REVIEW AND SYNTHESIS: CHAPTERS 6–8

Review Exercises

1. (a) **Strategy** Multiply the extension per mass by the mass to find the maximum extension required.

   **Solution**
   \[
   \left( \frac{1.0 \text{ mm}}{25 \text{ g}} \right) \left( 5.0 \text{ kg} \right) \left( \frac{1000 \text{ g}}{1 \text{ kg}} \right) \left( \frac{1 \text{ m}}{1000 \text{ mm}} \right) = 0.20 \text{ m}
   \]

   (b) **Strategy** Set the weight of the mass equal to the magnitude of the force due to the spring scale. Use Hooke’s law.

   **Solution**
   
   Weight = \(mg = kx\), so \(k = \frac{mg}{x} = \frac{(5.0 \text{ kg})(9.80 \text{ N/kg})}{0.20 \text{ m}} = 250 \text{ N/m}\).

2. **Strategy** Plot force on the \(y\)-axis and the spring length on the \(x\)-axis. Use the graph to answer the questions.

   **Solution**
   Graph the data.

   ![Graph of Force vs. Spring Length](image)

   (a) Determine the slope of the line to find \(k\), since \(F = kx\).
   \[
   k = \frac{1.20 \text{ N} - 0 \text{ N}}{20.0 \text{ cm} - 12.0 \text{ cm}} = \frac{1.20 \text{ N}}{8.0 \text{ cm}} = 0.15 \text{ N/cm}
   \]

   (b) The force on the spring is zero when the spring is relaxed, so from the figure, \(x_0 = 12 \text{ cm}\).

3. **Strategy** Use conservation of energy and Newton’s second law.

   **Solution** Relate the speed to the length of the cord.
   \[
   \Delta K = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2 = -\Delta U = -mg\Delta y = mg \frac{L}{2}, \text{ so } v^2 = gL.
   \]

   Use Newton’s second law and solve for the tension.
   \[
   \sum F_y = T - mg = ma_y = m \frac{v^2}{r} = m \frac{gL}{L} = mg, \text{ so } T = 2mg.
   \]
4. **Strategy** The work done by the muscles is 22% of the energy expended. The gravitational potential energy gained by the person is equal to the work done by the muscles.

**Solution** 
\[ E = W = \Delta U = mgh, \] 
\[ E = \frac{mgh}{0.22} = \frac{(80.0 \text{ kg})(9.80 \text{ N/kg})(15 \text{ m})}{0.22} = \boxed{53 \text{ kJ}}. \]

5. (a) **Strategy** Use the conservation of energy.

**Solution** 
Find the work done by friction.

\[ W_{\text{total}} = W_{\text{friction}} + W_{\text{grav}} = W_{\text{friction}} + mgd \sin \theta = \Delta K = 0 - \frac{1}{2} m v_1^2, \] 
\[ W_{\text{friction}} = -\frac{1}{2} m v_1^2 - mgd \sin \theta = -m \left( \frac{1}{2} v_1^2 + gd \sin \theta \right) \]
\[ = -(100 \text{ kg}) \left[ \frac{1}{2} (2.00 \text{ m/s})^2 + (9.80 \text{ m/s}^2)(1.50 \text{ m}) \sin 30.0^\circ \right] \]
\[ = -940 \text{ J}. \]
Thus, the energy dissipated by friction was \boxed{940 \text{ J}}.

(b) **Strategy** Use Newton’s second law.

**Solution** 
Find the normal force on the crate.

\[ \Sigma F_y = N - mg \cos \theta = 0, \] 
so \[ N = mg \cos \theta. \]

Since \( v_{\text{fx}}^2 - v_{\text{ix}}^2 = 0 - v_1^2 = 2a_x \Delta x = 2a_x d, \) the acceleration of the crate is \(-v_1^2/(2d)\).

Find the force of sliding friction.

\[ \Sigma F_x = -f_k + mg \sin \theta = -\mu_k mg \cos \theta + mg \sin \theta = ma_x = -m \frac{v_1^2}{2d}, \] 
\[ \mu_k = \tan \theta + \frac{v_1^2}{2dg \cos \theta} = \tan 30.0^\circ + \frac{(2.00 \text{ m/s})^2}{2(1.50 \text{ m})(9.80 \text{ m/s}^2) \cos 30.0^\circ} = \boxed{0.734}. \]

6. **Strategy** Use conservation of energy.

**Solution** 
Find the speed of the packing carton at the bottom of the inclined plane.

\[ \Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_1^2 = -\Delta U = -mg \Delta y, \] 
so
\[ v_f = \sqrt{v_1^2 - 2g \Delta y} = \sqrt{(4.0 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(0 - (2.0 \text{ m}) \sin 30.0^\circ)} \]
\[ = \boxed{6.0 \text{ m/s}}. \]

7. **Strategy** Use conservation of energy.

**Solution** 
Find the maximum height of the swing.

\[ \Delta K = 0 - \frac{1}{2} m v_1^2 = -\Delta U = mgh_i - mgh_{\text{max}}, \] 
so \[ h_{\text{max}} = \frac{v_1^2}{2g} + h_i = \frac{(6.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} + 0.50 \text{ m} = \boxed{2.3 \text{ m}}. \]
8. **Strategy** Use conservation of energy and Newton’s second law.

**Solution** Find the normal force on the crate.

\[ \Sigma F_y = N - mg \cos \theta = 0, \text{ so } N = mg \cos \theta. \]

Find the force of sliding friction.

\[ \Sigma F_x = -f_k + mg \sin \theta = -\mu_k mg \cos \theta + mg \sin \theta = ma_x, \text{ so } \]

\[ a_x = -\mu_k g \cos \theta + g \sin \theta = -0.70(9.80 \text{ m/s}^2) \cos 53^\circ + (9.80 \text{ m/s}^2) \sin 53^\circ = 3.7 \text{ m/s}^2. \]

Therefore, the acceleration of the block is \[ 3.7 \text{ m/s}^2 \text{ down the ramp.} \]

9. **Strategy** The collision is inelastic. Use conservation of momentum and energy.

**Solution** Write equations using conservation of momentum and energy.

momentum: \[ m v_i = (m + M) v_f \]

energy: \[ \frac{1}{2} (m + M) v_f^2 = (m + M) g y \]

Find the initial speed of the putty.

\[ \frac{1}{2} (m + M) \left( \frac{mv_i}{m + M} \right)^2 = (m + M) g y \]

\[ \left( \frac{m}{m + M} \right)^2 v_i^2 = 2g y \]

\[ v_i = \sqrt{2g y \left( \frac{m + M}{m} \right)} = \sqrt{2(9.8 \text{ m/s}^2)(1.50 \text{ m}) (0.50 \text{ kg} + 2.30 \text{ kg})} = 30 \text{ m/s} \]

10. **Strategy** Use conservation of energy. The rotational inertia of a hollow cylinder is \[ I = mr^2. \]

**Solution** Find \( d \), the distance the cylinder travels up the incline.

\[ 0 = \Delta K + \Delta U = -\frac{1}{2} m v_i^2 - \frac{1}{2} I \omega^2 + m g d \sin \theta = -\frac{1}{2} m v_i^2 - \frac{1}{2} m_2^2 \left( \frac{v_i}{r} \right)^2 + m g d \sin \theta, \]

so \[ d = \frac{v_i^2}{g \sin \theta} = \frac{(3.00 \text{ m/s})^2}{(9.80 \text{ m/s}^2) \sin 37^\circ} = 1.53 \text{ m}. \]

11. **Strategy** The rotational inertia of a wheel about its central axis is \( I = \frac{1}{2} MR^2 \). Use the rotational form of Newton’s second law.

