Final Exam: 04/28/12 10:00am-noon

• Room Assignments:

<table>
<thead>
<tr>
<th>Last Name</th>
<th>Room</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-F</td>
<td>TUR L007</td>
</tr>
<tr>
<td>G-L</td>
<td>BRY 130</td>
</tr>
<tr>
<td>M-Q</td>
<td>LIT 109</td>
</tr>
<tr>
<td>R-S</td>
<td>TUR L005</td>
</tr>
<tr>
<td>T-Z</td>
<td>FLG 280</td>
</tr>
</tbody>
</table>

• Breakdown of the 20 Problems

<table>
<thead>
<tr>
<th>Material</th>
<th># of Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Your Exam 1</td>
<td>4</td>
</tr>
<tr>
<td>Your Exam 2</td>
<td>4</td>
</tr>
<tr>
<td>Chapter 11.1-12.8</td>
<td>4</td>
</tr>
<tr>
<td>Chapter 1.1-10.8</td>
<td>8</td>
</tr>
</tbody>
</table>

• Crib Sheet: You may bring two hand written formula sheet on 8½ x 11 inch paper (both sides).

• Calculator: You should bring a calculator (any type).

• Scratch Paper: We will provide scratch paper.
Make-Up Exam: 04/23/12 5:10-7:00pm

For anyone who had to miss an exam during the semester.

- **Room:** 1220 NPB.

- **Breakdown of the 20 Problems:** Similar to the Final Exam.

- **Crib Sheet:** You may bring two hand written formula sheet on 8½ x 11 inch paper (both sides).

- **Calculator:** You should bring a calculator (any type).

- **Scratch Paper:** We will provide scratch paper.
Final Exam Fall 2010: Problem 9

Near the surface of the Earth a startled armadillo leaps vertically upward at time \( t = 0 \), at time \( t = 0.5 \) s it is a height of 0.98 m above the ground. At what time does it land back on the ground?

Answer: 0.9 s
% Right: 47%

\[
y(t) = v_o t - \frac{1}{2} gt^2 = t(v_o - \frac{1}{2} gt)
\]

\[
y(t_f) = 0 = t_f (v_o - \frac{1}{2} gt_f)
\]

\[
t_f = \frac{2v_o}{g}
\]

\[
y(t_h) = h = v_o t_h - \frac{1}{2} gt_h^2
\]

\[
v_o = \frac{h + \frac{1}{2} gt_h^2}{t_h}
\]

\[
t_f = \frac{2v_o}{g} = \frac{2(h + \frac{1}{2} gt_h^2)}{gt_h} = \frac{2h}{gt_h} + t_h = \frac{2(0.98m)}{(9.8m/s^2)(0.5s)} + (0.5s) = (0.4s) + (0.5s) = 0.9s
\]
Final Exam Spring 2011: Problem 14

A projectile hits the ground at an angle of 30° with respect to the vertical at a speed of 30 m/s. The horizontal component of its velocity at launch (in m/s) was:

Answer: 15.0
% Right: 53%

\[ v_x(t) = v_{xo} = v_{xf} = v_f \sin \phi = (30 \text{ m/s}) \sin(30°) = 15 \text{ m/s} \]
Near the surface of the Earth, a block of mass M is at rest on a plane inclined at angle $\theta$ to the horizontal. If the coefficient of static friction between the block and the surface of the plane is 0.7, what is the largest angle $\theta$ without the block sliding?

Answer: $35^\circ$

% Right: 83%

\[ Mg \sin \theta - f_s = Ma_x = 0 \]

\[ F_N - Mg \cos \theta = Ma_y = 0 \]

\[ Mg \sin \theta = f_s = \mu_s F_N = \mu_s Mg \cos \theta \]

\[ \tan \theta = \mu_s = 0.7 \]

$\theta \approx 35^\circ$
Final Exam Spring 2011: Problem 12

- A uniform disk with mass $M$, radius $R = 0.50 \text{ m}$, and moment of inertia $I = MR^2/2$ rolls without slipping along the floor at 2 revolutions per second when it encounters a long ramp angled upwards at $45^\circ$ with respect to the horizontal. How high above its original level will the center of the disk get (in meters)?

