Exam 4 Solutions

1. Wire A and Wire B are made out of copper with their radii related by \( r_A = 3r_B \) | \( 4r_B \) | \( 5r_B \). If the drift velocities are the same, how does the current \( i_A \) in wire A compared to the current \( i_B \) in wire B?

**Answer:** \( i_A = 9i_B \) | \( i_A = 16i_B \) | \( i_A = 25i_B \)

**Solution:** The current is given by the equation \( i = n_e A v_d \), where \( n_e \) is the free electron density, \( A \) is the cross sectional area and \( v_d \) is the drift velocity. Since \( A = \pi r^2 \) we see that the current increases by a factor of \( N^2 \) when the radius increases by a factor of \( N \).

2. A positive charge \( Q = +2\mu C \) | \( +3\mu C \) | \( +4\mu C \) is placed on the x-axis at \( x = 1 \) m and a charge \( -Q \) is placed on the x-axis at \( x = -1 \) m. What is the potential difference \( V_A - V_B \) between the points \( A = (2 \text{ m}, 0) \) and \( B = (0, -1 \text{ m}) \)?

**Answer:** 12 kV | 18 kV | 24 kV

**Solution:** For the two charges \( +Q \) and \( -Q \), the potential at a specified point is given by \( V = kQ(1/r_1 - 1/r_2) \), where \( r_1 \) is the distance from \( +Q \) and \( r_2 \) is the distance from \( -Q \). Since \( V_B = 0 \) (equidistant from each charge), we just have to work out \( V_A \) which gives the answers shown.

3. An electron circles a Nitrogen | Oxygen | Fluorine nucleus (7 | 8 | 9 protons, 7 | 8 | 9 neutrons) at a distance of 0.0302 | 0.0265 | 0.0235 nanometers. What is its velocity in km/s?

**Answer:** 7650 | 8730 | 9840

**Solution:** We obtain the velocity by equating the centripetal force to the electrostatic force, or \( m_e v^2 / r = kZe^2 / r^2 \), where \( Z, m_e \) is the electron mass, \( e \) is the fundamental charge is the atomic number and \( r \) is the radius of the orbit. Solving yields \( v = \sqrt{kZe^2 / rm_e} \) which yields the values shown.
4. A wire having mass 2.6 g and length 3.6 cm carries a 2.10 | 4.2 | 6.3 A current horizontally in a horizontal magnetic field oriented perpendicularly to the wire. For what magnitude of the magnetic field will the wire move vertically upward with a net acceleration of 2.0 m/s^2? (Note that g = 9.8 m/s^2.)

Answer: 0.41 T | 0.20 T | 0.14 T
Solution: Since gravity pulls the wire down with a force of mg, the force caused by the magnetic field is \( ma = iLB \), where \( a = 9.8 + 2 = 11.8 \text{ m/s}^2 \). Solving for \( B \) yields the values shown.

5. In the circuit shown the capacitors are initially uncharged. If the switch is closed at \( t = 0 \) how long will it take the current in the circuit to reach 0.58 | 0.31 | 0.21 A?

Answer: 35 \( \mu \text{s} \) | 50 \( \mu \text{s} \) | 60 \( \mu \text{s} \)
Solution: The RC time of the circuit, after combining resistances and capacitances, is \( 8 \times 3 = 24 \mu \text{s} \). The current in the circuit falls according to \( i = i_{\text{max}} e^{-t/RC} \), with \( i_{\text{max}} = V/R = 20/8 = 2.5 \text{ A} \), which can be solved for \( t \).

6. Protons are accelerated through a potential difference of 2500 V and enter a magnetic field region at an angle of 30° relative to the field direction where they move in a helical path with a period of 3.509 | 1.754 | 1.170 msec. What is the period of helical motion for \(^{14}\text{C}\) nuclei (6 protons, 8 neutrons) that are accelerated by the same potential in the same direction?

Answer: 8.20 msec | 4.10 msec | 2.73 msec
Solution: We discussed this situation in class. The radius is given by \( r = m_p v_{\perp} / qB \), where \( v_{\perp} \) is the perpendicular velocity and \( q = e \) for a proton. However, the period \( T \) is calculated from \( T = 2\pi r / v_{\perp} = 2\pi m_p / qB \), which depends only on the charge and mass of the particle. The ratio \( T'/T = (q'/q)(m'/m_p) = 14/6 = 2.333 \) allows us to determine the period \( T' \) for \(^{14}\text{C}\), which has a charge of 6e and mass 14\( m_p \).

7. Three light bulbs rated at 36 | 54 | 72 W at 120 V are connected in series and the combination is then connected to a 120 V source. What is the total power output of the combination?

