Exam 3 Solutions

1. An electron and a proton, located far apart and initially at rest, accelerate toward each other in a location undisturbed by any other fields or particles. Just before they collide, the ratio of their kinetic energies, $K_e / K_p$, is approximately

Answer: 1830

Solution: Because of equal and opposite electrical forces, we have conservation of momentum, $m_e v_e = m_p v_p$, using obvious notation. Recall that the kinetic energy can be written as $K = \frac{1}{2} m v^2 = \frac{p^2}{2m}$. Thus the ratio of kinetic energies is, using equality of momentum, $K_e / K_p = m_p / m_e = 1830$.

2. Four particles of charge $Q$ are fixed at the corners of a square with sides of length $L$, as shown in the figure. One particle is moved from its original position to the center of the square by an external force. The work performed by the external force is:

Answer: $\left(\frac{5}{\sqrt{2} - 2}\right) kQ^2 / L$

Solution: The initial potential energy of particle 4 (caused by particles 1, 2 and 3) is $\left(1 / \sqrt{2} + 2\right) kQ^2 / L$. The final potential energy is $6kQ^2 / \sqrt{2}L$. The work done by the external force is $W_{ext} = -W_E = \Delta U = \left(\frac{5}{\sqrt{2} - 2}\right) kQ^2 / L$.

3. A resistor in the form of a solid cylinder of material is connected across the terminals of an ideal battery and the current is measured to be 6.0 A. A second resistor made of the same material and having the same volume but two times the length is put in parallel with the first resistor. What is the total current in the new circuit?

Answer: 7.5 A

Solution: Let the original resistance be called $R$. The second resistor has twice the length and half the cross sectional area, so its resistance is $4R$. The total resistance is thus $0.8R$ and the current is $6.0 / 0.8 = 7.5$ A.
4. What is the charge on the 20 \( \mu \)F capacitor if the battery has an emf of 30 V?

**Answer:** 225 \( \mu \)C

**Solution:** The total capacitance of the 4 capacitors to the right of the 20 \( \mu \)F capacitor is 12 \( \mu \)F (8 \( \mu \)F + 4 \( \mu \)F). Thus the total capacitance in the circuit is 7.5 \( \mu \)F. The charge on the 20 \( \mu \)F capacitor is the same as that on the equivalent capacitance, so \( Q = CV = 7.5 \times 30 = 225 \mu \)C.

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5. A 12.0-V truck battery is rated at 239.0 ampere-hours and has an internal resistance of 0.9 \( \Omega \). Approximately how long can it supply current to a 3.0 \( \Omega \) load?

**Answer:** None of these

**Solution:** The current in the circuit is 12.0 / 3.9 = 3.08 A. The total lifetime is thus 78 hours.

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6. A conducting rod slides at 4.0 m/s on metal rails separated by 2.0 m. If a current of 40 mA is generated in the circuit with resistance \( R = 0.36 \Omega \), what is the magnetic field between the rails (in mT)?

**Answer:** 1.8

**Solution:** The induced emf is obtained from Faraday’s law using \( \mathcal{E} = vLB \), with an induced current of \( i = vLB / R \). Solving yields \( B = 1.8 \) mT.

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7. A series LR circuit includes a 9.0 V battery, a resistance of 0.50 \( \Omega \), and an inductance of 0.80 H. The elements are in series with an open switch. What is the voltage across the inductor 2.0 s after the switch is closed?

**Answer:** 2.6 V

**Solution:** The emf across the inductor in an LR circuit is given by \( \Delta V_L = \mathcal{E}e^{-t/\tau} \), where \( \tau = L / R = 1.6 \) s. Solving yields an emf of 2.6 V.

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8. A proton cyclotron has protons circulating in a magnetic field of 0.20 T that extends out to a radius of 0.40 m. What is the maximum energy of protons in this machine?

**Answer:** \( 4.9 \times 10^{-14} \) J

**Solution:** The maximum energy occurs when the proton is circulating at the largest radius possible, or \( R = 0.40 \) m. The centripetal force in a B field gives the relation \( mv^2 / R = evB \) from which we can find the momentum \( p = mv = eBR \). The kinetic energy is thus \( K = p^2 / 2m = (eBR)^2 / 2m \), which yields \( K = 4.9 \times 10^{-14} \) J.
9. Two long parallel wires are aligned vertically 30 cm apart as shown in the figure. The left wire carries a 6.0 A current upwards while the right wire carries a 15.0 A current downwards. What is the magnetic field (in µT) at a point between the wires at point P, a distance of 10 cm from the left wire?

**Answer:** 27

**Solution:** From the right hand rule, we see that the B fields add in the downward direction, not subtract. The total B field is then

\[ B = \frac{\mu_0 i_1}{2\pi d_1} + \frac{\mu_0 i_2}{2\pi d_2}, \]

or

\[ 12 \mu T + 15 \mu T = 27 \mu T. \]

10. A circular loop of wire with radius 10.0 cm and resistance 0.50 Ω lies in the plane of the page within a magnetic field directed into the page. The magnetic field strength changes at a rate of 10.0 T/sec. Which of the following answers describes the induced current? (cw = clockwise, ccw = counterclockwise)

**Answer:** 0.63 A, ccw

**Solution:** Since the magnetic field is increasing into the page, the induced current must generate a B field upwards out of the page, which means the current is counterclockwise. The magnitude of the induced emf is given by Faraday’s law or

\[ E = \pi r^2 \Delta B / \Delta t = 3.14 \times 0.10^2 \times 10 = 0.314. \]

With a resistance of 0.50 Ω the induced current is then 0.628 A.

