QUICK QUIZZES

1. (c). The corrective lens for a farsighted eye is a converging lens, while that for a nearsighted eye is a diverging lens. Since a converging lens is required to form a real image of the Sun on the paper to start a fire, the campers should use the glasses of the farsighted person.

2. (a). We would like to reduce the minimum angular separation for two objects below the angle subtended by the two stars in the binary system. We can do that by reducing the wavelength of the light — this in essence makes the aperture larger, relative to the light wavelength, increasing the resolving power. Thus, we would choose a blue filter.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Diffraction of light as it passes through, or reflects from, the objective element of a telescope can cause the images of two sources having a small angular separation to overlap and fail to be seen as separate images. The minimum angular separation two sources must have in order to be seen as separate sources is inversely proportional to the diameter of the objective element. Thus, using a large diameter objective element in a telescope increases its resolution, making (c) the correct choice.

2. The power of a lens in diopters equals the reciprocal of the focal length when that focal length is expressed in meters. Hence, the power of a lens having a focal length of 25 cm is

\[
P = \frac{1}{f} = \frac{1}{0.25 \text{ m}} = 4.0 \text{ diopters}
\]

and choice (b) is the correct answer.

3. The amount of light focused on the film by a camera is proportional to the area of the aperture through which the light enters the camera. Since the area of a circular opening varies as the square of the diameter of the opening, the light reaching the film is proportional to the square of the diameter of the aperture. Thus, increasing this diameter by a factor of 3 increases the amount of light by a factor of 9, and (c) is the correct choice.

4. When the eye is shorter than normal, the lens-cornea system fails to bring light from near objects to a focus by the time it reaches the retina, resulting in a blurry image. Light rays entering the pupil from distant objects are less divergent than those from near objects, and the lens-cornea system can focus them on the retina. Such an eye is farsighted, or has hyperopia, and needs a converging corrective lens to help bring rays from near objects to focus sooner. The correct choice is (c).

5. When the eye is longer than normal, the lens-cornea system will bring light from distant objects to focus before it reaches the retina. Rays from near objects are more divergent and the lens-cornea system brings them to focus farther from the lens, on the retina. This means that the eye can see near objects clearly, but is unable to focus on distant objects. Such an eye is nearsighted (myopia), and needs a diverging corrective lens to make the rays from distant objects more divergent before they enter the eye. Choice (b) is the correct answer.
6. Stars are very distant and the reciprocal of the object distance in the lens or mirror equation
\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \]
is essentially zero. This means that \( q = f \), or the images are formed at a distance equal to the focal length from the objective element. Thus, the angular separation of the images, and hence the stars, is
\[ \alpha = \frac{s}{f} \times \frac{20.0 \times 10^{-3} \text{ m}}{2.00 \text{ m}} = 1.00 \times 10^{-2} \text{ rad} \left( \frac{360^\circ}{2\pi \text{ rad}} \right) = 0.573^\circ \]
and we see that choice (d) is the correct answer.

7. The corrective lens must form an upright, virtual image located 55 cm in front of the lens \((q = -55 \text{ cm})\) when the object is 25 cm in front of the lens \((p = +25 \text{ cm})\). The thin lens equation then gives the required focal length as
\[ f = \frac{pq}{p+q} = \frac{(25 \text{ cm})(-55 \text{ cm})}{25 \text{ cm} - 55 \text{ cm}} = +46 \text{ cm} \]
and the correct choice is (c).

8. When a compound microscope is adjusted for most relaxed viewing (i.e., the final image formed by the eyepiece is at infinity), the approximate overall magnification produced by the microscope is given by the expression
\[ m = -\frac{L}{f_o} \left( \frac{f_e}{25 \text{ cm}} \right) \]
where \( L \) is the length of the microscope, \( f_o \) is the focal length of the objective lens, and \( f_e \) is the focal length of the eyepiece. With the microscope described, the approximate magnification is
\[ m = -\frac{15 \text{ cm}}{4.0 \text{ cm}} \left( \frac{25 \text{ cm}}{0.80 \text{ cm}} \right) = -1.2 \times 10^2 \quad \text{or} \quad |m| = 120 \]
and the correct answer is choice (d).

9. The angular separation of the two stars is \( \theta = \frac{s}{r} = (10^{-5} \text{ ly})/(200 \text{ ly}) = 5 \times 10^{-8} \) radians. The limiting angle of resolution for a circular aperture is \( \theta_{\text{max}} = 1.22(\lambda/D) \). Requiring that \( \theta_{\text{max}} = \theta \), and assuming a wavelength at the center of the visible spectrum (550 nm), the required diameter of the aperture is found to be
\[ D = \frac{1.22\lambda}{\theta} = \frac{1.22(550 \times 10^{-9} \text{ m})}{5 \times 10^{-8} \text{ rad}} = 1 \times 10^4 \text{ m} = 10 \text{ m} \]
so (c) is the best choice of the listed possible answers.