Answer: 3.0  
% Right: 39%

\[
E_i = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 + MgR = E_f = Mg(H + R)
\]

\[
H = \frac{1}{2g} \left( v^2 + \frac{I}{M} \omega^2 \right) = \frac{1}{2g} \left( R^2 \omega^2 + \frac{I}{M} \omega^2 \right)
\]

\[
= \frac{R^2}{2g} \left( 1 + \frac{I}{MR^2} \right) \omega^2 = \frac{2\pi^2 R^2}{g} \left( 1 + \frac{I}{MR^2} \right) f^2 = \frac{2\pi^2 (0.5m)^2}{9.8m/s^2} \left( \frac{3}{2} \right) (2/s)^2 \approx 3.0m
\]
Example: Ball & Hoop

- A thin hoop (\(I = MR^2\)) and a solid spherical ball (\(I = 2MR^2/5\)) start from rest and roll without slipping a distance \(d\) down an incline ramp as shown in the figure. If it takes 1 s for the ball to roll down the incline, how long does it take for the hoop (in s)?

Answer: 1.2

\[
Mg \sin \theta - f = Ma_x \\
v = R\omega \\
\tau = I\alpha = \frac{I}{R} a_x = Rf \\
\alpha = R a_x \\
Mg \sin \theta - \frac{I}{R^2} a_x = Ma_x \\
a_x = \frac{g \sin \theta}{\left(1 + \frac{I}{MR^2}\right)} \\
d = \frac{1}{2} a_x t^2 \\
t = \sqrt{\frac{2d}{a_x}}
\]

\[
a_x (\text{hoop}) = \frac{1 + \frac{I_{\text{ball}}}{MR^2}}{1 + \frac{I_{\text{hoop}}}{MR^2}} = \frac{1 + \frac{2}{5}}{1 + 1} = \frac{7}{10} = 0.7 \\
a_x (\text{ball}) = \frac{1 + \frac{I_{\text{ball}}}{MR^2}}{1 + \frac{I_{\text{ball}}}{MR^2}} = \frac{7}{10} = 0.7 \\
t(\text{hoop}) = \sqrt{\frac{a_x (\text{ball})}{a_x (\text{hoop})}} = \frac{1}{\sqrt{0.7}} = 1.1952 \\
t(\text{hoop}) = 1.1952 \times t(\text{ball}) \approx 1.2 \text{s}
\]
Exam 2 Fall 2011: Problem 17

- A cloth tape is wound around the outside of a non-uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the figure. The cylinder is held with the tape vertical and then released from rest. If the acceleration of the center-of-mass of the cylinder is 3g/5, what is its moment of inertia about its symmetry axis?

Answer: $\frac{2}{3}MR^2$

% Right: 48%

\[ Mg - F_T = Ma_y \]

\[ F_T = M(g - a_y) \]

\[ \tau = RF_T = I\alpha = \frac{Ia_y}{R} \]

\[ I = \frac{R^2 F_T}{a_y} = \left(\frac{g - a_y}{a_y}\right)MR^2 = \left(\frac{g - \frac{3}{5}g}{\frac{3}{5}g}\right)MR^2 = \frac{2}{3}MR^2 \]
Example Problem: Pulley

- The ideal mechanical advantage is defined to be the ratio of the weight \( W \) to the force of the pull \( F_p \) for equilibrium (i.e. \( W/F_p \) in equilibrium). Assuming that the pulleys rotate without friction and without the rope slipping, what is the ideal mechanical advantage of the combination of pulleys shown in the figure?

Answer: 7

\[
T_1 = F_p \\
T_2 = 2T_1 \\
T_3 = 2T_2 \\
T_1 + T_2 + T_3 = W
\]

\[
\frac{W}{F_p} = \frac{T_1 + T_2 + T_3}{T_1} = \frac{T_1 + 2T_1 + 4T_1}{T_1} = 7
\]
Problem 10

A cannon on a railroad car is facing in a direction parallel to the tracks as shown in the figure. The cannon can fires a 100-kg cannon ball at a muzzle speed of 150 m/s at an angle of $\theta$ above the horizontal as shown in the figure. The cannon plus railway car have a mass of 5,000 kg. If the cannon and one cannon ball are travelling to the right on the railway car at a speed of $v = 2$ m/s, at what angle $\theta$ must the cannon be fired in order to bring the railway car to rest? Assume that the track is horizontal and there is no friction.