11. Light polarized along the vertical direction is incident on four polarizing sheets with their transmission axes oriented at 20°, 30°, 50° and 60° relative to the vertical. What fraction of the initial light intensity is transmitted through this set of polarizers?

**Answer:** 0.73

**Solution:** The light is initially polarized and the polarizers are oriented 20°, 10°, 20° and 10° relative to one another. The fraction of the initial intensity that makes it through is then

\[ I_f = \cos(20°)^2 \cos(10°)^2 \cos(20°)^2 \cos(10°)^2 = 0.73I_0. \]

12. A light ray in the core \((n = 1.37)\) of a cylindrical optical fiber travels at an angle \(\theta_1 = 47.9°\) with respect to the axis of the fiber. A ray is transmitted through the cladding \((n = 1.25)\) and into the air. What angle \(\theta_2\) does the exiting ray make with the outside surface of the cladding?

**Answer:** 23.3°

**Solution:** From Snell’s law and the given angles, we use

\[ n_{\text{fiber}} \sin(90 - \theta_1) = n_{\text{air}} \sin(90 - \theta_2) \]

(the specified angles are relative to the surface not the normal). This gives \(\theta_2 = 23.3°\).
13. As shown in the figure, a light ray is incident normal to one face of a triangular block of material with index of refraction \( n_2 = 1.65 \) that is immersed in water \( (n_1=1.333) \). What is the maximum angle \( \alpha \) for which total internal reflection occurs at \( P \)?

**Answer:** 36.1°

**Solution:** When the incoming ray strikes the hypotenuse of the triangle it will totally internally reflect as long as the angle \( \alpha \) is small enough. Its maximum value is determined using the critical angle obtained from Snell’s law and geometry, \( n_2 \sin(90 - \alpha) = n_1 \). Solving yields \( \alpha = 36.1° \).

14. A dental hygienist uses a concave mirror to see the back of her patient’s teeth. When the mirror is 1.5 cm from a tooth, the image is upright and 3.0 times as large as the tooth. What is the focal length of the mirror?

**Answer:** 2.25 cm

**Solution:** The magnification is given by \( M = -q / p = +3.0 \), while the lens formula is 
\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.
\]
The conditions give \( p = +1.5 \) cm and \( q = -4.5 \) cm, yielding \( f = 2.25 \) cm.

15. A laser beam of wavelength 632.8 nm has intensity 13.3 W/m\(^2\). The amplitudes of the associated electric and magnetic fields, in V/m and µT, respectively are:

**Answer:** 100, 0.33

**Solution:** We know that \( I = c\varepsilon_0 E_{\text{rms}}^2 \) = 13.3. Solving yields \( E_{\text{rms}} = 70.8 \) V \( \) or an amplitude of \( E_{\text{peak}} = \sqrt{2} \times 70.8 = 100 \) V \( \). The B field strength is obtained from \( B_{\text{peak}} = E_{\text{peak}} / c = 0.33 \) µT \( \).

16. Two converging lenses, the first with focal length 25 cm and the second with focal length 20 cm, are separated by 30 cm. A 4.0 cm tall object is placed 50 cm in front of the first lens. Where is the final image?

**Answer:** 10 cm after the second lens

**Solution:** From the lens formula, the image after the first lens is located 50 cm downstream, or 20 past the second lens. Thus \( p' = -20 \) cm and we calculate \( q' = +10 \) cm for the second lens. Thus the image is 10 cm after the second lens.
17. A person has his nearsightedness corrected by lenses of $-5.00$ diopter. Without his glasses, his near point was 12.5 cm. With his glasses on, what is his near point? Assume the lenses are next to his eyes.

**Answer:** 33.0 cm

**Solution:** With the lens on, an object placed at the new near point must have the image located at the eye’s natural near point (so it can be seen clearly), or $q = -12.5$ cm. Using the lens equation $\frac{1}{p} + \frac{1}{q} = -5.00$, we obtain $p = 1/3.0$ m = 33.0 cm.

18. A thin film of soapy water, $n = 1.37$, is held vertically. Light of wavelength 600 nm is reflected from the film and horizontal dark lines appear. What is the difference in thickness of the soap film from one band to the next?

**Answer:** 219 nm

**Solution:** The soapy film thickens from top to bottom. The condition for a dark band for a soap film is $2d = m\lambda / n$ (recall that there is a phase shift at one surface and not at the other), where $m$ is a positive integer. The difference in film thickness from one band to the next is thus $\Delta d = \lambda / 2n = 600 / (2 \times 1.37) = 219$ nm.

19. A grating with 8,000 slits spaced over 2.54 cm is illuminated by light of a wavelength of 546 nm. What is the angle corresponding to the third order maximum?

**Answer:** 31.1°

**Solution:** The condition for a maximum is $d \sin \theta = m\lambda$, where $d$ is the slit spacing and $m$ is the order. For 3rd order, this gives $d = 0.0254 / 8000 = 3175$ nm and $\theta = 31.1°$.

20. A binary star in the constellation Orion has an angular separation between the stars of $10^{-5}$ radians. Assuming a wavelength of 500 nm, what is the smallest aperture (diameter) telescope that will just resolve the two stars?

**Answer:** 6.1 cm

**Solution:** The diffraction-limited resolving angle $\theta$ of a telescope is given by $\theta = 1.22\lambda / D$, where $D$ is the telescope aperture. Solving yields $D = 6.1$ cm.