**Solution**

(a) \[ I = \frac{1}{2} MR^2 = \frac{1}{2} (20.0 \text{ kg})(0.224 \text{ m})^2 = 0.502 \text{ kg} \cdot \text{m}^2 \]

(b) The torque required to overcome the friction must be added to that necessary to accelerate the wheel to 1200 rpm in 4.00 s in the absence of friction to get the net torque necessary to accelerate the wheel to 1200 rpm in 4.00 s. Find the torque.

\[ \Sigma \tau = I \alpha + I \alpha = I(\alpha + \alpha_r) = \frac{1}{2} MR^2 \left[ \frac{\Delta \omega}{\Delta t} + \left( \frac{\Delta \omega}{\Delta t} \right) r \right] \]

\[ = \frac{1}{2} (20.0 \text{ kg})(0.224 \text{ m})^2 \left( \frac{1200 \text{ rpm}}{4.00 \text{ s}} + \frac{1200 \text{ rpm}}{60.0 \text{ s}} \right) \left( \frac{2\pi \text{ rad}}{1 \text{ min}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 17 \text{ N} \cdot \text{m} \]
12. **Strategy** Use the work-kinetic energy theorem. The rotational inertia of a thin hoop is \( I = mr^2 \). The distance \( d \) the bike travels while slowing is equal to the distance the friction force is applied to each wheel.

**Solution** The force of friction on one of the wheels due to one brake pad is \( f_k = \mu_k N = 0.90N \). Assuming constant acceleration, the distance the bike travels in the time \( \Delta t = 4.5 \) s is \( \Delta x = \frac{1}{2}(v_{fx} + v_{ix})\Delta t = (1/2)v_t\Delta t \). Find the normal force on the wheel due to one brake pad.

\[
W_{\text{total}} = W_{\text{friction}} = -4f_kd = -4(0.90)N\left(\frac{v_t\Delta t}{2}\right) = -1.8Nv_t\Delta t = \Delta K = 0 - \frac{1}{2}m_bv_t^2 - 2\left(\frac{1}{2}I_w\omega^2\right) = -\frac{1}{2}m_bv_t^2 - m_wv_t^2, \text{ so } N = \frac{1}{2}m_bv_t + m_wv_t = \frac{(7.5 \text{ m/s})(1.1 \text{ kg}) + 1.3 \text{ kg}}{1.8\Delta t} = \boxed{6.3 \text{ N}}.
\]

13. **Strategy** Use conservation of energy. Let \( d = 2.05 \text{ m} \). Then, the ramp rises \( h = d \sin 5.00^\circ \). The rotational inertia of a uniform sphere is \( \frac{2}{5}mr^2 \).

**Solution** Find the speed of the ball when it reaches the top of the ramp.

\[
0 = \Delta K + \Delta U = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega^2 - \frac{1}{2}mv_i^2 - \frac{1}{2}I\omega_i^2 + mgh
= \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mv_i^2\right)\left(\frac{v_f}{r}\right)^2 - \frac{1}{2}mv_i^2 - \frac{1}{2}\left(\frac{2}{5}mv_i^2\right)\left(\frac{v_f}{r}\right)^2 + mgh
= \frac{7}{10}mv_i^2 + mgh, \text{ so }

v_f = \sqrt{\frac{10}{7}gh} = \sqrt{(2.20 \text{ m/s})^2 - \frac{10}{7}(9.80 \text{ m/s}^2)(2.05 \text{ m})\sin 5.00^\circ} = \boxed{1.53 \text{ m/s}}.
\]

14. **Strategy** Use conservation of angular momentum, Eq. (8-1), Eq. (8-14), and the relationship between period and angular velocity.

**Solution**

(a) Since the angular momentum is conserved, the ratio is \( \boxed{1} \).

(b) Since the rotational inertia is proportional to the square of the radius, \( \omega = \frac{L}{I} \propto \frac{L}{r^2} \).

Find the ratio of the angular velocities.

\[
\frac{\omega_f}{\omega_i} = \left(\frac{r_i^2}{r_f^2}\right) = \left(\frac{1}{1.0 \times 10^{-4}}\right)^2 = \boxed{1.0 \times 10^8}
\]

(c) The rotational kinetic energy is \( K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{L}{\omega}\right)\omega^2 = \frac{1}{2}L\omega \). Find the ratio of the rotational kinetic energies.

\[
\frac{K_f}{K_i} = \frac{\omega_f}{\omega_i} = \boxed{1.0 \times 10^8}
\]

(d) The period is related to the angular velocity by \( T = \frac{2\pi}{\omega} \).

Find the period of the star after collapse.

\[
\frac{T_f}{T_i} = \frac{\omega_i}{\omega_f}, \text{ so } T_f = \frac{\omega_i}{\omega_f}T_i = (1.0 \times 10^{-8})(1.0 \times 10^7 \text{ s}) = \boxed{0.10 \text{ s}}.
\]
15. **Strategy**  Assume the collision time between the dart and the block is short so that the block’s motion during the collision can be neglected. Let the dart be fired to the right and let the positive \( x \)-direction be to the right. Let the origin be at the original position of the block. Use conservation of momentum during the collision and conservation of energy after.

**Solution**  Find the speed \( v \) of the dart and block just after the collision.

\[
p_i = m_d v_d = p_f = (m_d + m_b) v_f, \quad \text{so} \quad v = \frac{m_d}{m_d + m_b} v_d = \frac{0.122 \, \text{kg}}{0.122 \, \text{kg} + 5.00 \, \text{kg}} \times (132 \, \text{m/s}) = 3.144 \, \text{m/s}.
\]

Find the compression of the spring.

\[
W_{\text{total}} = W_{\text{friction}} + W_{\text{spring}} = -f_k x - \frac{1}{2} k x^2 = -\mu_k N x - \frac{1}{2} k x^2 = -\mu_k (m_d + m_b) g x - \frac{1}{2} k x^2 = \Delta K = 0 - \frac{1}{2} (m_d + m_b) v^2,
\]

so \( 0 = kx^2 + 2\mu_k (m_d + m_b) g x - (m_d + m_b)v^2 \). Solve for \( x \).

\[
x = \frac{-2\mu_k (m_d + m_b) g \pm \sqrt{(2\mu_k (m_d + m_b) g)^2 - 4k[-(m_d + m_b)v^2]}}{2k}
\]

\[
= \frac{-(0.630)(5.122 \, \text{kg})(9.80 \, \text{m/s}^2) \pm \sqrt{((0.630)(5.122 \, \text{kg})(9.80 \, \text{m/s}^2))^2 + (8.56 \, \text{N/m})(5.122 \, \text{kg})(3.144 \, \text{m/s})^2}}{8.56 \, \text{N/m}}
\]

\[
= -3.69 \, \text{m} \pm 4.42 \, \text{m} = 0.73 \, \text{m} \text{ or } -8.11 \, \text{m}
\]

Since \( x > 0 \), the maximum compression is \( 0.73 \, \text{m} \).

16. **Strategy**  Use the conditions of equilibrium.

**Solution**  Find the vertical components of the forces on each hinge.

\[
\Sigma F_y = 2F_v - mg = 0, \quad \text{so} \quad F_v = \frac{mg}{2} = \frac{(5.60 \, \text{kg})(9.80 \, \text{m/s}^2)}{2} = 27.4 \, \text{N}.
\]

Let the axis of rotation be the midpoint of the left edge of the door. The only horizontal forces are the horizontal components of the forces on the hinges, therefore, these forces are equal and opposite.

\[
\Sigma \tau = (0.735 \, \text{m}) F_h - (0.380 \, \text{m})(5.60 \, \text{kg})(9.80 \, \text{m/s}^2) + (0.735 \, \text{m})F_h = 0, \quad \text{so} \quad F_h = \frac{(0.380 \, \text{m})(5.60 \, \text{kg})(9.80 \, \text{m/s}^2)}{2(0.735 \, \text{m})} = 14.2 \, \text{N}.
\]

The upper and lower horizontal forces on the hinges are 14.2 N away from the door and 14.2 N toward the door, respectively.
17. **Strategy** Use conservation of energy. The energy delivered to the fluid in the beaker plus the kinetic energies of the pulley, spool, axle, paddles, and the block are equal to the work done by gravity on the block, which is negative the change in the block’s gravitational potential energy. The rotational inertia of the pulley (uniform solid disk) is $\frac{1}{2}mr^2$.

**Solution** Let the energy delivered to the fluid be $E$, the distance the block falls be $h$, and the rotational inertia of the spool, axle, and paddles be $I_s = 0.00140 \text{ kg} \cdot \text{m}^2$. Since the radii of the pulley and the spool are the same ($r$), their tangential speeds are the same, so let $v_p = v_s = v$.