Answer: 12 W | 18 W | 24 W
Solution: We get the resistance \( R \) for each bulb from \( P = V^2 / R \). When they are put in series, the total resistance is tripled, so the current is 1/3 the original current. Thus the power dissipated by each bulb is 1/9 of the original power dissipation and the total power is 3 \times 1/9 = 1/3 of the original power.
8. A 50 Ω | 200 Ω | 100 Ω resistor and a 0.45 H inductor are connected in series to a 120 Vrms 60 Hz power source. What is the average power dissipated in the resistor and the inductor, respectively in W?

**Answer:** 23 and 0 | 42 and 0 | 37 and 0

**Solution:** The rms current \( i_{\text{rms}} \) is found from the impedance \( Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2} \) by the expression \( i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \), where \( \omega = 2\pi f = 120\pi \) is the angular frequency and \( L \) is the inductance. The average power dissipated by the resistor is \( P_R = i_{\text{rms}}^2 R = \left(\frac{V_{\text{rms}}}{Z}\right)^2 R \) while the power \( P_L \) dissipated by the inductor is zero (inductors store and release energy, they don’t dissipate it).

9. Two coils (A and B) made out of the same wire are in a uniform magnetic field with the coil axes aligned in the field direction. Coil A has 10 | 20 | 20 turns of loop with a radius of 5 cm and coil B has 5 | 5 | 10 turns with an unknown radius. Now the field strength is doubled in 2 s and it is found that the ratio of the emf induced in each coil (emf_A:emf_B) is 1:2 | 1:4 | 1:2. What is the radius of coil B?

**Answer:** 10 cm | 20 cm | 10 cm

**Solution:** The induced emf in coil \( i \) is \( N_i A_i \Delta \Phi_B / \Delta t \). The ratio of emfs is thus independent of the field change and depends only on the area and number of turns. Since the area is \( \pi r^2 \), the emf ratio is \( (N_A / N_B)(A_A / A_B) = (N_A / N_B)(r_A / r_B)^2 \), from which we can determine \( r_B \).

10. Microwaves of frequency 5.50 GHz travel through a particular material of length 25.4 m. The index of refraction of this material is 1.09 | 1.21 | 1.15. Approximately how many complete wavelengths fit inside the material?

**Answer:** 508 | 563 | 536

**Solution:** The wavelength in the material is \( \lambda / n \), where \( n \) is the index of refraction and the vacuum wavelength \( \lambda \) is \( \lambda = c / f \). If \( L \) is the length of the material, then the number of waves in the material is \( N = Ln / \lambda = Ln f / c \).

11. It is desired to use a 60 cm focal length diverging lens to form an image of an object. The image is to be 1/3 | 1/2 | 2/3 of the object size. How far should the object be placed from the lens?

**Answer:** 120 cm | 60 cm | 30 cm

**Solution:** The magnification \( M \) is given by \( M = -q / p \), where \( p \) is the object distance and \( q \) is the image distance. We are given \( M \), so we solve for \( q \) using the lens equation \( 1/p + 1/q = 1/f \) to obtain \( M = (f - p)/p \).
12. A linearly polarized beam of light (vertically polarized) is incident upon a group of three polarizing sheets which are arranged so that the transmission axis of each sheet is rotated by $\theta = 45^\circ | 30^\circ | 60^\circ$ with respect to the preceding sheet starting from the vertical polarizer as shown in the figure. What fraction of the incident intensity is transmitted?

**Answer:** 0.25 | 0.56 | 0.06

**Solution:** The first polarizer has no effect on the beam because it’s already polarized in the vertical direction. The next two polarizers reduce the intensity by $\cos^2 \theta \times \cos^2 \theta = \cos^4 \theta$.

13. Two lenses are both converging and each has a 25 | 20 | 20 cm focal length. They are placed 50 | 40 | 60 cm apart with their optical axes aligned with one another. A 3.0 | 2.0 | 3.0 cm tall object is placed 75 | 60 | 90 cm in front of the first lens. Find the final image size.

**Answer:** 3.0 cm | 2.0 cm | 3.0 cm

**Solution:** Let’s work out the problem for the first set of values. Using the lens equation with $f = 25$ and $p = 75$ yields $q = 37.5$ downstream of the first lens. The magnification from this lens is thus $M = -q/p = -0.5$. This image is 12.5 cm in front of the second lens, so we have for that lens $f = 25$ and $p' = 12.5$, giving $q' = -25$ cm. The magnification for that lens is thus $M' = -q'/p' = -2.0$. The total magnification is thus $M_{\text{tot}} = MM' = +1.0$ so the image size is the same as the first one.

14. Bats emit high frequency sound to detect insects. If bats need to detect individual insects separated from one another by at least 1.0 | 1.5 | 2.0 cm at a distance of 2 m, and their ears are approximately 2.0 cm in diameter, approximately what minimum sound frequency must they emit? The speed of sound is 343 m/s.

