Capacitance
Purpose of Capacitors

- **Storage of charge**: $Q = CV$
  - Used in DC and AC circuits

- **Storage of energy**
  - Can provide energy to circuits

- **Used in DC and AC circuits**
  - Timing in DC circuits
  - Resonance in AC circuits
  - (Later in course)

\[
U = \frac{Q^2}{2C} = \frac{1}{2}CV^2
\]
Capacitors in Parallel

\[ V_1 = V_2 = V_3 \text{ (same potential top and bottom)} \]

\[ \text{Total charge: } q_{tot} = q_1 + q_2 + q_3 \]

\[ C_{eq}V = C_1V + C_2V + C_3V \]

Basic law for combining capacitors in parallel

Works for N capacitors
Capacitors in Series

$q_1 = q_2 = q_3$ (same current charges all capacitors)

Total potential: $V = V_1 + V_2 + V_3$

$q/C_{eq} = q/C_1 + q/C_2 + q/C_3$

Basic law for combining capacitors in series

Works for $N$ capacitors
ConcepTest

Two identical parallel plate capacitors are shown in an end-view in Figure A. Each has a capacitance of $C$.

If the two are joined together at the edges as in Figure B, forming a single capacitor, what is the final capacitance?

- (a) $C/2$
- (b) $C$
- (c) $2C$
- (d) 0
- (e) Need more information

Area is doubled

A

B
ConcepTest

⇒ Each capacitor is the same in the three configurations. Which configuration has the lowest equivalent capacitance?

◆ (1) A
◆ (2) B  \[ C/2 \text{ (series)} \]
◆ (3) C
◆ (4) They all have identical capacitance

[Diagrams of configurations A, B, and C]
Capacitors in Circuits

- Find total capacitance $C_{eq}$ between (a) and (b)
  - Use multi-step process
- $C_{23} = 2 + 3$ in series
  - $1/C_{23} = 1/C + 1/C = 2/C$
  - $C_{23} = C/2$
- $C_1 + C_{23}$ in parallel
  - $C_{eq} = C + C/2 = 3C/2$
Another Example

→ Assume all capacitors = 10 \( \mu \)F. Find total capacitance
  
  ◆ \( C_3 \) and \( C_4 \) in parallel: \( C_{34} = 10 + 10 = 20 \) \( \mu \)F
  ◆ \( C_1, C_{34}, C_2 \) in series

\[
\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{10} = \frac{5}{20}
\]

\[C_{eq} = 4.0 \mu \text{F} \]

→ How much charge provided by battery to fully charge capacitors? Assume \( V = 20 \).

◆ \( Q = C_{eq} \times V = 4 \times 20 = 80 \) \( \mu \)C
Another Example

- Assume all capacitors = 10 μF. Find total capacitance
  - $C_{234} = 30 \mu F$ (parallel)
  - $C_{56} = 20 \mu F$ (parallel)
  - $C_1, C_{234}, C_{56}$ (series)

$$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{30} + \frac{1}{20} = \frac{11}{60}$$

$$C_{eq} = 5.45 \mu F$$

- How much charge provided by battery to fully charge capacitors? Assume $V = 20$.
  - $Q = C_{eq} \times V = 5.45 \times 20 = 109 \mu C$
Find Charges on Series Capacitors

Let \( V = 10 \), \( C_1 = 6 \mu F \), \( C_2 = 12 \mu F \)

- Find charges, voltages on \( C_1, C_2 \)

Combine series capacitances
- This is what battery “sees”!

Find \( q_{eq} \), then use \( q_{eq} = q_1 = q_2 \)
- \( q_{eq} = C_{eq} V = 4 \times 10 = 40 \mu C \)

Find \( V_1, V_2 \)
- \( V_1 = q_1 / C_1 = 40 / 6 = 6.67 \) V
- \( V_2 = q_2 / C_2 = 40 / 12 = 3.33 \) V
- \( V_1 + V_2 = 10 \), as expected
Example: Find $q_i$ and $V_i$ on All Capacitors

$C_1$ is charged in position A, then S is thrown to B position

- Initial voltage across $C_1$: $V_0 = 12$
- Initial charge on $C_1$: $q_{10} = 12 \times 4 = 48 \mu C$

After switch is thrown to B:

- Charge flows from $C_1$ to $C_2$ and $C_3$
- $V_1 = V_{23}$ (parallel branches)

