Centripetal Acceleration

- Centripetal refers to “center-seeking”
- Quantifies the change in direction of the velocity
- The acceleration is directed toward the center of the circle of motion
Centripetal Acceleration, final

- The magnitude of the centripetal acceleration is given by
  \[ a_c = \frac{v_t^2}{r} \]
  - This direction is toward the center of the circle
Centripetal Acceleration and Angular Velocity

- The angular velocity and the linear velocity are related \((v_t = \omega r)\)
- The centripetal acceleration can also be related to the angular velocity

\[
a_c = \omega^2 r
\]
Total Acceleration

- The tangential component of the acceleration is due to changing speed.
- The centripetal component of the acceleration is due to changing direction.
- Total acceleration can be found from these components:

\[ a = \sqrt{a_t^2 + a_C^2} \]
Forces Causing Centripetal Acceleration

- Newton’s Second Law says that the centripetal acceleration is accompanied by a force

\[ F = m \frac{v^2}{r} \]

- \( F = ma \) \[ \Rightarrow \]

- \( F \) stands for any force that keeps an object following a circular path
  - Tension in a string
  - Gravity
  - Force of friction
  - others
Examples

- Ball at the end of revolving string (m=0.5kg, r=1.20m, \( \omega = 0.8 \) rev/s). Tension, Gravity?
- Fast car rounding a curve on flat road, friction!
Example

A roller coaster has a loop-the-loop circular portion in its track with a radius of 10 m. How fast must the cart be moving if the passengers in the cart are to be just on the verge of falling out at the top?
Which figure shows the correct direction of the centripetal acceleration (arrow)?

A. Figure 1
B. Figure 2
C. Figure 3
D. Figure 4

Correct answer: B. Figure 2
Ex. An engineer wishes to design a curved exit ramp for a toll road in such a way that a car will not have to rely on friction to round the curve without skidding. He does so by banking the road in such a way that the force causing the centripetal acceleration will be supplied by the component of the normal force toward the center of the circular path. (a) Show that, for a given speed $v$ and a radius $r$, the curve must be banked at the angle $\theta$ such that $\tan \theta = v^2 / rg$. (b) Find the angle at which the curve should be banked if a typical car rounds it at a 50.0-m radius and a speed of 13.4 m/s.
Which figure shows the correct direction of the centripetal acceleration (fat arrow)?

A. Figure 1
B. Figure 2
C. Figure 3

Answer: C. Figure 3
The sum of all the forces MUST point in the direction of acceleration.

Draw your coordinate axes such that x-axis is along the acceleration direction.
Which way would the friction point if the car were going faster than v?

A. Figure 1

✓ B. Figure 2

C. Figure 3
Newton’s Law of Universal Gravitation

Every particle in Universe attracts every other particle with force:
- **directly** proportional to product of masses
- **inversely** proportional to square of distance between them.

\[ F = G \frac{m_1 m_2}{r^2} \]

**inverse square law**

\( G = \text{universal gravitational constant} \)
\[ = 6.673 \times 10^{-11} \text{ N m}^2 /\text{kg}^2 \]
- Uniform sphere
- Acceleration due to gravity $g$ will vary with altitude

$$F = G \frac{M_E m}{r^2}$$

- $r$ is the distance from Earth’s center
- $g$ is not constant

$$g = G \frac{M_E}{r^2}$$
\[ \Delta PE = mgh \]

\[ PE_f - PE_i = mgh \]

\[ PE_f = mgh + PE_i \]
$PE = -GM_E \frac{m}{r}$
Gravitational Potential Energy

- PE = mgy is valid only near the earth’s surface
- For objects high above the earth’s surface, an alternate expression is needed

\[ PE = -G \frac{M_E m}{r} \]

- Zero reference level is infinitely far from the earth
Escape Speed

- speed needed for an object “escape” from planet

\[ v_{\text{esc}} = \sqrt{\frac{2GM_E}{R_E}} \]

- For the earth, \( v_{\text{esc}} \) is about 11.2 km/s
- Note, \( v \) is independent of the mass of the object