10. When using light of wavelength \( \lambda \), the resolving power needed to distinguish two closely spaced spectral lines having a difference in wavelength of \( \Delta \lambda \) is \( R = \lambda/\Delta \lambda \). Thus, if two lines in the visible spectrum differ in wavelength by \( \Delta \lambda = 0.1 \text{ nm} \), the minimum resolving power of a diffraction grating that might be used to separate them is
\[ R_{\text{min}} = \frac{\lambda}{\Delta \lambda} = \frac{400 \text{ nm}}{0.1 \text{ nm}} = 4 \times 10^3 = 4000 \]
and the correct choice is (b).
ANSWERS TO CONCEPTUAL QUESTIONS

2. The objective lens of the microscope must form a real image just inside the focal point of the eyepiece lens. In order for this to occur, the object must be located just outside the focal point of the objective lens. Since the focal length of the objective lens is typically quite short (~1 cm), this means that the microscope can focus properly only on objects close to the end of the barrel and will be unable to focus on objects across the room.

4. For a lens to operate as a simple magnifier, the object should be located just inside the focal point of the lens. If the power of the lens is +20.0 diopters, its focal length is

\[ f = \frac{(1.00 \text{ m})}{\varphi} = \frac{(1.00 \text{ m})}{+20.0} = 0.050 \text{ m} = 5.00 \text{ cm} \]

The object should be placed slightly less than 5.00 cm in front of the lens.

6. The aperture of a camera is a close approximation to the iris of the eye. The retina of the eye corresponds to the film of the camera, and a close approximation to the cornea of the eye is the lens of the camera.

8. You want a real image formed at the location of the paper. To form such an image, the object distance must be greater than the focal length of the lens.

10. Under low ambient light conditions, a photoflash unit is used to insure that light entering the camera lens will deliver sufficient energy for a proper exposure to each area of the film. Thus, the most important criterion is the additional energy per unit area (product of intensity and the duration of the flash, assuming this duration is less than the shutter speed) provided by the flash unit.

12. The angular magnification produced by a simple magnifier is \( m = \frac{25 \text{ cm}}{f} \). Note that this is proportional to the optical power of a lens, \( \varphi = \frac{1}{f} \), where the focal length \( f \) is expressed in meters. Thus, if the power of the lens is doubled, the angular magnification will also double.

PROBLEM SOLUTIONS

25.1 The \( f \)-number (or focal ratio) of a lens is defined to be the ratio of focal length of the lens to its diameter. Therefore, the \( f \)-number of the given lens is

\[
\text{\( f \)-number} = \frac{f}{D} = \frac{28 \text{ cm}}{4.0 \text{ cm}} = 7.0
\]

25.2 If a camera has a lens with focal length of 55 mm and can operate at \( f \)-numbers that range from \( f/1.2 \) to \( f/22 \), the aperture diameters for the camera must range from

\[
D_{\text{min}} = \frac{f}{(f\text{-number})_{\text{max}}} = \frac{55 \text{ mm}}{22} = 2.5 \text{ mm}
\]
to

\[
D_{\text{max}} = \frac{f}{(f\text{-number})_{\text{min}}} = \frac{55 \text{ mm}}{1.2} = 46 \text{ mm}
\]
25.3 The thin lens equation, \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \), gives the image distance as

\[
q = \frac{pf}{p - f} = \frac{(100 \text{ m})(52.0 \text{ mm})}{100 \text{ m} - 52.0 \times 10^{-3} \text{ m}} = 52.0 \text{ mm}
\]

From the magnitude of the lateral magnification, \( |M| = h'/h = \frac{|q|}{p} \), where the height of the image is \( h' = 0.0920 \text{ m} = 92.0 \text{ mm} \), the height of the object (the building) must be

\[
h = h' - \frac{p}{q} = (92.0 \text{ mm}) - \frac{100 \text{ m}}{52.0 \text{ mm}} = 177 \text{ m}
\]

25.4 Consider rays coming from opposite edges of the object and passing undeviated through the center of the lens as shown at the right. For a very distant object, the image distance equals the focal length of the lens. If the angular width of the object is \( \theta \), the full image width on the film is

\[
h = 2 \left[ f \tan \left( \frac{\theta}{2} \right) \right] = 2 (55.0 \text{ mm}) \tan \left( \frac{20^\circ}{2} \right) = 19 \text{ mm}
\]

so the image easily fits within a 23.5 mm by 35.0 mm area.

25.5 The exposure time is being reduced by a factor of

\[
\frac{t_2}{t_1} = \frac{1/256}{1/32} = 8
\]

Thus, to maintain correct exposure, the intensity of the light reaching the film should be increased by a factor of 8. This is done by increasing the area of the aperture by a factor of 8, so in terms of the diameter, \( \pi D_2^2 / 4 = 8 (\pi D_1^2 / 4) \) or \( D_2 = \sqrt{8} D_1 \).

The new f-number will be

\[
(f\text{-number})_2 = \frac{f}{D_2} = \frac{f}{\sqrt{8} D_1} = \frac{(f\text{-number})_1}{\sqrt{8}} = \frac{4.0}{\sqrt{8}} = 1.4 \quad \text{or} \quad f/1.4
\]
25.6 (a) The intensity is a measure of the rate at which energy is received by the film per unit area of the image, or \( I \sim \frac{1}{A_{\text{image}}} \). Consider an object with horizontal and vertical dimensions \( h_x \) and \( h_y \), as shown at the right. If the vertical dimension intercepts angle \( \theta \), the vertical dimension of the image is \( h'_y = q \theta \), or \( h'_y \approx q \). Similarly for the horizontal dimension, \( h'_x \approx q \), and the area of the image is \( A_{\text{image}} \approx h'_y h'_x \).