$E = m_bgh + \frac{1}{2}I_p\omega_p^2 + \frac{1}{2}I_s\omega_s^2 + E = \frac{1}{2}m_bv_b^2 + \frac{1}{2}r^2 \left( \frac{v}{r} \right)^2 + \frac{1}{2}I_s \left( \frac{v}{r} \right)^2 + E$

The tangential speeds of the pulley and spool are equal to the speed of the block.

$E = m_bgh + \frac{1}{4}m_bv^2 + \frac{1}{2}I_s \left( \frac{v^2}{r^2} \right) + E = \frac{1}{2}m_bv_b^2 + \frac{1}{4}m_bv^2 + \frac{1}{2}I_s \left( \frac{v^2}{r^2} \right) + E$, so

$E = m_bgh - \frac{v^2(2m_b + m_p + 2I_s/r^2)}{4}$

$= (0.870 \text{ kg})(9.80 \text{ m/s}^2)(2.50 \text{ m}) - \frac{(3.00 \text{ m/s})^2(2(0.870 \text{ kg}) + 0.0600 \text{ kg} + 2(0.00140 \text{ kg} \cdot \text{m}^2))/(0.0300 \text{ m}^2)}{4}$

$= 10.3 \text{ J}$.

18. **Strategy** Use conservation of linear momentum, the work-kinetic energy theorem, and Newton’s second law.

**Solution** According to Newton’s second law, the normal force of the ground on the players is $N = (m_1 + m_2)g$, where $m_1 = 85 \text{ kg}$ and $m_2 = 95 \text{ kg}$. The force of friction is opposite the players direction of motion and has a magnitude of $f_k = \mu_k N = \mu_k (m_1 + m_2)g$. Find the initial speed $v_2$ of the two-player combination.

$p_i = m_1v_1 = p_f = (m_1 + m_2)v_2$, so $v_2 = \frac{m_1v_1}{m_1 + m_2}$.

Find the distance $d$ the players slide.

$W_{\text{total}} = W_{\text{friction}} = -f_kd = -\mu_k(m_1 + m_2)gd = \Delta K = 0 - \frac{1}{2}(m_1 + m_2)v_2^2 = -\frac{1}{2}(m_1 + m_2) \left( \frac{m_1v_1}{m_1 + m_2} \right)^2$, so

$d = \frac{m_1^2v_1^2}{2\mu_k(m_1 + m_2)^2g} = \frac{(85 \text{ kg})^2(8.0 \text{ m/s})^2}{2(0.70)(85 \text{ kg} + 95 \text{ kg})^2(9.80 \text{ m/s}^2)} = 1.0 \text{ m}$.
19. **Strategy** Since the collision is elastic, kinetic energy is conserved. Use conservation of linear momentum and conservation of energy.

**Solution** The kinetic energy of bob A just before it strikes bob B is \( \frac{1}{2}m_A v_{Ai}^2 = m_A g h \), so \( v_{Ai} = \sqrt{2gh} \), where \( h \) is the height fallen by bob A, 5.1 m. Since the mass of bob A is half that of bob B, let \( m = m_A \) and \( 2m = m_B \). Since kinetic energy is conserved, we have

\[
m_A gh = \frac{1}{2} m_A v_{Ai}^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m v_A^2 + m v_B^2 = mgh \quad (1),
\]

where \( v_A \) and \( v_B \) are the speeds of the bobs just after the collision.

Use conservation of linear momentum to find \( v_A \) in terms of \( v_B \).

\[
p_i = m v_{Ai} = p_f = m v_A + 2m v_B, \text{ so } v_A = v_{Ai} - 2v_B = \sqrt{2gh} - 2v_B. \text{ Substitute this into (1) and solve for } v_B.
\]

\[
\frac{1}{2} m (\sqrt{2gh} - 2v_B)^2 + m v_B^2 = \frac{1}{2} m (2gh - 4v_B \sqrt{2gh} + 4v_B^2) + m v_B^2 = mgh, \text{ so } v_B = \frac{2}{3} \sqrt{2gh}. \text{ Thus, we have,}
\]

\[
v_A = \sqrt{2gh} - 2v_B = \sqrt{2gh} - \frac{4}{3} \sqrt{2gh} = -\frac{\sqrt{2gh}}{3}. \text{ Now, we use conservation of energy to find how high each bob rises after the collision.}
\]

\[
mgh_A = \frac{1}{2} m v_A^2 = \frac{1}{2} m \left( -\frac{\sqrt{2gh}}{3} \right)^2 = \frac{mgh}{9}, \text{ so } h_A = \frac{h}{9} = \frac{5.1 \text{ m}}{9} = 0.57 \text{ m}.
\]

\[
2mgh_B = \frac{1}{2} (2m)v_B^2 = m \left( \frac{2\sqrt{2gh}}{3} \right)^2 = \frac{8mgh}{9}, \text{ so } h_B = \frac{4h}{9} = \frac{4(5.1 \text{ m})}{9} = 2.3 \text{ m}.
\]
20. **Strategy** Since the collision is elastic, kinetic energy is conserved. Use conservation of linear momentum. Let the positive \( y \)-direction be along the shooter’s original velocity.

**Solution**

(a) Let the mass of the marble be \( m \), then the mass of the shooter is \( 3m \). After the collision, let the speed of the marble be \( v \) and the speed of the shooter be \( V \).  
\[
\frac{1}{2}(3m)v^2 = \frac{1}{2}(3m)V_x^2 + \frac{1}{2}mV_y^2, \quad \text{so} \quad 3V_x^2 = 3V_y^2 + v^2 + v^2 \quad (1).
\]
\[
p_{iy} = 0 = p_{ix} = 3mV_x + mv_x, \quad \text{so} \quad 3V_x = v \sin 40^\circ \quad (2).
\]
\[
p_{iy} = 3mV_y = p_{iy} = 3mV_y + mv_y, \quad \text{so} \quad 3V_y = V_y + v \cos 40^\circ \quad (3).
\]

We have three equations and three unknowns \( (V_x, V_y, \text{and } v) \). From (2), we have \( v = \frac{3V_x}{\sin 40^\circ} \quad (4) \).

Substituting this into (3) and solving for \( V_y \) gives \( V_y = V_i - V_x \cot 40^\circ \quad (5) \). Substitute (4) and (5) into (1) and solve for \( V_x \).

\[
3V_i^2 = 3V_x^2 + 3(V_i - V_x \cot 40^\circ)^2 + \left( \frac{3V_x}{\sin 40^\circ} \right)^2
\]
\[
3V_i^2 = 3V_x^2 + 3(V_i - 2V_i \cot 40^\circ + V_x \cot 40^\circ)^2 + \frac{9V_x^2}{\sin^2 40^\circ}
\]
\[
0 = V_x^2 - 2V_i \cot 40^\circ + V_x \cot 40^\circ \cot 40^\circ + \frac{3V_x^2}{\sin^2 40^\circ}
\]
\[
0 = V_x \sin^2 40^\circ - 2V_i \cot 40^\circ \sin 40^\circ \sin 40^\circ + V_x \cot^2 40^\circ + 3V_x
\]
\[
0 = V_x^2 \sin^2 40^\circ + 2V_i \cot 40^\circ \sin 40^\circ \sin 40^\circ + 3V_x - 2V_i \cos 40^\circ \sin 40^\circ
\]
\[
0 = V_x - 2V_i \cos 40^\circ \sin 40^\circ + V_x \cot 40^\circ \sin 40^\circ
\]
\[
4V_x = 2V_i \cos 40^\circ \sin 40^\circ = V_i \sin 80^\circ
\]
\[
V_x = \frac{V_i \sin 80^\circ}{4}
\]

Use this result and (5) to find \( V \).

\[
V = \sqrt{V_x^2 + V_y^2} = \sqrt{V_x^2 + (V_i - V_x \cot 40^\circ)^2} = \sqrt{\left( \frac{V_i \sin 80^\circ}{4} \right)^2 + \left( V_i - \frac{V_i \sin 80^\circ}{4} \cot 40^\circ \right)^2}
\]
\[
= V_i \sqrt{\frac{\sin^2 80^\circ}{16} + \left[ 1 - \frac{\sin 80^\circ \cot 40^\circ}{4} \right]^2} = (3.2 \text{ m/s}) \sqrt{\frac{\sin^2 80^\circ}{16} + \left[ 1 - \frac{\sin 80^\circ \cot 40^\circ}{4} \right]^2} = 2.4 \text{ m/s}
\]