**Answer:** $48.2^\circ$

% Right: 57%

\[
(p_x)_i = (M_{\text{car}} + M_{\text{Ball}})V_i \quad (p_x)_f = M_{\text{Ball}}V_{\text{ball}} = M_{\text{Ball}}(V_i + V_{\text{muzzle}} \cos \theta)
\]

\[
(p_x)_i = (p_x)_f \quad (M_{\text{car}} + M_{\text{Ball}})V_i = M_{\text{Ball}}(V_i + V_{\text{muzzle}} \cos \theta)
\]

\[
\cos \theta = \frac{M_{\text{car}}V_i}{M_{\text{Ball}}V_{\text{muzzle}}} = \frac{(5000 \text{kg})(2 \text{ m/s})}{(100 \text{ kg})(150 \text{ m/s})} = 0.6667 \quad \theta \approx 48.2^\circ
\]
Previous Problem: Equivalent

- A cannon on a railroad car is facing in a direction parallel to the tracks as shown in the figure. The cannon can fire a 100-kg cannon ball at a muzzle speed of 150 m/s at an angle of $\theta$ above the horizontal as shown in the figure. The cannon plus railway car have a mass of 5,000 kg. If the cannon, one cannon ball, and the railway car are initially at rest, at what angle $\theta$ must the cannon be fired so that the recoil speed of the railway car plus cannon is 2 m/s toward the left? Assume that the track is horizontal and there is no friction.

Answer: 48.2°

$$ (p_x)_i = 0 \quad (p_x)_f = M_{\text{Ball}} V_{\text{ball}} - M_{\text{car}} V_f = M_{\text{Ball}} V_{\text{muzzle}} \cos \theta - M_{\text{car}} V_f $$

$$ (p_x)_i = (p_x)_f \quad M_{\text{Ball}} V_{\text{muzzle}} \cos \theta - M_{\text{car}} V_f = 0 $$

$$ \cos \theta = \frac{-M_{\text{car}} V_f}{M_{\text{Ball}} V_{\text{muzzle}}} = \frac{-(5000\text{kg})(-2\text{m/s})}{(100\text{kg})(150\text{m/s})} = 0.6667 \quad \theta \approx 48.2° $$
Exam 2 Fall 2011: Problem 31

- A uniform solid disk with mass M and radius R is mounted on a vertical shaft with negligible rotational inertia and is initially rotating with angular speed \( \omega \). A non-rotating uniform solid disk with mass M and radius R/2 is suddenly dropped onto the same shaft as shown in the figure. The two disks stick together and rotate at the same angular speed. What is the new angular speed of the two disk system?

Answer: \( \frac{4\omega}{5} \)

% Right: 47%

\[
L_i = I_i \omega_i = \frac{1}{2} MR_1^2 \omega = L_f = I_f \omega_f = \left( \frac{1}{2} MR_1^2 + \frac{1}{2} MR_2^2 \right) \omega_f
\]

\[
\omega_f = \frac{R_1^2}{R_1^2 + R_2^2} \omega = \frac{R^2}{R^2 + (R/2)^2} \omega = \frac{4}{5} \omega
\]
Final Exam Spring 2011: Problem 5

- A toy merry-go-round consists of a uniform disk of mass M and radius R that rotates freely in a horizontal plane about its center. A mouse of the same mass, M, as the disk starts at the rim of the disk. Initially the mouse and disk rotate together with an angular velocity of \( \omega \). If the mouse walks to a new position a distance \( r \) from the center of the disk the new angular velocity of the mouse-disk system is \( 2\omega \). What is the new distance \( r \)?

Answer: \( R/2 \)

% Right: 65%

\[
L_i = I_i \omega_i = (MR^2 + \frac{1}{2} MR) \omega_i = L_f = (Mr^2 + \frac{1}{2} MR) \omega_f
\]

\[
r = R \sqrt{\frac{3 \omega_i}{2 \omega_f}} - \frac{1}{2} = R \sqrt{\frac{3}{4} - \frac{1}{2}} = \frac{R}{2}
\]
Exam 2 Fall 2011: Problem 36

- Two small balls are simultaneously released from rest in a deep pool of water (with density $\rho_{\text{water}}$). The first ball is released from rest at the surface of the pool and has a density three times the density of water (i.e. $\rho_1 = 3\rho_{\text{water}}$). A second ball of unknown density is released from rest at the bottom of the pool. If it takes the second ball the same amount of time to reach the surface of the pool as it takes for the first ball to reach the bottom of the pool, what is the density of the second ball? Neglect hydrodynamic drag forces.

Answer: $3\rho_{\text{water}}/5$

% Right: 22%
• What is the minimum radius (in m) that a spherical helium balloon must have in order to lift a total mass of $m = 10$-kg (including the mass of the empty balloon) off the ground? The density of helium and the air are, $\rho_{HE} = 0.18$ kg/m$^3$ and $\rho_{air} = 1.2$ kg/m$^3$, respectively.