The intensity of the light reaching the film is also proportional to the cross-sectional area of the lens and hence to the square of the diameter of that lens, or \( I \sim D^2 \). Combining this with our earlier conclusion gives

\[
I \sim \frac{D^2}{f^2} = \frac{1}{(f/D)^2} \quad \text{or} \quad I \sim \frac{1}{(f\text{-number})^2}
\]

(b) The total light energy hitting the film is proportional to the product of intensity and exposure time, \( I t \). Thus, to maintain correct exposure, this product must be kept constant, or \( I_1 t_1 = I_2 t_2 \), giving

\[
t_2 = \left( \frac{I_2}{I_1} \right) t_1 = \left( \frac{(f\text{-number})_1^2}{(f\text{-number})_2^2} \right) t_1 = \left( \frac{4.0}{1.8} \right)^2 \left( \frac{1}{500} \right) \text{ s} = \frac{1}{100} \text{ s}
\]

25.7 Since the exposure time is unchanged, the intensity of the light reaching the film must be doubled if the energy delivered is to be doubled. Using the result of Problem 6 (part a), we obtain

\[
(f\text{-number})_1 = \left( \frac{I_1}{I_2} \right) (f\text{-number})_2 = \left( \frac{1}{2} \right) (11) = 61, \quad \text{or} \quad f\text{-number} = \sqrt{61} = 7.8
\]

Thus, you should use the \( f/8.0 \) setting on the camera.

25.8 The image must always be focused on the film, so the image distance is the distance between the lens and the film. From the thin lens equation, \( 1/f = 1/p + 1/q \), the object distance is \( p = q f / (q - f) \), and the range of object distances this camera can work with is from

\[
p_{\text{min}} = \frac{q_{\text{max}} f}{q_{\text{max}} - f} = \frac{(210 \text{ mm})(175 \text{ mm})}{210 \text{ mm} - 175 \text{ mm}} = 1.05 \times 10^3 \text{ mm} = 1.05 \text{ m}
\]

to

\[
p_{\text{max}} = \frac{q_{\text{max}} f}{q_{\text{min}} - f} = \frac{(180 \text{ mm})(175 \text{ mm})}{180 \text{ mm} - 175 \text{ mm}} = 6.30 \times 10^3 \text{ mm} = 6.30 \text{ m}
\]
25.9 The corrective lens must form an upright, virtual image at the near point of the eye (i.e., \( q = -60.0 \text{ cm} \) in this case) for objects located 25.0 cm in front of the eye \(( p = +25.0 \text{ cm} )\). From the thin lens equation, \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \), the required focal length of the corrective lens is

\[ f = \frac{pq}{p+q} = \frac{(25.0 \text{ cm})(-60.0 \text{ cm})}{25.0 \text{ cm} - 60.0 \text{ cm}} = +42.9 \text{ cm} \]

and the power (in diopters) of this lens will be

\[ P = \frac{1}{f_{\text{meters}}} = \frac{1}{+0.429 \text{ m}} = +2.33 \text{ diopters} \]

25.10 (a) The person is farsighted, able to see distant objects but unable to focus on objects at the normal near point for a human eye.

(b) With the corrective lens 2.00 cm in front of the eye, the object distance for an object 20.0 cm in front of the eye is \( p = 20.0 \text{ cm} - 2.00 \text{ cm} = 18.0 \text{ cm} \).

(c) The upright, virtual image formed by the corrective lens will serve as the object for the eye, and this object must be 40.0 cm in front of the eye. With the lens 2.00 cm in front of the eye, the magnitude of the image distance for the lens will be \( |q| = 40.0 \text{ cm} - 2.00 \text{ cm} = 38.0 \text{ cm} \).

(d) The image must be located in front of the corrective lens, so it is a virtual image and the image distance is negative. Thus, \( q = -38.0 \text{ cm} \).

(e) From the thin lens equation, \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \), the required focal length of the corrective lens is

\[ f = \frac{pq}{p+q} = \frac{(18.0 \text{ cm})(-38.0 \text{ cm})}{18.0 \text{ cm} - 38.0 \text{ cm}} = +34.2 \text{ cm} \]

(f) The power of the corrective lens is then

\[ P = \frac{1}{f_{\text{meters}}} = \frac{1}{+0.342 \text{ m}} = +2.92 \text{ diopters} \]

(g) With a contact lens, the lens to eye distance would be zero, so we would have \( p = 20.0 \text{ cm} \), \( q = -40.0 \text{ cm} \), giving a required focal length of

\[ f = \frac{pq}{p+q} = \frac{(20.0 \text{ cm})(-40.0 \text{ cm})}{20.0 \text{ cm} - 40.0 \text{ cm}} = +40.0 \text{ cm} \]

and a power in diopters of

\[ P = \frac{1}{f_{\text{meters}}} = \frac{1}{+0.400 \text{ m}} = +2.50 \text{ diopters} \]

25.11 His lens must form an upright, virtual image of a very distant object \(( p \approx \infty )\) at his far point, 80.0 cm in front of the eye. Therefore, the focal length is \( f = q = -80.0 \text{ cm} \).