(b) Substituting the result for \( V_x \) in (4) gives

\[
v = \frac{3V_x}{\sin 40^\circ} = \frac{3V_i \sin 80^\circ}{4 \sin 40^\circ} = \frac{3(3.2 \text{ m/s}) \sin 80^\circ}{4 \sin 40^\circ} = \frac{3.7 \text{ m/s}}{4 \sin 40^\circ}
\]

(c) According to the way we set up the coordinate system, the tangent of \( \theta \) is equal to \( V_x \) divided by \( V_y \) instead of the usual \( V_y \) divided by \( V_x \).

\[
\theta = \tan^{-1} \frac{V_x}{V_y} = \tan^{-1} \frac{V_i \sin 80^\circ / 4}{V_i - \frac{V_i \sin 80^\circ}{4} \cot 40^\circ} = \tan^{-1} \frac{\sin 80^\circ}{4 - \frac{\sin 80^\circ \cot 40^\circ}{4}} = 19^\circ
\]
21. **Strategy** Use energy conservation to find the speed of Jones just before he grabs Smith. Then, use momentum conservation to find the speed of both just after. Finally, again use energy conservation to find the final height.

**Solution** Find Jones’s speed, \( v_J \).

\[
\frac{1}{2} m_J v_J^2 = m_J g h_J, \text{ so } v_J = \sqrt{2g h_J}.
\]

Find the speed of both, \( v \).

\[
p_f = m_J v_J = p_i = (m_J + m_S) v, \text{ so } v = \frac{m_J v_J}{m_J + m_S} = \frac{m_J \sqrt{2g h_J}}{m_J + m_S}.
\]

Find the final height, \( h \).

\[
(m_J + m_S) g h = \frac{1}{2} (m_J + m_S) \left( \frac{m_J \sqrt{2g h_J}}{m_J + m_S} \right)^2, \text{ so } h = \frac{m_J^2 h_J}{(m_J + m_S)^2} = \frac{(78.0 \text{ kg})^2 (3.70 \text{ m})}{(78.0 \text{ kg} + 55.0 \text{ kg})^2} = 1.27 \text{ m}.
\]

22. (a) **Strategy** Use the definition of angular acceleration.

**Solution**

\[
\alpha = \frac{\Delta \omega}{\Delta t} = \frac{11 \text{ rad/s} - 0}{0.20 \text{ s}} = \frac{55 \text{ rad/s}^2}{\text{.}}
\]

(b) **Strategy** Use Newton’s second law for rotation.

**Solution** Find the torque.

\[
\Sigma \tau = I \alpha = I \frac{\Delta \omega}{\Delta t} = (1.5 \text{ kg} \cdot \text{m}^2) \frac{11 \text{ rad/s} - 0}{0.20 \text{ s}} = 83 \text{ N} \cdot \text{m}.
\]

(c) **Strategy** Use Eq. (5-21).

**Solution** Let \( \Delta \theta_1 \) be the angle during spin-up and \( \Delta \theta_2 \) be the angle during spin-down.

\[
\omega_i^2 - \omega_1^2 = \omega_2^2 - 0 = 2 \alpha_1 \Delta \theta_1, \text{ so } \Delta \theta_1 = \frac{\omega_2^2}{2 \alpha_1}. \quad \omega_i^2 - \omega_2^2 = 0 - \omega_2^2 = 2 \alpha_2 \Delta \theta_2, \text{ so } \Delta \theta_2 = \frac{\omega_2^2}{2 \alpha_2}.
\]

Find \( \Delta \theta_1 + \Delta \theta_2 \).

\[
\Delta \theta_1 + \Delta \theta_2 = \frac{\omega_2^2}{2 \alpha_1} + \frac{-\omega_2^2}{2 \alpha_2} = \frac{\omega_2^2}{2} \left( \frac{1}{\alpha_1} - \frac{1}{\alpha_2} \right) = \left( \frac{11 \text{ rad/s}}{2} \right) \left( \frac{1}{55 \text{ rad/s}^2} - \frac{1}{-9.8 \text{ rad/s}^2} \right) = 7.3 \text{ rad}.
\]

(d) **Strategy** Use Eq. (5-18) and the relationship between angular speed and linear speed.

**Solution** Find the speed of a point halfway along the radius of the disk 0.20 s after the accelerating torque is removed.

\[
\omega_i - \omega_1 = \frac{v}{r} - \omega_1 = \alpha \Delta t, \text{ so }
\]

\[
v = r (\alpha \Delta t + \omega_i) = \frac{0.115 \text{ m}}{2} \left[ (-9.8 \text{ rad/s}^2) (0.20 \text{ s}) + (11 \text{ rad/s}) \right] = 0.52 \text{ m/s}.
\]
23. **Strategy** Use the work-kinetic energy theorem and Newton’s second law.

**Solution** \( \Sigma F_y = N - mg \cos \theta = 0, \) so \( N = mg \cos \theta. \) Thus, \( f_k = \mu_k mg \cos \theta. \)

\[
W_{\text{total}} = W_{\text{grav}} + W_{\text{friction}} = mgd \sin \theta - \mu_k mg \cos \theta d = \Delta K = \frac{1}{2} mv_f^2, \text{ so}
\]

\[
v_f = \sqrt{2gd(\sin \theta - \mu_k \cos \theta)} = \sqrt{2(9.80 \text{ m/s}^2)(0.300 \text{ m}) \sin 60.0^\circ - 0.38 \cos 60.0^\circ}) = 2.0 \text{ m/s}. 
\]

24. **Strategy** The cylinder falls a vertical distance \( h = d \sin \theta = (0.300 \text{ m}) \sin 60.0^\circ \) as it rolls down the incline. The rotational inertia of a uniform solid cylinder is \( \frac{1}{2} mr^2. \) Use conservation of energy.

**Solution** Find the cylinder’s final speed.

\[
0 = \Delta K + \Delta U = \frac{1}{2} mv_f^2 + \frac{1}{2} I \omega_f^2 - 0 + 0 - mgh = \frac{1}{2} mv_f^2 + \frac{1}{2} \left( \frac{1}{2} mr^2 \right) \left( \frac{v_f}{r} \right)^2 - mgd \sin \theta
\]

\[
= \frac{3}{4} mv_f^2 - mgd \sin \theta, \text{ so } v_f = \sqrt{\frac{4}{3} gd \sin \theta} = \sqrt{\frac{4}{3}(9.80 \text{ m/s}^2)(0.300 \text{ m}) \sin 60.0^\circ} = 1.84 \text{ m/s}. 
\]

25. **Strategy** Use conservation of linear momentum.

**Solution**

\[
p_{fx} = m_b v_{b_f} = p_{fx} = (m_b + m_c)v_{f_x}, \text{ so } v_{f_x} = \frac{m_b v_b}{m_b + m_c}.
\]

\[
p_{fy} = m_c v_{c_f} = p_{fy} = (m_b + m_c)v_{f_y}, \text{ so } v_{f_y} = \frac{m_c v_c}{m_b + m_c}.
\]

Compute the magnitude of the final velocity.

\[
v = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left( \frac{m_b v_b}{m_b + m_c} \right)^2 + \left( \frac{m_c v_c}{m_b + m_c} \right)^2} = \frac{\sqrt{(m_b v_b)^2 + (m_c v_c)^2}}{m_b + m_c}
\]

\[
= \sqrt{[(2.00 \text{ kg})(2.70 \text{ m/s})]^2 + [(1.50 \text{ kg})(-3.20 \text{ m/s})]^2} = 2.06 \text{ m/s}
\]

Compute the angle.

\[
\theta = \tan^{-1} \frac{v_{fy}}{v_{fx}} = \tan^{-1} \frac{m_c v_c}{m_b v_b} = \frac{m_c v_c}{m_b v_b} = \frac{\tan^{-1} \left( \frac{1.50 \text{ kg})(-3.20 \text{ m/s})}{2.00 \text{ kg}(2.70 \text{ m/s})} \right)}{\text{kg}} = -41.6^\circ
\]

The velocity of the block and the clay after the collision is \( 2.06 \text{ m/s at } 41.6^\circ \text{ S of E}. \)
26. (a) **Strategy** Use conservation of angular momentum.