**Answer:** 4.2 MHz | 2.8 MHz | 2.1 MHz

**Solution:** Sound waves diffract the same way as all other waves. Each ear acts as a telescope with a diameter $D = 2$ cm. The angular resolution is given by $\theta = d/L = 1.22\lambda/D$, where $d$ is the separation of the insects, $L$ is the distance to the insects and $\lambda = c/f$, where $c$ is the speed of sound and $f$ is the frequency. Given $d$, $L$ and $D$ we can solve for $\lambda$ and thus $f$.

15. A layer of an unknown liquid lies on top of a pool of water. Light emitted from a laser source under water totally reflects at an angle of $62^\circ | 58^\circ | 55^\circ$ from the normal. What is the index of refraction of the liquid? ($n_{\text{H}_2\text{O}} = 1.33$)

**Answer:** 1.17 | 1.13 | 1.09

**Solution:** The critical angle is given by $\sin \theta = n/n_{\text{H}_2\text{O}}$, allowing us to solve for $n$. 
16. An individual has a near point of 13 cm and a far point of 40 cm. What is her new near point when her nearsightedness is corrected?

**Answer:** 19 cm | 21 cm | 18 cm

**Solution:** Nearsightedness is corrected by making the image of an object at infinity appear at the person’s far point. Let’s use the first set of values to illustrate the general solution. The lens equation gives us for \( p = \infty \) and \( q = -40 \text{ cm} \), a solution \( f = -40 \text{ cm} \). With the correction lens on, the person’s new near point is the closest location of an object whose image will appear at \( q = -13 \text{ cm} \) (virtual image). This yields \( p = 19 \text{ cm} \).

17. Two thin lenses, with focal lengths of 25.0 cm and -30.0 cm are placed in contact in an orientation so that their optic axes coincide. What is the focal length of the two in combination? Hint: A thin lens is one whose thickness is negligible.

**Answer:** +150 cm | -60 cm | +30 cm

**Solution:** We learned that when thin lenses are put next to one another the total focal length is found from \( \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \). This gives the values shown. You can also work it out manually. If an object is placed a distance \( p \) from \( f_1 \), then the image location \( q \) is found from \( \frac{1}{q} = \frac{1}{f_1} - \frac{1}{p} \). This acts as an object for the second lens \( f_2 \), but the distance is negative so we write \( \frac{1}{q'} + (-\frac{1}{f_1} + \frac{1}{p}) = \frac{1}{f_2} \), for which we see \( \frac{1}{p} + \frac{1}{q'} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} \).

18. Plane polarized light is sent through two consecutive polarizers, the first having its plane of polarization in the same direction as the incident light and the second having its plane at 90° to the original plane of polarization. A third polarizer, with plane of polarization at 30° | 20° | 40° to the original plane of polarization is placed between the two other polarizers. What fraction of the original intensity now gets through?

**Answer:** 0.19 | 0.10 | 0.24

**Solution:** When the last polarizer is inserted, the angle it makes with the first polarizer is \( \theta \) and with the second is \( 90° - \theta \). Thus the fraction of total intensity getting through, noting that the initial beam is aligned with the polarization axis of the first polarizer, is \( \cos^2 \theta \cos^2(90° - \theta) = \cos^2 \theta \sin^2 \theta \).

19. At what angle will the highest order maximum appear for a wavelength 650 nm using a grating with 700 lines/mm?

**Answer:** 66° | 54° | 82°

**Solution:** The angles of the intensity maxima for a diffraction grating are given by \( \sin \theta = m\lambda / d \), where \( d \) is the spacing of the slits in the grating (for 700 lines/mm \( d = 10^{-6} / 700 \text{ in nm} \)). We just need to determine the largest value of \( m \) for which \( \sin \theta \leq 1 \).
20. A certain telescope has an objective of focal length 1460 | 1560 | 1420 cm. If the Moon is used as an object, a 1.3 | 1.1 | 0.7 cm long image formed by the objective corresponds to what distance, in km, on the Moon? Assume $3.8 \times 10^8$ m for the Earth-Moon distance.

**Answer:** 340 | 270 | 190

**Solution:** Because of the distance to the moon, the image formed by the objective is essentially at the focal point, or $q = f$. A simple way to solve the problem is to use similar triangles with rays passing through the center of the lens and write the proportion $h' / h = f / p$, where $h$ is the size of the feature, $h'$ is size of its image, and $p = 3.8 \times 10^8$ m is the moon distance. We could also use the magnification, $M = q / p = -f / p$ and multiply it by $h$ to get the image size $h'$ (we didn’t ask for the sign here).