If this lens is to form a virtual image at his near point \(( q = -18.0 \text{ cm} )\), the object distance must be

\[ p = \frac{qf}{q-f} = \frac{(-18.0 \text{ cm})(-80.0 \text{ cm})}{-18.0 \text{ cm} - (-80.0 \text{ cm})} = 23.2 \text{ cm} \]
25.12  (a) When the child clearly sees objects at her far point \( p_{\text{max}} = 125 \text{ cm} \), the lens-cornea combination has assumed a focal length suitable for forming the image on the retina \( (q = 2.00 \text{ cm}) \). The thin lens equation gives the optical power under these conditions as

\[
\frac{1}{f_{\text{in meters}}} = \frac{1}{p} + \frac{1}{q} = \frac{1}{1.25 \text{ m}} + \frac{1}{0.0200 \text{ m}} = 50.8 \text{ diopters}
\]

When the eye is focused \( (q = 2.00 \text{ cm}) \) on objects at her near point \( (p_{\text{min}} = 10.0 \text{ cm}) \), the optical power of the lens-cornea combination is

\[
\frac{1}{f_{\text{in meters}}} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.100 \text{ m}} + \frac{1}{0.0200 \text{ m}} = 60.0 \text{ diopters}
\]

(b) If the child is to see very distant objects \( (p \to \infty) \) clearly, her eyeglass lens must form an erect virtual image at the far point of her eye \( (q = -125 \text{ cm}) \). The optical power of the required lens is

\[
\frac{1}{f_{\text{in meters}}} = \frac{1}{p} + \frac{1}{q} = 0 + \frac{1}{-1.25 \text{ m}} = -0.800 \text{ diopters}
\]

Since the power, and hence the focal length, of this lens is negative, it is diverging.

25.13  (a) The lens should form an upright, virtual image at the far point \( (q = -50.0 \text{ cm}) \) for very distant objects \( (p \to \infty) \). Therefore, \( f = q = -50.0 \text{ cm} \), and the required power is

\[
\frac{1}{f} = \frac{1}{-0.500 \text{ m}} = -2.00 \text{ diopters}
\]

(b) If this lens is to form an upright, virtual image at the near point of the unaided eye \( (q = -13.0 \text{ cm}) \), the object distance should be

\[
p = \frac{qf}{q - f} = \frac{(-13.0 \text{ cm})(-50.0 \text{ cm})}{-13.0 \text{ cm} - (-50.0 \text{ cm})} = 17.6 \text{ cm}
\]

25.14  (a) Yes, a single lens can correct the patient’s vision. The patient needs corrective action in both the near vision (to allow clear viewing of objects between 45.0 cm and the normal near point of 25 cm) and the distant vision (to allow clear viewing of objects more than 85.0 cm away). A single lens solution is for the patient to wear a bifocal or progressive lens. Alternately, the patient must purchase two pairs of glasses, one for reading, and one for distant vision.

(b) To correct the near vision, the lens must form an upright, virtual image at the patient’s near point \( (q = -45.0 \text{ cm}) \) when a real object is at the normal near point \( (p = +25.0 \text{ cm}) \). The thin lens equation gives the needed focal length as

\[
f = \frac{pq}{p + q} = \frac{(25.0 \text{ cm})(-45.0 \text{ cm})}{25.0 \text{ cm} - 45.0 \text{ cm}} = +56.3 \text{ cm}
\]

so the required power in diopters is

\[
\frac{1}{f_{\text{in meters}}} = \frac{1}{0.563 \text{ m}} = +1.78 \text{ diopters}
\]

continued on next page
To correct the distant vision, the lens must form an upright, virtual image at the patient’s far point \((q = -85.0 \text{ cm})\) for the most distant objects \((p \to \infty)\). The thin lens equation gives the needed focal length as \(f = q = -85.0 \text{ cm}\), so the needed power is

\[
\varphi = \frac{1}{f_{\text{infin}}} = \frac{1}{-0.850 \text{ m}} = -1.18 \text{ dioptrers}
\]

25.15 Considering the image formed by the cornea as a virtual object for the implanted lens, we have \(p = -(2.80 \text{ cm} + 2.53 \text{ cm}) = -5.33 \text{ cm}\) and \(q = +2.80 \text{ cm}\). The thin lens equation then gives the focal length of the implanted lens as

\[
f = \frac{pq}{p + q} = \frac{(-5.33 \text{ cm})(2.80 \text{ cm})}{-5.33 \text{ cm} + 2.80 \text{ cm}} = +5.90 \text{ cm}
\]

so the power is \(\varphi = \frac{1}{f} = \frac{1}{+0.059 \text{ m}} = +17.0 \text{ dioptrers}\).

25.16 (a) The upper portion of the lens should form an upright, virtual image of very distant objects \((p \to \infty)\) at the far point of the eye \((q = -1.5 \text{ m})\). The thin lens equation then gives \(f = q = -1.5 \text{ m}\), so the needed power is

\[
\varphi = \frac{1}{f} = \frac{1}{-1.5 \text{ m}} = -0.67 \text{ dioptrers}
\]

(b) The lower part of the lens should form an upright, virtual image at the near point of the eye \((q = -30 \text{ cm})\) when the object distance is \(p = 25 \text{ cm}\). From the thin lens equation,

\[
f = \frac{pq}{p + q} = \frac{(25 \text{ cm})(-30 \text{ cm})}{25 \text{ cm} - 30 \text{ cm}} = +1.5 \times 10^3 \text{ cm} = +1.5 \text{ m}
\]

Therefore, the power is \(\varphi = \frac{1}{f} = \frac{1}{+1.5 \text{ m}} = +0.67 \text{ dioptrers}\).