**Solution** Find the tangential speed of the skaters after they grab the rods.

\[ L_i = I_1\omega_1 + I_2\omega_2 = m_1r_1^2\frac{v_1}{r} + m_2r_2^2\frac{v_2}{r} = m_1rv_1 + m_2rv_2 = L_f = I_f\omega_f = m_1rv + m_2rv, \]

so

\[ m_1v_1 + m_2v_2 = (m_1 + m_2)v. \]

Solve for the tangential speed.

\[ (m_1 + m_2)v = m_1v_1 + m_2v_2, \quad v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(60.0 \text{ kg})(6.0 \text{ m/s}) + (30.0 \text{ kg})(2.0 \text{ m/s})}{60.0 \text{ kg} + 30.0 \text{ kg}} = 4.7 \text{ m/s}. \]

(b) **Strategy and Solution** According to the RHR, the angular momentum is **upward, away from the ice** before and after the collision. Angular momentum is conserved in magnitude and direction.

27. (a) **Strategy** Use conservation of angular momentum, since no external torques act on the two-disk system.

**Solution** Find the final angular velocity.

\[ L_f = I_f\omega_f = L_i = I_i\omega_i, \quad \text{so} \quad \omega_f = \frac{I_i\omega_i}{I_f} = \frac{I_i\omega_i}{I_1 + \frac{1}{2}mr^2} = \frac{\omega_i}{\frac{1}{1 + \frac{mr^2}{(MR)^2}}}. \]

(b) **Strategy and Solution** The total angular momentum does not change, since no external torques act on the system.

(c) **Strategy** Compute the initial and final total kinetic energies and compare their values.

**Solution**

\[ K_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2 = \frac{1}{4}MR^2\omega_i^2 \]

\[ K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}\left(\frac{1}{2}MR^2 + \frac{1}{2}mr^2\right)\left(\frac{\omega_i}{\frac{1}{1 + \frac{mr^2}{(MR)^2}}}\right)^2 = \frac{1}{2}\left(\frac{1}{2}MR^2 + \frac{1}{2}mr^2\right)\left(\frac{\frac{1}{2}MR^2\omega_i}{\frac{1}{2}MR^2 + \frac{1}{2}mr^2}\right)^2 \]

\[ = \frac{\frac{1}{2}(\frac{1}{2}MR^2)^2}{\frac{1}{2}MR^2 + \frac{1}{2}mr^2} \omega_i^2 = \frac{\frac{1}{4}MR^2\omega_i^2}{\frac{1}{1 + \frac{mr^2}{(MR)^2}}} = K_i \]

So, \( K_f \neq K_i \). Therefore, the answer is **yes, the kinetic energy changes**.

28. (a) **Strategy** The candy is released with a horizontal speed equal to the tangential speed of the pocket of the rotating wheel. Use the relationship between angular and tangential speed and the equations of motion for a changing velocity.

**Solution** Find the time it takes for the candy to land.

\[ \Delta y = \frac{1}{2}a_\text{y}(\Delta t)^2 = \frac{1}{2}(-g)(\Delta t)^2, \quad \text{so} \quad \Delta t = \frac{-2\Delta y}{g}. \]

Find the candy’s distance from its starting point.

\[ v = r|\omega|, \quad \Delta x = v\Delta t = r|\omega|\sqrt{\frac{-2\Delta y}{g}} = (0.120 \text{ m})(1.60 \text{ Hz})(\frac{2\pi \text{ rad}}{\text{cycle}})\sqrt{\frac{2(-0.240 \text{ m})}{9.80 \text{ m/s}^2}} = 0.267 \text{ m}. \]

(b) **Strategy** Use the relationship between radial acceleration and angular speed.

**Solution** Find the radial acceleration of the candy.

\[ a_r = \omega^2r = (1.60 \text{ Hz})^2(2\pi \text{ rad/cycle})^2(0.120 \text{ m}) = 12.1 \text{ m/s}^2. \]
29. (a) **Strategy** Consider the work-kinetic energy theorem and the impulse momentum theorem.

**Solution** Since the Romulan ship is twice as massive as the Vulcan ship, the Romulan ship will not travel as far as the Vulcan ship for the same engine force, since \( \Delta x = (1/2) a (\Delta t)^2 = (1/2) (F/m)(\Delta t)^2 \). Since \( W = F \Delta x = \Delta K \), the Vulcan ship will have the greater kinetic energy. Since \( \Delta p = F \Delta t \), the ships will have the same momentum.

(b) **Strategy** Consider the work-kinetic energy theorem and the impulse momentum theorem.

**Solution** Since the distances and the forces are the same, and since \( W = F \Delta x = \Delta K \), the ships will have the same kinetic energy. Since \( \Delta x = (1/2) a (\Delta t)^2 = (1/2) (F/m)(\Delta t)^2 \), the more massive Romulan ship will have to fire its engines longer than the Vulcan ship to travel the same distance. Since \( \Delta p = F \Delta t \) and the forces are the same, the Romulan ship will have the greater momentum.

(c) **Strategy** Refer to parts (a) and (b).

**Solution** For part (a), we have the following:

Vulcan:
\[
\Delta K = W = F \Delta x = F \left[ \frac{F}{2m} (\Delta t)^2 \right] = \frac{(9.5 \times 10^6 \text{ N})^2 (100 \text{ s})^2}{2(65,000 \text{ kg})} = 6.9 \times 10^{12} \text{ J}
\]

\[\Delta p = F \Delta t = (9.5 \times 10^6 \text{ N})(100 \text{ s}) = 9.5 \times 10^8 \text{ kg \cdot m/s}\]

Romulan:
\[
\Delta K = W = F \Delta x = F \left[ \frac{F}{2m} (\Delta t)^2 \right] = \frac{(9.5 \times 10^6 \text{ N})^2 (100 \text{ s})^2}{2(130,000 \text{ kg})} = 3.5 \times 10^{12} \text{ J}
\]

\[\Delta p = F \Delta t = (9.5 \times 10^6 \text{ N})(100 \text{ s}) = 9.5 \times 10^8 \text{ kg \cdot m/s}\]

In part (a), the momenta are the same, \(9.5 \times 10^8 \text{ kg \cdot m/s}\), but the kinetic energies differ: Vulcan at \(6.9 \times 10^{12} \text{ J}\) and Romulan at \(3.5 \times 10^{12} \text{ J}\).

For part (b), we have the following:

Vulcan:
\[
\Delta K = W = F \Delta x = (9.5 \times 10^6 \text{ N})(100 \text{ m}) = 9.5 \times 10^8 \text{ J}
\]

Since \( K = \frac{1}{2} m v^2 = \frac{p^2}{2m} \), \( p = \sqrt{2mK} = \sqrt{2(65,000 \text{ kg})(9.5 \times 10^8 \text{ J})} = 1.1 \times 10^7 \text{ kg \cdot m/s}\).

Romulan:
\[
\Delta K = W = F \Delta x = (9.5 \times 10^6 \text{ N})(100 \text{ m}) = 9.5 \times 10^8 \text{ J}
\]

\[p = \sqrt{2mK} = \sqrt{2(2(65,000 \text{ kg})(9.5 \times 10^8 \text{ J})} = 1.6 \times 10^7 \text{ kg \cdot m/s}\]

In part (b), the kinetic energies are the same, \(9.5 \times 10^8 \text{ J}\), but the momenta differ: Vulcan at \(1.1 \times 10^7 \text{ kg \cdot m/s}\) and Romulan at \(1.6 \times 10^7 \text{ kg \cdot m/s}\).
30. (a) **Strategy** Use conservation of energy.

**Solution** Let \( d \) be the distance moved along the incline by \( m_2 \). Both masses move the same distance and have the same speed, since they are connected by a rope.

\[
0 = \Delta K + \Delta U = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 - 0 - 0 + m_1 g (-d \sin \theta) + m_2 g d \sin \phi, \text{ so}
\]

\[
v = \sqrt{\frac{2gd (m_1 \sin \theta - m_2 \sin \phi)}{m_1 + m_2}} = \sqrt{\frac{2(9.80 \text{ m/s}^2)(2.00 \text{ m})[(6.00 \text{ kg}) \sin 36.9^\circ - (4.00 \text{ kg}) \sin 45.0^\circ]}{6.00 \text{ kg} + 4.00 \text{ kg}}} = 1.7 \text{ m/s}.
\]

(b) **Strategy** Use Newton’s second law.

