Exam 1 Solutions

1. Two identical conducting spheres A and B carry charges $2Q$ and $3Q$, respectively. They are separated by a constant distance much larger than their diameters. A third identical conducting sphere C is uncharged. Sphere C is first touched to A, then to B and finally removed. As a result, the magnitude of the electrostatic force between A and B, initially $F$, becomes

**Answer:** $F/3$

**Solution:** When C is touched to A, their charges equalize at $Q$ apiece. When C is then touched to B, their charges equalize at $2Q$ apiece. The force between A and B is proportional to the product of the charges, which changes from $6Q^2$ to $2Q^2$, so the force is $1/3$ of its original value.

2. Electrons in a particle beam have a velocity of $5 \times 10^6$ m/s along the $+x$ direction when they enter an electric field. What is the magnitude of the electric field that will cause an electron's velocity to reach three times its initial value over a distance of 0.15 m?

**Answer:** 3790 V/m

**Solution:** The velocity triples, so the final kinetic energy is 9 times larger and thus the work performed by the field is 8 times the initial kinetic energy. Thus the work is $W = 8K = Eed$, where $K = \frac{1}{2}m_ev^2 = 1.14 \times 10^{-17}$ J and $d = 0.15$ m. Solving for $E$ yields $E = 3790$ V/m.

3. Four charges having charge $q$ are placed at the corners of a square with sides of length $L$. What is the magnitude of the force acting on any of the charges?

**Answer:** $1.91kq^2 / L^2$

**Solution:** The forces on any charge from its nearest neighbors are both $kq^2 / L^2$, but at right angles, so the net force is $\sqrt{2}kq^2 / L^2$ along the diagonal. The force from the opposite charge is $kq^2 / (\sqrt{2}L)^2 = kq^2 / 2L^2$ along the diagonal in the same direction. Thus the total force is $(\sqrt{2} + \frac{1}{2})kq^2 / L^2 = 1.91kq^2 / L^2$. 
4. Refer to the previous problem. A charge $-q$ is placed at the center of the square of sides $L$. What is the change in the total potential energy of the system when the new charge is added?

**Answer:** $-4\sqrt{2}kq^2 / L$

**Solution:** The change in potential energy is the sum of the pairwise potential energies of the new change with the 4 corner charges. Each pairwise energy is $-\sqrt{2}kq^2 / L$ and there are 4 charges at the same distance, so the total change in potential energy is $-4\sqrt{2}kq^2 / L$.

5. Charges $+3Q$ and $-9Q$ are held in place at positions $x = 0$ m and $x = 2$ m, respectively. At what position in $x$ (in m) should a third charge be placed so that it is in equilibrium?

**Answer:** $-2.73$ m

**Solution:** We know from the relative magnitude and sign of the charges that the electric fields are canceled only in the region $x < 0$. Thus the equation for the $x$ component of the electric fields is

$$E_x = 0 = -\frac{3kQ}{x^2} + \frac{9kQ}{(L-x)^2} \Rightarrow \frac{3}{x^2} = \frac{9}{(L-x)^2}$$

where we use the fact that $k$ and $Q$ cancel. Solving for $x$ (making sure that we take positive square roots) yields $x = -2.73$.

6. In the figure, a small charged ball of mass 4.0 g hangs from a support and makes an angle 35° with the vertical under the influence of gravity and a horizontal electric field of magnitude $E = 2000$ V/m. What is the charge on the ball? ($g = 9.8$ m/s²)

**Answer:** 13.7 μC

**Solution:** Let $T$ be the tension in the string holding the ball. The equations describing the balance of forces in the horizontal and vertical directions are

$$T \sin \theta = Eq$$

$$T \cos \theta = mg$$

Dividing the two equations to eliminated $T$ yields $\tan \theta = Eq / mg$, which we solve for $q$ to obtain $q = 13.7$ μC.
7. A capacitor is charged by a battery in a circuit and then disconnected from the circuit, leaving it with charges $+Q$ and $-Q$ on the plates and a total energy $U$. A person then moves the capacitor plates to $1/5$ of their original separation. What is the work done by the person?

**Answer:** $-4U/5$

**Solution:** The work done by an external force is the negative of the work done by the electric field. The total potential energy of the capacitor is proportional to the plate separation $d$. Thus the external work is $W_{\text{external}} = +\Delta U = \frac{1}{5}U - U = -\frac{4}{5}U$. (Note that the plates attract one another and thus the applied force is in the opposite direction from the motion, giving negative work. Similarly, positive work would be needed to pull the plates further apart.)

8. Two protons approach one another head-on from a great distance. Initially each proton has kinetic energy 1.2 MeV ($1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$). What is the closest distance in fm ($1 \text{ fm} = 10^{-15} \text{ m}$) the protons will approach one another?

**Answer:** 0.6 fm

**Solution:** This is a problem of conservation of energy $K_i + U_i = K_f + U_f$. Here $K_i = 2 \times \frac{1}{2}m_p v_i^2$, $U_i = 0$, $K_f = 0$ and $U_f = ke^2 / d$, where $d$ is the closest distance. Solving yields $d = 0.6 \text{ fm}$.

9. The movement of a charge in an electric field from one point to another at constant speed without the expenditure of work by or against the field

**Answer:** Can only occur along an equipotential

**Solution:** The work done by the electric field is $W = -\Delta U = -q\Delta V$. So if the potential is constant the work is zero.