Answer: 1.33 m  
% Right: 37% 

\[
F_{buoyancy} - (M_{HE} + m)g = 0
\]

\[
\rho_{air} V_{ballon} g - (\rho_{HE} V_{ballon} + m)g = 0
\]

\[
V_{ballon} = \frac{4}{3} \pi r^3 = \frac{m}{\rho_{air} - \rho_{HE}}
\]

\[
r = \left( \frac{3m}{4\pi (\rho_{air} - \rho_{HE})} \right)^{\frac{1}{3}} = \left( \frac{3(10kg)}{4\pi (1.02kg / m^3)} \right)^{\frac{1}{3}} = (2.34m^3)^{\frac{1}{3}} \approx 1.33m
\]
Exam 2 Fall 2011: Problem 50

- An ideal spring-and-mass system is undergoing simple harmonic motion (SHM) with period \( T = 3.14 \) s. If the mass \( m = 0.5 \) kg and the total energy of the spring-and-mass system is \( 5 \) J, what is the speed (in m/s) of the mass when the displacement is \( 1.0 \) m?

Answer: 4.0
% Right: 34%

\[ E = \frac{1}{2} k A^2 = \frac{1}{2} mv_x^2 + \frac{1}{2} kx^2 \]

\[ \omega = \sqrt{\frac{k}{m}} \]

\[ \omega = \frac{2\pi}{T} \]

\[ v_x^2 = \frac{2E}{m} - \frac{k}{m} x^2 = \frac{2E}{m} - \omega^2 x^2 = \frac{2E}{m} - \frac{4\pi^2}{T^2} x^2 \]

\[ v_x = \sqrt{\frac{2E}{m} - \frac{4\pi^2}{T^2} x^2} = \sqrt{\frac{2(5J)}{0.5kg} - \frac{4\pi^2}{(3.14s)^2}} (1)^2 = \sqrt{(20m^2 / s^2) - (m^2 / s^2)} = 4m / s \]
Exam 2 Fall 2011: Problem 52

- In the figure, two blocks (m = 5 kg and M = 15 kg) and a spring (k = 196 N/m) are arranged on a horizontal frictionless surface. If the smaller block begins to slip when the amplitude of the simple harmonic motion is greater than 0.5 m, what is the coefficient of static friction between the two blocks? (Assume that the system is near the surface of the Earth.)

Answer: 0.5
% Right: 26%

\[ f_s = ma_x \leq \mu_s mg \quad a_{\max} = \mu_s g \quad a_{\max} = \omega_{\text{spring}}^2 A = \frac{k}{(M + m)} A \]

\[ \mu_s = \frac{k}{(M + m)g} A = \frac{196 \text{ N/m}}{(20 \text{ kg})(9.8 \text{ m/s})}(0.5 \text{ m}) = 0.5 \]
Final Exam Spring 2011: Problem 20

- An ambulance moving at a constant speed of 20 m/s with its siren on overtakes a car moving at a constant speed of 10 m/s in the same direction as the ambulance. As the ambulance approaches the car the driver of the car perceives the siren to have a frequency of 1200 Hz. What frequency does the driver of the car perceive after the ambulance has passed? This takes place on a cold night in Alaska where the speed of sound is 320 m/s.

Answer: 1127 Hz
% Right: 53%

\[ f_{\text{toward}} = \frac{v_{\text{sound}} - v_{\text{car}}}{v_{\text{sound}} - v_A} f_0 \]

\[ f_{\text{away}} = \frac{v_{\text{sound}} + v_{\text{car}}}{v_{\text{sound}} + v_A} f_0 \]

\[ \frac{f_{\text{away}}}{f_{\text{toward}}} = \frac{(v_{\text{sound}} + v_{\text{car}})(v_{\text{sound}} - v_A)}{(v_{\text{sound}} + v_A)(v_{\text{sound}} - v_{\text{car}})} \]

\[ f_{\text{away}} = \frac{(v_{\text{sound}} + v_{\text{car}})(v_{\text{sound}} - v_A)}{(v_{\text{sound}} + v_A)(v_{\text{sound}} - v_{\text{car}})} f_{\text{toward}} = \frac{(320 + 10)(320 - 20)}{(320 + 20)(320 - 10)} (1200 Hz) \approx 1127 Hz \]