$q_2$ and $q_3$ in series: $q_2 = q_3 = q_{23}$ ($C_{23} = 2 \mu F$)

- Charge conservation: $q_{10} = q_1 + q_{23}$
- $48 = C_1 V_1 + C_{23} V_1$ ($V_1 = V_{23}$)
- Find $V_1$: $V_1 = 48 / (C_1 + C_{23}) = 8 V$
- Find $q_1$: $q_1 = C_1 V_1 = 32 \mu C$
- $q_{23} = 48 - 32 = q_2 = q_3 = 16 \mu C$
- $V_2 = q_2 / C_2 = 2.67 V$
- $V_3 = q_3 / C_3 = 5.33 V$
Another Example

- Each capacitor has $C = 10\mu F$. Find the total capacitance.

- Do it in stages:
  - $2 \& 3 \Rightarrow C_{23} = 5 \mu F$
  - Add 4 $\Rightarrow C_{234} = 15 \mu F$
  - Add 5 $\Rightarrow C_{2345} = 6 \mu F$
  - Add 1 $\Rightarrow C_{12345} = 16 \mu F$

- Charge supplied by battery (20V)
  - $q_{tot} = C_{12345} \times V = 16 \times 20 = 320 \mu C$
Find Charges, Voltages on All Capacitors

- Each capacitor has capacitance $10\mu F$. $V = 20$ volts
  - $q_1 = C_1 V = 10 \times 20 = 200 \mu C$

- $C_{2345} = 6\mu F$, $q_{2345} = 6 \times 20 = 120 \mu C$
  - $q_{2345} = q_{234} = q_5 = 120 \mu C$ (series)
  - $V_5 = q_5 / C_5 = 120 / 10 = 12$

- Find $q_4$, $V_4 = V_{23}$
  - $V_{234} = V_4 = 20 - 12 = 8$
  - $q_4 = C_4 V_4 = 10 \times 8 = 80 \mu C$

- Find $q_2$, $q_3$, $V_2$, $V_3$ ($C_{23} = 5\mu F$)
  - $q_2 = q_3 = q_{23} = C_{23} \times V_{23} = 5 \times 8 = 40 \mu C$
  - $V_2 = q_2 / C_2 = 40 / 10 = 4$
  - $V_3 = q_3 / C_3 = 40 / 10 = 4$
  - Check: $V_2 + V_3 = 8$ ($= V_{23}$)
Capacitor Monster

All voltages = 4V, all capacitors = 2μF. What is the charge on C? Can you find the charge on all capacitors?
Capacitor Monster

All voltages = 4V, all capacitors = 2µF. What is the charge on C?

\[ q = CV = 2 \times 4 = 8\mu C \]
Energy in a Capacitor

- Capacitors have energy associated with them
  - Grab a charged capacitor with two hands and find out!

- Calculation of stored energy
  - Proof requires simple calculus derivation
  - Energy = work moving charge from – to + surface

\[ U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 \]

- Capacitors store and release energy as they acquire and release charge
  - This energy is available to drive circuits
Example of Capacitor Energy

\[ U = \frac{1}{2} CV^2 = \frac{1}{2} \times \left( 5 \times 10^{-6} \right) \times 200^2 = 0.1 \text{ J} \]

\[ U = \frac{1}{2} CV^2 = \frac{1}{2} \times \left( 5 \times 10^{-6} \right) \times 20000^2 = 1000 \text{ J} \]
Where is the Energy Stored?

Answer: Energy is stored in the electric field itself!!

Example: Find energy density of two plate capacitor

- E field is constant

\[ u = \frac{U}{Ad} = \frac{CV^2}{2Ad} = \frac{\varepsilon_0 (A/d)(Ed)^2}{2Ad} = \frac{1}{2} \varepsilon_0 E^2 \]

Energy density depends only on E field!