25.17 The corrective lens should form an upright, virtual image at the woman’s far point \((q = -40.0 \text{ cm})\) for a very distant object \((p \to \infty)\). The thin lens equation gives the required focal length as \(f = q = -40.0 \text{ cm} = -0.400 \text{ m}\). Since \(f < 0\), it is a \textbf{diverging lens}, and the required power is

\[
\varphi = \frac{1}{f_{\text{infin}}} = \frac{1}{-0.400 \text{ m}} = -2.50 \text{ dioptrers}
\]

25.18 (a) \(f = \frac{1}{\varphi} = \frac{1}{-0.250 \text{ m}} = -4.00 \text{ dioptrers}\)

(b) The corrective lens form virtual images of very distant objects \((p \to \infty)\) at \(q = f = -25.0 \text{ cm}\). Thus, the person must be very \textbf{nearsighted}, unable to see objects clearly when they are over \((25.0 + 2.00) \text{ cm} = 27.0 \text{ cm}\) in front of the eye.

(c) If contact lenses are to be worn, the far point of the eye will be \(27.0 \text{ cm}\) in front of the lens, so the needed focal length will be \(f = q = -27.0 \text{ cm}\), and the power is

\[
\varphi = \frac{1}{f_{\text{infin}}} = \frac{1}{-0.270 \text{ m}} = -3.70 \text{ dioptrers}
\]
25.19 (a) The simple magnifier (a converging lens) is to form an upright, virtual image located 25 cm in front of the lens \( q = -25 \text{ cm} \). The thin lens equation then gives

\[
p = \frac{qf}{q - f} = \frac{(-25 \text{ cm})(7.5 \text{ cm})}{-25 \text{ cm} - 7.5 \text{ cm}} = +5.8 \text{ cm}
\]

so the stamp should be placed 5.8 cm in front of the lens.

(b) When the image is at the near point of the eye, the angular magnification produced by the simple magnifier is

\[
m = m_{\text{max}} = 1 + \frac{25 \text{ cm}}{f} = 1 + \frac{25 \text{ cm}}{7.5 \text{ cm}} = 4.3
\]

25.20 (a) The maximum magnification of a simple magnifier is \( m_{\text{max}} = 1 + (25 \text{ cm})/f \). Thus, if \( m_{\text{max}} = +6.0 \), the focal length of the lens is

\[
f = \frac{25 \text{ cm}}{m_{\text{max}} - 1} = \frac{25 \text{ cm}}{6.0 - 1} = 5.0 \text{ cm}
\]

(b) While using a simple magnifier, the eye is most relaxed if the lens forms the virtual image at infinity (so parallel rays emerge from the lens) rather than at the near point of the eye. Under these conditions, the magnification produced is

\[
m = \frac{25 \text{ cm}}{f} = \frac{25 \text{ cm}}{5.0 \text{ cm}} = +5.0
\]

25.21 (a) From the thin lens equation,

\[
f = \frac{pq}{p + q} = \frac{(3.50 \text{ cm})(-25.0 \text{ cm})}{3.50 \text{ cm} - 25.0 \text{ cm}} = +4.07 \text{ cm}
\]

(b) With the image at the normal near point, the angular magnification is

\[
m = m_{\text{max}} = 1 + \frac{25.0 \text{ cm}}{f} = 1 + \frac{25.0 \text{ cm}}{4.07 \text{ cm}} = +7.14
\]

25.22 (a) When the object is at the focal point of the magnifying lens, a virtual image is formed at infinity and parallel rays emerge from the lens. Under these conditions, the eye is most relaxed and the magnification produced is

\[
m = \frac{25 \text{ cm}}{f} = \frac{25 \text{ cm}}{5.0 \text{ cm}} = +5.0
\]

(b) When the object is positioned so the magnifier forms a virtual image at the near point of the eye \( q = -25 \text{ cm} \), maximum magnification is produced and this is

\[
m_{\text{max}} = 1 + \frac{25 \text{ cm}}{f} = 1 + \frac{25 \text{ cm}}{5.0 \text{ cm}} = +6.0
\]

(c) From the thin lens equation, the object distance needed to yield the maximum magnification computed in part (b) above is

\[
p = \frac{qf}{q - f} = \frac{(-25 \text{ cm})(5.0 \text{ cm})}{-25 \text{ cm} - 5.0 \text{ cm}} = 4.2 \text{ cm}
\]
25.23 (a) From the thin lens equation, a real inverted image is formed at an image distance of

\[ q = \frac{pf}{p - f} = \frac{(71.0 \text{ cm})(39.0 \text{ cm})}{71.0 \text{ cm} - 39.0 \text{ cm}} = +86.5 \text{ cm} \]

so the lateral magnification produced by the lens is

\[ M = \frac{h'}{h} = -\frac{q}{p} = -\frac{86.5 \text{ cm}}{71.0 \text{ cm}} = -1.22 \quad \text{and the magnitude is } |M| = 1.22 \]

(b) If \(|h|\) is the actual length of the leaf, the small angle approximation gives the angular width of the leaf when viewed by the unaided eye from a distance of \(d = 126 \text{ cm} + 71.0 \text{ cm} = 197 \text{ cm}\) as

\[ \theta_0 = \frac{|h|}{d} = \frac{|h|}{197 \text{ cm}} \]