10. A conducting sphere of 5 cm radius is charged until the electric field just outside its surface is 2000 V/m. The electric potential of this sphere is:

**Answer:** 100 V

**Solution:** The electric field magnitude for a spherical charge distribution is $E = kQ / R^2$ and the potential is $V = kQ / R$. Thus for this case, $V = R \times E = 0.05 \times 2000 = 100$.
11. A particle of charge +0.04 C starts at point E moving left. How much kinetic energy must the particle have in order to reach point A?

**Answer:** 2.8 J  
**Solution:** The positive particle has to climb from 40 V to 110 V, a net change of 70 V (there are no higher humps in between), so the required kinetic energy is \( K = 70 \times 0.04 = 2.8 \) J.

12. Two metal spheres of radius 1.5 cm are initially separated by a distance of 1.2 cm and a power supply maintains a constant potential difference of 15,000 V between them. The spheres are brought closer to each other until a spark flies between them. If the dielectric strength of dry air is \( 3.0 \times 10^6 \) V/m, what is the distance between the surfaces of the spheres at this time? Assume that the charge distribution of each sphere is unaffected by the other sphere.

**Answer:** 0.50 cm  
**Solution:** The electric field is \( E = \Delta V / d \). Solving for \( d \) with the numbers shown yields 0.50 cm.

13. A potential difference is applied between the electrodes in a gas discharge tube. During a period of 1.6 s, \( 2.8 \times 10^{16} \) electrons and \( 1.1 \times 10^{16} \) singly charged positive ions move in opposite directions through a surface perpendicular to the length of the tube. What is the current in the tube?

**Answer:** 3.9 mA  
**Solution:** The ions are oppositely charged and move in the opposite direction, thus their currents add. The total current is \( I_T = (2.8 \times 10^{16} + 1.1 \times 10^{16})(1.6 \times 10^{-19}) / 1.6 = 0.0039 \) A.

14. If a 92.0 V emf is connected to the terminals A and B and the current in the 4.0 Ω resistor is 16.4 A, what is the value of the unknown resistor \( R \)?

**Answer:** 7.25 Ω  
**Solution:** The current going through the battery is 16.4 A so the total resistance is \( R_T = 92.0 / 16.4 = 5.61 \) Ω and the rest of the resistance is therefore \( 5.61 - 4.0 = 1.61 \) Ω. We can solve for \( R \) using the fact that \( R + 1.0 \) in parallel to 2.0 Ω gives 1.61 Ω, yielding \( R = 7.25 \) Ω.
15. In the multi-loop circuit shown the current through the 2.0 kΩ resistor is

**Answer**: 1.2 mA  
**Solution**: This is a two loop circuit. The equations describing the left ($i_1$ clockwise) and right handed ($i_2$ counterclockwise) loops, using currents in mA and resistances in kΩ, are

\[
\begin{align*}
6 - 6i_1 - 2(i_1 + i_2) & = 0 \\
6 - 3i_2 - 2(i_1 + i_2) - 3i_2 & = 0
\end{align*}
\]

Solving yields $i_1 = i_2 = \frac{3}{5}$, so the current going through the 2.0 kΩ resistor is $i_1 + i_2 = 1.2$ mA.

16. When two capacitors are connected in parallel their equivalent capacitance is 12 µF. When the same two capacitors are connected in series their equivalent capacitance is $\frac{35}{12}$ µF. What is the capacitance of the smaller of the two capacitors?

**Answer**: 5.0 µF  
**Solution**: The equations to be solved are

\[
\begin{align*}
C_1 + C_2 & = 12 \quad \text{Parallel} \\
\frac{1}{C_1} + \frac{1}{C_2} & = \frac{12}{35} \quad \text{Series}
\end{align*}
\]

Solving yields $C_1 = 5 \mu F$ and $C_2 = 7 \mu F$.

17. What is the current in the 4.0 Ω resistor in the circuit shown in the figure?

**Answer**: 1.5 A  
**Solution**: The 6 Ω and 12 Ω resistors have a combined resistance of 4.0 Ω. Thus each major branch has 8 Ω of resistance in parallel (4 Ω total) and thus equal currents. The total current is $\frac{12}{4} = 3$ A, so the current going through the 4.0 Ω resistor is 1.5 A.
18. A uniform length of wire with total resistance $R$ is made into a square loop (see figure). What is the resistance between points A and B?

**Answer:** 0.19 $R$  
**Solution:** We have $R/4$ and $3R/4$ in parallel, yielding $3R/16 = 0.19R$.

19. In the figure, the capacitance $C$ is 3.4 $\mu$F. What is the capacitance across A and B?

**Answer:** 1.64 $\mu$F  
**Solution:** This is a problem involving parallel and series capacitances. $C$ and 1.0 $\mu$F are in series (0.773 $\mu$F) and together are in parallel with 2.0 $\mu$F (2.773 $\mu$F) and that combination is in series with 4.0 $\mu$F (1.64 $\mu$F).

20. A material shaped as a cylinder has a resistance $R$ measured from one end of the cylinder to the other. If the material is now stretched to form a cylinder 4 times longer (with the same volume) what is the resistance of the new shape?

**Answer:** 16$R$  
**Solution:** The new cylinder is 4 times longer and must have $1/4$ the cross sectional area to maintain constant volume. Since the resistance is proportional to length and inversely proportional to cross sectional area, the new resistance is 16 times larger than before.