**Solution** Let the positive direction be along the inclines from left to right.

For \( m_1 \): \( \Sigma F = -T + m_1 g \sin \theta = m_1 a \), so \( T = m_1 g \sin \theta - m_1 a \).

For \( m_2 \): \( \Sigma F = T - m_2 g \sin \phi = m_2 a \), so \( T = m_2 g \sin \phi + m_2 a \).

Solve for the acceleration.

\[
a = \frac{g(m_1 \sin \theta - m_2 \sin \phi)}{m_1 + m_2} = \frac{(9.80 \text{ m/s}^2)[(6.00 \text{ kg}) \sin 36.9^\circ - (4.00 \text{ kg}) \sin 45.0^\circ]}{6.00 \text{ kg} + 4.00 \text{ kg}} = 0.76 \text{ m/s}^2.
\]

Find the speed.

\[
v_f^2 - v_i^2 = 2ad, \text{ so } v = \sqrt{2ad} = \sqrt{2(0.76 \text{ m/s}^2)(2.00 \text{ m})} = 1.7 \text{ m/s}.
\]

31. **Strategy** Refer to the figure and use conservation of energy.

**Solution**

(a) According to the graph, the particle’s potential energy is \(-550 \text{ J}\). Since \( E = K + U \), the kinetic energy of the particle is \( K = E - U = -100 \text{ J} - (-550 \text{ J}) = 450 \text{ J} \).

(b) The total energy is as given, \(-100 \text{ J}\). According to the graph, the potential energy is \(-100 \text{ J}\). The kinetic energy is \( K = E - U = -100 \text{ J} - (-100 \text{ J}) = 0 \).

(c) The kinetic energy is \( K = E - U = -100 \text{ J} - (-300 \text{ J}) = 200 \text{ J} \).

(d) The particle has a kinetic energy of 450 J at \( t = 0 \), and we are told the motion is to the left. The particle will continue moving left but the kinetic energy will decrease by 450/4.5 J for every cm of travel until it reaches \( x = 1 \text{ cm} \). At this point \( K = 0 \), and the particle has stopped instantaneously. It will next move to the right with an increasing \( K \) until it reaches \( x = 5.5 \text{ cm} \). At this point \( K = 450 \text{ J} \), and this kinetic energy will be maintained as it continues moving right until it reaches \( x = 11 \text{ cm} \). At this point, its kinetic energy will decrease by 450/2.5 J for every cm of travel until it reaches \( x = 13.5 \text{ cm} \). At this point \( K = 0 \), and the particle has again stopped instantaneously. It will then turn around again.
32.  (a) **Strategy**  Use conservation of angular momentum at the moment of impact.

**Solution** \( L_i = I \omega_i = I \omega_f + rp \), where \( I \) is the rotational inertia of the blade, \( r \) is the distance from the center of the blade to the location of impact, and \( p = mv \) is the momentum of the stone just after it is struck. 

Find the speed of the stone.

\[
L_i = I \omega_i \quad \Rightarrow \quad I(\omega_i - \omega_f) = rmv_{\tan}
\]

\[
v_{\tan} = \frac{I(\omega_i - \omega_f)}{rm} = \frac{1}{12} ML^2 (\omega_i - \omega_f) = \frac{1}{12} M(2r)^2 (\omega_i - \omega_f) = \frac{Mr(\omega_i - \omega_f)}{3m}
\]

\[
= \frac{(2.0 \text{ kg})(0.25 \text{ m})[2\pi(60 \text{ rev/s} - 55 \text{ rev/s})]}{3(0.10 \text{ kg})} = 52 \text{ m/s}
\]

(b) **Strategy**  Find time it takes for the stone to reach the house. Then use this time to find the distance the stone falls just before it reaches the window.

**Solution**  Find the time.

\[
\Delta x = v \Delta t, \quad \Rightarrow \quad \Delta t = \frac{\Delta x}{v_{\tan}} = \frac{10.0 \text{ m}}{52.4 \text{ m/s}} = 0.191 \text{ s}
\]

Find the distance the stone falls.

\[
\Delta y = \frac{1}{2} g (\Delta t)^2 = \frac{1}{2} (9.8 \text{ m/s}^2)(0.191 \text{ s})^2 = -0.18 \text{ m}
\]

Since 0.18 m is less than half of 1.00 m (0.50 m), the stone hits the window.

33.  **Strategy**  Use conservation of energy. \( m \) is the mass of one wheel. \( M \) is the total mass of the system. \( v \) is the speed of the center of mass of the system (which is the same as the speed of a point on either wheel).

**Solution**

(a)  \( K_{rot} = \frac{1}{2} I \omega^2 = \frac{1}{2} mv^2 = K_{trans} \) for one wheel.  \( K_{rot, total} = 2 \cdot \frac{1}{2} mv^2 = mv^2 \) and  \( K_{trans, total} = \frac{1}{2} Mv^2 \).

\[
K_{total} = U_i
\]

\[
mv^2 + \frac{1}{2} Mv^2 = MgH
\]

\[
\nu^2 (2m + M) = 2MgH
\]

\[
v = \frac{2MgH}{2m + M} = \sqrt{\frac{2(80.0 \text{ kg})(9.8 \text{ m/s}^2)(20.0 \text{ m})}{2(1.5 \text{ kg}) + 80.0 \text{ kg}}} = 19 \text{ m/s}
\]

(b)  Since the speed depends upon the combined total mass of the system, the speed at the bottom would not be the same for a less massive rider.  The answer is \[\text{no}\].
34. **Strategy** Find the change in height from the initial height to the height at which the vine breaks; then use the change in height to find Tarzan’s speed when the vine breaks. Use Newton’s second law to find the tension when the vine breaks.

**Solution**

(a) Find $L$, the length of the vine.

\[
\sin \theta_i = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5.00 \text{ m}}{L}, \quad \text{so} \quad L = \frac{5.00 \text{ m}}{\sin \theta_i} = \frac{5.00 \text{ m}}{\sin 60^\circ} = 5.77 \text{ m}.
\]

Find the change in height.

\[
\Delta y = y_f - y_i = L \cos \theta_f - L \cos \theta_i = \frac{5.00 \text{ m}}{\sin \theta_i} (\cos \theta_f - \cos \theta_i)
\]

Find Tarzan’s speed when the vine breaks.

\[
v_f^2 - v_i^2 = v^2 - 0 = 2g [\Delta y] = 2gL(\cos \theta_f - \cos \theta_i) = 2g \frac{5.00 \text{ m}}{\sin \theta_i} (\cos \theta_f - \cos \theta_i)
\]

\[
v = \sqrt{\frac{2gL(\cos \theta_f - \cos \theta_i)}{\sin \theta_i}} = \sqrt{\frac{2(9.80 \text{ m/s}^2)(5.00 \text{ m})}{\sin 60.0^\circ} (\cos 20.0^\circ - \cos 60.0^\circ)} = 7.05 \text{ m/s}
\]

Find the tension.

\[
\Sigma F = T - mg \cos \theta_i = \frac{mv^2}{r}, \quad \text{so}
\]

\[
T = \frac{mv^2}{r} - mg \cos \theta_i = \frac{mgv^2}{gL} + mg \cos \theta_i = mg \left(\frac{v^2}{gL} + \cos \theta_i\right)
\]

\[
= (900.0 \text{ N}) \left[\frac{(7.05 \text{ m/s})^2}{(9.80 \text{ m/s}^2)(5.77 \text{ m})} + \cos 20^\circ\right] = 1.64 \text{ kN}
\]

(b) At the moment of the vine breaking, the distance to ground level is

\[8.00 \text{ m} - L \cos \theta_f = 8.00 \text{ m} - (5.77 \text{ m}) \cos 20.0^\circ = 2.58 \text{ m};\]

and the distance to the river’s edge is

\[L \sin \theta_f = (5.77 \text{ m}) \sin 20.0^\circ = 1.97 \text{ m}.
\]

The time it takes for Tarzan to reach ground level is given by

\[
\Delta y = -v \sin \theta_i \Delta t - \frac{1}{2} g(\Delta t)^2, \quad \text{or} \quad 0 = \frac{1}{2} g(\Delta t)^2 + v \sin \theta_i \Delta t + \Delta y.
\]

Using the quadratic formula, we find $\Delta t = 0.52 \text{ s}$. The horizontal distance traveled in this time is

\[
\Delta x = v \cos \theta_i \Delta t = (7.05 \text{ m/s}) \cos 20.0^\circ (0.52 \text{ s}) = 3.4 \text{ m}.\]

Since $3.4 \text{ m} > 1.97 \text{ m}$, Tarzan lands safely on the other side. The answer is **yes**.
35. **Strategy** Use the relationship between energy and work to find the boy’s speed just before his friend lands on the sled and the speed when the two reach the bottom. Use conservation of momentum to find the speed of the boys just after the friend lands on the sled.