The length of the image formed by the lens is \(|h'| = |M h| = 1.22|h|\), and its angular width when viewed from a distance of \(d' = 126 \text{ cm} - q = 39.5 \text{ cm}\) is

\[ \theta = \frac{|h'|}{d'} = \frac{1.22|h|}{39.5 \text{ cm}} \]

The angular magnification achieved by viewing the image instead of viewing the leaf directly is

\[ \frac{\theta}{\theta_0} = \frac{1.22|h|/39.5 \text{ cm}}{|h|/197 \text{ cm}} = \frac{1.22(197 \text{ cm})}{39.5 \text{ cm}} = 6.08 \]

25.24 (a) With the image at the normal near point \((q = -25 \text{ cm})\), the angular magnification is

\[ m = 1 + \frac{25 \text{ cm}}{f} = 1 + \frac{25 \text{ cm}}{25 \text{ cm}} = +2.0 \]

(b) When the eye is relaxed, parallel rays enter the eye and

\[ m = \frac{25 \text{ cm}}{f} = \frac{25 \text{ cm}}{25 \text{ cm}} = +1.0 \]

25.25 The overall magnification is \(m = M_m = M_1 \left( \frac{25 \text{ cm}}{f_o} \right)\)

where \(M_1\) is the lateral magnification produced by the objective lens. Therefore, the required focal length for the eyepiece is

\[ f_o = \frac{M_1(25 \text{ cm})}{m} = \frac{(-12)(25 \text{ cm})}{-140} = 2.1 \text{ cm} \]

25.26 The approximate overall magnification of a compound microscope is given by

\[ m = -(L/f_o)(25.0 \text{ cm}/f_e) \]

where \(L\) is the distance between the objective and eyepiece lenses, while \(f_o\) and \(f_e\) are the focal lengths of the objective and eyepiece lenses, respectively. Thus, the described microscope should have an approximate overall magnification of

\[ m = \frac{L}{f_o} \left( \frac{25.0 \text{ cm}}{f_e} \right) = \frac{20.0 \text{ cm}}{0.500 \text{ cm}} \left( \frac{25.0 \text{ cm}}{1.70 \text{ cm}} \right) = -588 \]
25.27 The magnitude of the magnification of a telescope is \( m = f_o / f_e \), where \( f_o \) and \( f_e \) are the focal lengths of the objective element and the eyepiece, respectively. Thus, if \( m = 45 \) and \( f_e = 4.0 \) cm, the focal length of the objective must be \( f_o = m f_e = (45)(4.0 \text{ cm}) = 180 \text{ cm} \). The overall length of the telescope will therefore be

\[
L = f_o + f_e = 180 \text{ cm} + 4.0 \text{ cm} = 184 \text{ cm} = 1.84 \text{ m}
\]

25.28 It is specified that the final image the microscope forms of the blood cell is 29.0 cm in front of the eye and that the diameter of this image intercepts an angle of \( \theta = 1.43 \) mrad. The diameter of this final image must then be

\[
h_r = r\theta = (29.0 \times 10^{-2} \text{ m})(1.43 \times 10^{-3} \text{ rad}) = 4.15 \times 10^{-4} \text{ m}
\]

At this point, it is tempting to use Equation (25.7) from the textbook for the overall magnification of a compound microscope, and compute \( h = h_r / m \) as the size of the blood cell serving as the object for the microscope. However, the derivation of that equation is based on several assumptions, one of which is that the eye is relaxed and viewing a final image located an infinite distance in front of the eyepiece. This is clearly not true in this case, and the use of Equation (25.7) would introduce considerable error. Instead, we shall return to basics and use the thin lens equation to find the size of the original object.

The image formed by the objective lens is the object for the eyepiece, and we label the size of this image as \( h' \). The lateral magnification of the objective lens is \( M_o = h'/h = -q_o/p_o \), and that of the eyepiece is \( M_e = h_r/h' = -q_r/p_r \). The overall magnification produced by the microscope is

\[
M = h_r / h = \left( \frac{h'}{h} \right) \left( \frac{h}{h'} \right) = 1
\]

which gives the size of the original object as \( h = h_r / |M| \)

From the thin lens equation, the required object distance for the eyepiece is

\[
p_o = \frac{q_o f_o}{q_o - f_o} = \frac{-29.0 \text{ cm}(0.950 \text{ cm})}{-29.0 \text{ cm} - 0.950 \text{ cm}} = 0.920 \text{ cm}
\]

and the magnification produced by the eyepiece is

\[
M_e = \frac{-q_r}{p_r} = \frac{-29.0 \text{ cm}}{0.920 \text{ cm}} = -31.5
\]

The image distance for the objective lens is then

\[
q_o = L - p_o = 29.0 \text{ cm} - 0.920 \text{ cm} = 28.1 \text{ cm}
\]

and the object distance for this lens is

\[
p_i = \frac{q_i f_o}{q_i - f_o} = \frac{28.1 \text{ cm}(1.622 \text{ cm})}{28.1 \text{ cm} - 1.622 \text{ cm}} = 1.72 \text{ cm}
\]

The magnification by the objective lens is

\[
M_o = \frac{-q_i}{p_i} = \frac{-28.1 \text{ cm}}{1.72 \text{ cm}} = -16.3
\]

and the overall lateral magnification is \( M = M_o M_e = (-16.3)(31.5) = -513 \).