- A general result, independent of geometry
- Can be shown more generally by Maxwell’s equations

\[ u = \frac{1}{2} \varepsilon_0 E^2 \]
Dielectric Materials and Capacitors

- Insulating material that can be polarized in E field

- Induced charges at dielectric surface partially cancel E field
  - $E \rightarrow \frac{E}{\kappa}$ $\kappa > 1$ is “dielectric constant”
  - $V \rightarrow \frac{V}{\kappa}$ (since $V = Ed$)
  - $C \rightarrow \kappa C$ (since $C = \frac{Q}{V}$)

- But “good” dielectric requires more than high $\kappa$ value
  - Good insulator (no charge leakage)
  - High breakdown voltage (no arcing at high voltage)
  - Low cost (affordable)
Dielectric Mechanism is Due to Polarization

$E = 0$, Dipoles randomly aligned

- $E$ applied, partially aligns dipoles
- Aligned dipoles induce surface charges
- Surface charges partially cancel $E$ field

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/dielec.html
# Dielectric Constants of Some Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric constant $\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.0</td>
</tr>
<tr>
<td>Air</td>
<td>1.00059</td>
</tr>
<tr>
<td>Bakelite</td>
<td>4.9</td>
</tr>
<tr>
<td>Strontium titanate</td>
<td>310</td>
</tr>
<tr>
<td>Water</td>
<td>80</td>
</tr>
<tr>
<td>Ethanol</td>
<td>24</td>
</tr>
<tr>
<td>Mica</td>
<td>5.4</td>
</tr>
<tr>
<td>Barium Titanate</td>
<td>100 – 1250</td>
</tr>
<tr>
<td>Paper</td>
<td>3.7</td>
</tr>
<tr>
<td>Beeswax</td>
<td>2.7 – 3.0</td>
</tr>
<tr>
<td>Silica glass</td>
<td>3.8</td>
</tr>
</tbody>
</table>
Example of Dielectric Use

Simple capacitor: \( A = 2 \text{m}^2 \), \( d = 1 \mu \text{m} \) (no dielectric)

- Place 200 volts across \( C \)

\[
C = \varepsilon_0 \frac{A}{d} = 8.85 \times 10^{-12} \frac{2}{10^{-6}} = 1.77 \times 10^{-5} = 17.7 \ \mu \text{F}
\]

\[
q = CV = 17.7 \ \mu \text{F} \times 200 = 3540 \ \mu \text{C}
\]

\[
U = \frac{1}{2} CV^2 = \frac{1}{2} \left(1.77 \times 10^{-5}\right)(200)^2 = 0.35 \ \text{J}
\]

Now disconnect \( C \) from circuit

- Insert strontium titanate dielectric (\( \kappa = 310 \)) into capacitor
- Charge is conserved, calculate new \( C \), \( V \) and \( U \)

\[
C_{\text{new}} = \kappa C = 310 \times 17.7 = 5490 \ \mu \text{F}
\]

\[
V_{\text{new}} = \frac{V}{\kappa} = \frac{200}{310} = 0.65 \ \text{volts}
\]

\[
U_{\text{new}} = \frac{U}{\kappa} = \frac{0.35}{310} = 0.0011 \ \text{J} \quad \left(= \frac{1}{2} C_{\text{new}} V_{\text{new}}^2 \right)
\]
Similar Example

- Same capacitor as before, but this time insert dielectric while \( C \) is in the circuit
  - \( C_{\text{new}} = \kappa C = 5490 \, \mu\text{F} \)
  - \( V \) is still 200 volts (maintained by battery)

- Calculate new \( q \) and \( U \) in this example

\[
q_{\text{new}} = C_{\text{new}} V = 5490 \, \mu\text{F} \times 200 = 1.1 \, \text{C}
\]

\[
U_{\text{new}} = \frac{1}{2} C_{\text{new}} V^2 = \frac{1}{2} \left( 5490 \times 10^{-6} \right) (200)^2 = 110 \, \text{J}
\]

- Notice how \( q \) and \( U \) are increased by factor \( \kappa \)
ConcepTest

Two identical capacitors are given the same charge $Q$, then disconnected from a battery.

After $C_2$ has been charged and disconnected it is filled with a dielectric. Compare the voltages of the two capacitors.

- (1) $V_2 < V_1$
- (2) $V_2 > V_1$
- (3) $V_2 = V_1$

Charge is unchanged, but dielectric reduces $E$ to $E/\kappa$ and $V$ to $V/\kappa$
ConcepTest

When we fill the capacitor with the dielectric, what is the amount of work required to fill the capacitor?

1. $W > 0$
2. $W < 0$
3. $W = 0$

Energy is reduced from $U$ to $U/\kappa$, so work is negative.

If $U$ is total energy in capacitor

- Positive work: One “pushes in” dielectric $\Delta U > 0$
- Negative work: Capacitor “sucks in” dielectric $\Delta U < 0$