}^{\theta} \sin \theta \, d\theta' = -pE \cos \theta = -\vec{p} \cdot \vec{E} \]

At point A (\( \theta = 0 \)) \( U \) has a minimum value \( U_{\text{min}} = -pE \)
It is a position of stable equilibrium

At point B (\( \theta = 180^\circ \)) \( U \) has a maximum value \( U_{\text{max}} = +pE \)
It is a position of unstable equilibrium

\[ U = -pE \cos \theta \]

\[ U = -\vec{p} \cdot \vec{E} \]
Work done by an external agent to rotate an electric dipole in a uniform electric field

Consider the electric dipole in Fig.a. It has an electric dipole moment $\vec{p}$ and is positioned so that $\vec{p}$ is at an angle $\theta_i$ with respect to a uniform electric field $\vec{E}$.

An external agent rotates the electric dipole and brings it in its final position shown in Fig.b. In this position $\vec{p}$ is at an angle $\theta_f$ with respect to $\vec{E}$.

The work $W$ done by the external agent on the dipole is equal to the difference between the initial and final potential energy of the dipole:

$$W = U_f - U_i = -pE \cos \theta_f - (-pE \cos \theta_i)$$

$$W = pE \left( \cos \theta_i - \cos \theta_f \right)$$

(22-16)
Summary for today

Electric Fields of a continuous charge distribution
Field lines

Electric flux and Gauss’ theorem

Dipoles and Torques