**Solution** Find the speed of the boy.

\[
\Delta K = \frac{1}{2} m_1 v_1^2 - 0 = -\Delta U + W_{\text{friction}} = m_1 g h_1 - f d_1 = m_1 g d_1 \sin \theta - \mu m_1 g \cos \theta d_1,
\]

so

\[
v_1 = \sqrt{2 g d_1 (\sin \theta - \mu \cos \theta)} = \sqrt{2(9.8 \text{ m/s}^2)(20 \text{ m})(\sin 15^\circ - 0.12\cos 15^\circ)} = 7.48 \text{ m/s}.
\]

Find the initial speed of the two boys.

\[
p_i = m_1 v_1 = p_f = (m_1 + m_2)v_2, \quad \text{so} \quad v_2 = \frac{m_1 v_1}{m_1 + m_2} = \frac{(60 \text{ kg})(7.48 \text{ m/s})}{60 \text{ kg} + 50 \text{ kg}} = 4.08 \text{ m/s}.
\]

Find the final speed of the two boys.

\[
\Delta K = -\Delta U + W_{\text{friction}}
\]

\[
\frac{1}{2} (m_1 + m_2) v_3^2 - \frac{1}{2} (m_1 + m_2) v_2^2 = (m_1 + m_2) g d_2 - f d_2 = (m_1 + m_2) gd_2 \sin \theta - \mu (m_1 + m_2) g \cos \theta d_2
\]

\[
v_3^2 = 2 g d_2 \sin \theta - 2 \mu g \cos \theta d_2 + v_2^2
\]

\[
v_3 = \sqrt{2 g d_2 (\sin \theta - \mu \cos \theta) + v_2^2}
\]

\[
= \sqrt{2(9.8 \text{ m/s}^2)(50 \text{ m})(\sin 15^\circ - 0.12\cos 15^\circ) + (4.08 \text{ m/s})^2} = 13 \text{ m/s}
\]

36. **Strategy** Use conservation of momentum and the equations for motion with a constant acceleration.

**Solution**

(a) The banana will fall at the same rate as the monkey; therefore, you should throw the banana directly at the monkey.

\[
\tan \theta = \frac{3.33 \text{ m} + 1.67 \text{ m}}{3.00 \text{ m}} \quad \text{so} \quad \theta = \tan^{-1} \frac{5.00}{3.00} = 59.0^\circ \text{ above the horizontal}.
\]

(b) Since the banana will fall at the same rate as the monkey, regardless of the launch speed of the banana, the launch angle is the same for all launch speeds. Relatively high launch speeds will reach the monkey relatively sooner (and higher); relatively low launch speeds will reach the monkey relatively later (and lower).

(c) Find the time it takes the banana to reach the monkey.

\[
\Delta y = -\frac{1}{2} g (\Delta t)^2, \quad \text{so} \quad \Delta t = \sqrt{\frac{2 \Delta y}{g}} = \sqrt{\frac{2(-1.67 \text{ m})}{9.80 \text{ m/s}^2}} = 0.5838 \text{ s} = t.
\]

The banana reaches the monkey when it has traveled 3.00 m.

\[
\Delta x = v_x \Delta t = v \cos \theta \Delta t, \quad \text{so} \quad v = \frac{\Delta x}{\Delta t \cos \theta} = \frac{3.00 \text{ m}}{(0.5838 \text{ s}) \cos 59.0^\circ} = 9.98 \text{ m/s}.
\]
(d) The speed of the monkey just before the collision is given by $v_{my} = -gt$ and $v_{mx} = 0$. The speed of the banana at this time is given by $v_{by} = v \sin \theta - gt$ and $v_{bx} = v \cos \theta$. Use conservation of momentum.

$$mv_{bx} = (m + M)v_{fx}, \text{ so}$$

$$v_{fx} = \frac{m}{m + M}v_{bx} = \frac{m}{m + M}v \cos \theta = \frac{0.20 \text{ kg}}{0.20 \text{ kg} + 3.00 \text{ kg}}(9.98 \text{ m/s}) \cos 59.0^\circ = 0.321 \text{ m/s}.$$  

$$mv_{by} + Mv_{my} = (m + M)v_{fy}, \text{ so}$$

$$v_{fy} = \frac{mv_{by} + Mv_{my}}{m + M} = \frac{m(v \sin \theta - gt) + M(-gt)}{m + M} = \frac{m}{m + M}v \sin \theta - gt$$

$$= \frac{0.20 \text{ kg}}{0.20 \text{ kg} + 3.00 \text{ kg}}(9.98 \text{ m/s}) \sin 59.0^\circ - (9.80 \text{ m/s}^2)(0.5835 \text{ s}) = -5.18 \text{ m/s}.$$  

The time it takes for the monkey to hit the ground is given by

$$\Delta t = v_{fy}t_2 - \frac{1}{2}gt_2^2, \text{ or } 0 = \frac{1}{2}gt_2^2 - v_{fy}t_2 + \Delta t = (4.90 \text{ m/s}^2)t_2^2 + (5.18 \text{ m/s})t_2 - 5.33 \text{ m}.$$  

Using the quadratic formula, we find $t = 0.64 \text{ s}$. The horizontal distance is

$$d = v_{fy}t = (0.321 \text{ m/s})(0.64 \text{ s}) = [0.21 \text{ m}].$$

MCAT Review

1. **Strategy** Use conservation of momentum.

**Solution**

$p_i = mv_i = p_f = mv_f + p_{\text{wall}}$, so $p_{\text{wall}} = m(v_i - v_f) = (0.2 \text{ kg})(2.0 \text{ m/s} - (-1.0 \text{ m/s})) = 0.6 \text{ kg} \cdot \text{m/s}.$

The correct answer is **D**.

2. **Strategy** Use Hooke’s law.

**Solution** Let up be the positive direction. The gravitational force on the mass is

$$F = mg = (0.10 \text{ kg})(-9.80 \text{ m/s}^2) = -0.98 \text{ N}.$$  

Solving for the spring constant in Hooke’s law, we have

$$k = \frac{F}{x} = \frac{-0.98 \text{ N}}{0.15 \text{ m}} = 6.5 \text{ N/m}.$$  

Thus, the correct answer is **D**.

3. **Strategy** The net torque is zero.

**Solution**

$$\Sigma \tau = 0 = F(0.60 \text{ m}) - (1.0\times10^{-7} \text{ kg})(9.80 \text{ m/s}^2)(0.40 \text{ m}), \text{ so}$$

$$F = \frac{(1.0\times10^{-7} \text{ kg})(9.80 \text{ m/s}^2)(0.40 \text{ m})}{0.60 \text{ m}} = 6.5\times10^{-7} \text{ N}.$$  

The correct answer is **B**.
4. **Strategy** Determine the speed of the first ball just before it collides with the second. The collision is completely inelastic; that is, the balls stick together. Use conservation of momentum to find the speed of the balls after the collision.