The size of the red blood cell serving as the original object is

\[
h = \frac{h_r}{|M|} = \frac{4.15 \times 10^{-4} \text{ m}}{513} = 8.09 \times 10^{-7} \text{ m} = 0.809 \mu \text{m}
\]
25.29 Some of the approximations made in the textbook while deriving the overall magnification of a compound microscope are not valid in this case. Therefore, we start with the eyepiece and work backwards to determine the overall magnification.

If the eye is relaxed, the eyepiece image is at infinity \( q_e \to -\infty \), so the object distance is \( p_e = f_e = 2.50 \text{ cm} \), and the angular magnification by the eyepiece is

\[
m_e = \frac{25.0 \text{ cm}}{2.50 \text{ cm}} = 10.0
\]

The image distance for the objective lens is then

\[
q_o = L - p_e = 15.0 \text{ cm} - 2.50 \text{ cm} = 12.5 \text{ cm}
\]

and the object distance is

\[
p_o = -\frac{q_o f_e}{q_o - f_e} = \frac{(12.5 \text{ cm})(1.00 \text{ cm})}{12.5 \text{ cm} - 1.00 \text{ cm}} = 1.09 \text{ cm}
\]

The magnification by the objective lens is

\[
M_o = \frac{-q_o}{p_o} = \frac{-12.5 \text{ cm}}{1.09 \text{ cm}} = -11.5
\]

and the overall magnification of the microscope is

\[
m = M_o m_e = (-11.5)(10.0) = -115
\]

25.30 (a) For a refracting telescope, the overall length is \( L = f_o + f_e \), and the magnification produced is \( m = f_o / f_e \), where \( f_o \) and \( f_e \) are the focal lengths of the objective element and the eyepiece, respectively. Thus, we may write \( f_o = f_e / m \) to obtain

\[
L = f_e + \frac{f_e}{m} = f_e \left( 1 + \frac{1}{m} \right) = f_e \left( \frac{m + 1}{m} \right)
\]

(b) Using the result of part (a), the required change in the length of the telescope will be

\[
\Delta L = f_e \left( \frac{m' + 1}{m'} - \frac{m + 1}{m} \right) = (2.00 \text{ m}) \left( \frac{101}{100} - \frac{51.0}{50.0} \right) = -2.00 \times 10^{-2} \text{ cm} = -2.00 \text{ cm}
\]

or the telescope must be shortened by moving the eyepiece 2.00 cm forward toward the objective lens.

25.31 The length of the telescope is \( L = f_o + f_e = 92 \text{ cm} \) and the angular magnification is

\[
m = \frac{f_o}{f_e} = 45
\]

Therefore, \( f_o = 45 f_e \) and \( L = f_o + f_e = 45 f_e + f_e = 46 f_e = 92 \text{ cm} \), giving

\[
\begin{align*}
\frac{f_e}{2.0 \text{ cm}} & \quad \text{and} \quad f_e = 92 \text{ cm} - f_e \quad \text{or} \quad f_e = 90 \text{ cm}
\end{align*}
\]
25.32 The moon may be considered an infinitely distant object \( p \to \infty \) when viewed with this lens, so the image distance will be \( q = f_o = 1 \, 500 \, \text{cm} \).

Considering the rays that pass undeviated through the center of this lens as shown in the sketch, observe that the angular widths of the image and the object are equal. Thus, if \( w \) is the linear width of an object forming a 1.00 cm wide image, then

\[
\theta = \frac{w}{3.8 \times 10^8 \, \text{m}} = \frac{1.0 \, \text{cm}}{1500 \, \text{cm}}
\]

or

\[
w = (3.8 \times 10^8 \, \text{m}) \left( \frac{1.0 \, \text{cm}}{1500 \, \text{cm}} \right) \left( \frac{1 \, \text{mi}}{1609 \, \text{m}} \right) = 1.6 \times 10^{-1} \, \text{mi}
\]

25.33 (a) From the thin lens equation, \( q = \frac{pf}{p-f} \), so the lateral magnification by the objective lens is \( M = h'/h = -q/p = -f/(p-f) \). Therefore, the image size will be

\[
h' = Mh = -\frac{fh}{p-f} = \frac{fh}{f-p}
\]

(b) If \( p \gg f \), then \( p - f = p \) and \( h' = -\frac{fh}{p} \)

(c) Suppose the telescope observes the space station at the zenith.

Then, \( h' = -\frac{fh}{p} = \frac{(4.00 \, \text{m})(108.6 \, \text{m})}{407 \times 10^3 \, \text{m}} = -1.07 \times 10^{-3} \, \text{m} = -1.07 \, \text{mm} \)

25.34 Use the larger focal length (lowest power) lens as the objective element and the shorter focal length (largest power) lens for the eye piece. The focal lengths are

\[
f_o = \frac{1}{+1.20 \, \text{diop}} = +0.833 \, \text{m}, \quad \text{and} \quad f_e = \frac{1}{+9.00 \, \text{diop}} = +0.111 \, \text{m}
\]

(a) The angular magnification (or magnifying power) of the telescope is then

\[
m = \frac{f_o}{f_e} = \frac{+0.833 \, \text{m}}{+0.111 \, \text{m}} = 7.50
\]

(b) The length of the telescope is

\[
L = f_o + f_e = 0.833 \, \text{m} + 0.111 \, \text{m} = 0.944 \, \text{m}
\]
25.35 The lens for the left eye forms an upright, virtual image at \( q_L = -50.0 \text{ cm} \) when the object distance is \( p_L = 25.0 \text{ cm} \), so the thin lens equation gives its focal length as

\[
f_L = \frac{p_L q_L}{p_L + q_L} = \frac{(25.0 \text{ cm})(-50.0 \text{ cm})}{25.0 \text{ cm} - 50.0 \text{ cm}} = 50.0 \text{ cm}
\]

Similarly, for the other lens, \( q_R = -100 \text{ cm} \) when \( p_R = 25.0 \text{ cm} \), and \( f_R = 33.3 \text{ cm} \).