**Solution** Find the speed of the first ball just before the collision.

\[ v_{f_1} - v_{i_1} = v_1 - 0 = a_x \Delta t, \text{ so } v_1 = (10 \text{ m/s}^2)(2.0 \text{ s}) = 20 \text{ m/s}. \]

Find the speed \( v \) of the balls just after the collision.

\[ p_i = m_1 v_1 = p_f = (m_1 + m_2) v, \text{ so } v = \frac{m_1 v_1}{m_1 + m_2} = \frac{(0.50 \text{ kg})(20 \text{ m/s})}{0.50 \text{ kg} + 1.0 \text{ kg}} = 6.7 \text{ m/s}. \]

The correct answer is \[ \text{B} \].

5. **Strategy** Use Newton’s second law and Eq. (6-27).

**Solution** The gravitational force working against the motion of the car as it climbs the hill is \( mg \sin 10^\circ \), so the additional power required is

\[ P_{car} = -P_{grav} = -F \cos 180^\circ = (mg \sin 10^\circ) v = (1000 \text{ kg})(10 \text{ m/s}^2)(\sin 10^\circ)(15 \text{ m/s}) = 1.5 \times 10^5 \times \sin 10^\circ \text{ W}. \]

The correct answer is \[ \text{D} \].

6. **Strategy** Find the vertical distance the patient would have climbed had the treadmill been stationary (and very long). Then, find the work done by the patient on the treadmill.

**Solution** The “distance” walked along the incline is \( (2 \text{ m/s})(600 \text{ s}) = 1200 \text{ m} \). Thus, the vertical distance climbed is \( (1200 \text{ m}) \sin 30^\circ = 600 \text{ m} \). The work done is

\[ W = Fd = mgd = (90 \text{ kg})(10 \text{ m/s}^2)(600 \text{ m}) = 0.54 \text{ MJ}. \]

The correct answer is \[ \text{C} \].

7. **Strategy** Find the angle between the force exerted by the patient and the patient’s velocity. Use Eq. (6-27).

**Solution** The force due to gravity is down, so the force exerted by the patient is up. The velocity is directed at the angle of the incline, or \( 30^\circ \) above the horizontal, so the angle between the force and the velocity is \( 60^\circ \). Compute the mechanical power output of the patient.

\[ P = Fv \cos \theta = mgv \cos \theta = (100 \text{ kg})(10 \text{ m/s}^2)(3 \text{ m/s}) \cos 60^\circ = 1500 \text{ W}. \]

The correct answer is \[ \text{B} \].

8. **Strategy and Solution** The force pushing each friction pad is normal to the wheel; that is, it is the normal force \( f_k = \mu_k N \). Solve for the normal force.

\[ N = \frac{f_k}{\mu_k} = \frac{20 \text{ N}}{0.4} = 50 \text{ N} \]

This is the total force. The force pushing each friction pad is half this, or 25 N. The correct answer is \[ \text{B} \].
9. **Strategy** Find the average tangential speed at the friction pads. Then, use the relationship between tangential speed and radial acceleration.

**Solution**

The average tangential speed is \( v = \frac{4800 \text{ m}}{20 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 4.0 \text{ m/s} \). The radial acceleration is

\[
a_r = \frac{v^2}{r} = \frac{(4.0 \text{ m/s})^2}{0.3 \text{ m}} = 50 \text{ m/s}^2.
\]

The correct answer is \( \text{D} \).

10. **Strategy** Use the work-kinetic energy theorem.

**Solution** The work done by friction on the wheel is \( W = -f_k d \), where \( d \) is the linear distance the wheel passes between the pads before it stops. Relate \( d \) to the kinetic energy of the wheel.

\[
W_{\text{total}} = -f_k d = \Delta K = 0 - K_i, \quad \text{so} \quad d = \frac{K_i}{f_k}.
\]

Divide \( d \) by the circumference of a circle with radius 0.3 m to find the number of rotations.

\[
d = \frac{K_i}{2\pi f_k r} = \frac{30 \text{ J}}{2\pi(0.3 \text{ m})(20 \text{ N})} = 0.8 \text{ rotations}
\]

Since 0.8 < 1, the correct answer is \( \text{A} \).

11. **Strategy** Compute the average mechanical power output of the cyclist and compare it to the power consumed by the wheel at the friction pads.

**Solution** The metabolic power available for work is \( 535 \text{ W} - 85 \text{ W} = 450 \text{ W} \). Since the efficiency is 20%, the average mechanical power output of the cyclist is \( 0.20 \times 450 \text{ W} = 90 \text{ W} \). The average tangential speed of the wheel is

\[
v = \frac{4800 \text{ m}}{20 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 4.0 \text{ m/s}.
\]

Therefore, the power consumed by the friction pads is

\[
P = f_k v = (20 \text{ N})(4.0 \text{ m/s}) = 80 \text{ W}.
\]

Thus, the difference between the average mechanical power output of the cyclist and the power consumed by the wheel at the friction pads is \( 90 \text{ W} - 80 \text{ W} = 10 \text{ W} \). The correct answer is \( \text{B} \).

12. **Strategy and Solution** Increasing the force on the friction pads would increase the power consumed by the wheel at the friction pads (because \( P = Fv \)). So, if the cyclist is pedaling at the same rate and the power consumed by the friction pads increases, the difference between the two decreases and the fraction of mechanical power output of the cyclist consumed by the wheel at the friction pads increases. Thus, the correct answer is \( \text{D} \).

13. **Strategy** Relate the cyclist’s average metabolic rate to the energy released per volume of oxygen consumed, the time on the bike, and volume of oxygen consumed.

**Solution** The cyclist’s average metabolic rate while riding is 535 W. The total energy used during 20 minutes is

\[
(535 \text{ W})(20 \text{ min}) \frac{60 \text{ s}}{1 \text{ min}} = 642,000 \text{ J}.
\]

The total energy released by the consumption of oxygen is \( (20,000 \text{ J/L})V \), where \( V \) is the volume of oxygen consumed. Equating these two expressions and solving for \( V \) gives the number of liters of oxygen the cyclist consumes.

\[
(20,000 \text{ J/L})V = 642,000 \text{ J}, \quad \text{so} \quad V = \frac{642,000 \text{ J}}{20,000 \text{ J/L}} = 32 \text{ L} = 30 \text{ L}.
\]

The correct answer is \( \text{B} \).
14. **Strategy and Solution** Since the force has been reduced by 50% and the distance has been doubled, the cyclist does the same amount of work \[ W = 0.50F(2Δx) = FΔx \]. So, the energy transmitted in the second workout is equal to the energy transmitted in the first. The correct answer is \[ C \].

15. **Strategy** The circumference of a circle is \( C = 2πr \). A wheel moves a distance equal to its circumference during each rotation. The wheel rotates twice during each rotation of the pedals.

**Solution** The circumference of a circle with a radius of 0.15 m is \( 2π(0.15 \text{ m}) \). The circumference of a circle with a radius of 0.3 m is \( 2π(0.3 \text{ m}) \). During each rotation of the pedals, a point on the wheel at a radius of 0.3 m moves a distance \( 2[2π(0.3 \text{ m})] \). The ratio of the distance moved by a pedal to the distance moved by a point on the wheel located at a radius of 0.3 m in the same amount of time is \( \frac{2π(0.15 \text{ m})}{2[2π(0.3 \text{ m})]} = 0.25 \).

The correct answer is \[ A \].

16. **Strategy** Use the definition of power.

**Solution**

\[ P = \frac{ΔE}{Δt}, \text{ so } Δt = \frac{ΔE}{P} = \left( \frac{300 \text{ kcal}}{500 \text{ W}} \right) \left( \frac{4186 \text{ J}}{1 \text{ kcal}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 41.9 \text{ min}. \]

The correct answer is \[ D \].

17. **Strategy** Consider the distance a point on the wheel travels for each situation.

**Solution** The circumference of a circle with a radius of 0.3 m is \( 2π(0.3 \text{ m}) \). The circumference of a circle with a radius of 0.4 m is \( 2π(0.4 \text{ m}) \). During each rotation, a point on a wheel travels a distance equal to the circumference. The force on the wheel is the same in each case, but the distance traveled by a point on the wheel is greater for a greater radius. In this case, the distance is \( 0.4 \text{ m}/(0.3 \text{ m}) = 1.33 \) times farther or 33%. Since work is equal to the product of force times distance, the work done on the wheel per revolution is 33% more. Thus, the correct answer is \[ C \].

454