(a) Using the lens for the left eye as the objective,

\[
m = \frac{f_L}{f_R} = \frac{50.0 \text{ cm}}{33.3 \text{ cm}} = 1.50
\]

(b) Using the lens for the right eye as the eyepiece and, for maximum magnification, requiring that the final image be formed at the normal near point \( q_e = -25.0 \text{ cm} \) gives

\[
p_e = \frac{q_e f_e}{q_e - f_e} = \frac{(-25.0 \text{ cm})(33.3 \text{ cm})}{-25.0 \text{ cm} - 33.3 \text{ cm}} = +14.3 \text{ cm}
\]

The maximum magnification by the eyepiece is then

\[
m_e = 1 + \frac{25.0 \text{ cm}}{f_e} = 1 + \frac{25.0 \text{ cm}}{33.3 \text{ cm}} = +1.75
\]

and, the image distance for the objective is

\[q_i = L - p_e = 10.0 \text{ cm} - 14.3 \text{ cm} = -4.3 \text{ cm}\]

The thin lens equation then gives the object distance for the objective as

\[
p_i = \frac{q_i f_i}{q_i - f_i} = \frac{(-4.3 \text{ cm})(50.0 \text{ cm})}{-4.3 \text{ cm} - 50.0 \text{ cm}} = +4.0 \text{ cm}
\]

The magnification by the objective is then

\[M_i = -\frac{q_i}{p_i} = -\frac{(-4.3 \text{ cm})}{4.0 \text{ cm}} = +1.1\]

and the overall magnification is \( m = M_i m_e = (+1.1)(+1.75) = 1.9 \).

25.36 Note: We solve part (b) before answering part (a) in this problem.

(b) The objective forms a real, diminished, inverted image of a very distant object at \( q_1 = f_o \). The image is a virtual object for the eyepiece at \( p_e = \frac{1}{f_e} \), giving

\[
\frac{1}{q_e} = \frac{1}{p_e} - \frac{1}{f_e} = -\frac{1}{f_o} + \frac{1}{f_e} = 0
\]

and \( q_e \rightarrow \infty \)

continued on next page
(a) Parallel rays emerge from the eyepiece, so the eye observes a **virtual image**.

(c) The angular magnification is \( m = \frac{f_e}{|f_r|} = 3.00 \), giving \( f_r = 3.00|f_e| \). Also, the length of the telescope is \( L = f_e + f_r = 3.00|f_e| - |f_r| = 10.0 \text{ cm} \), giving

\[
f_e = -|f_r| = -\frac{10.0 \text{ cm}}{2.00} = -5.00 \text{ cm} \quad \text{and} \quad f_r = 3.00|f_e| = 15.0 \text{ cm}
\]

25.37 If just resolved, the angular separation is

\[
\theta = \theta_{\text{min}} = 1.22 \frac{\lambda}{D} = 1.22 \left( \frac{500 \times 10^{-9} \text{ m}}{0.300 \text{ m}} \right) = 2.03 \times 10^{-8} \text{ rad}
\]

Thus, the altitude is

\[
h = \frac{d}{\theta} = \frac{1.00 \text{ m}}{2.03 \times 10^{-8} \text{ rad}} = 4.92 \times 10^{5} \text{ m} = 492 \text{ km}
\]

25.38 For a narrow slit, Rayleigh’s criterion gives

\[
\theta_{\text{min}} = \frac{\lambda}{a} = \frac{500 \times 10^{-9} \text{ m}}{0.500 \times 10^{-3} \text{ m}} = 1.00 \times 10^{-3} \text{ rad} = 1.00 \text{ mrad}
\]

25.39 The limit of resolution in air is \( \theta_{\text{min} | \text{air}} = 1.22 \frac{\lambda}{D} = 0.60 \mu \text{rad} \)

In oil, the limiting angle of resolution will be

\[
\theta_{\text{min} | \text{oil}} = 1.22 \frac{\lambda_{\text{oil}}}{D} = 1.22 \left( \frac{\lambda}{n_{\text{oil}}} \right) \frac{1}{n_{\text{oil}}} = \frac{1.22 \lambda}{D} \frac{1}{n_{\text{oil}}}
\]

or

\[
\theta_{\text{min} | \text{oil}} = \frac{\theta_{\text{min} | \text{air}}}{n_{\text{oil}}} = \frac{0.60 \mu \text{rad}}{1.5} = 0.40 \mu \text{rad}
\]

25.40 (a) The wavelength of the light within the eye is \( \lambda_e = \frac{\lambda}{n} \). Thus, the limiting angle of resolution for light passing through the pupil (a circular aperture with diameter \( D = 2.00 \text{ mm} \)) is

\[
\theta_{\text{min}} = 1.22 \frac{\lambda_e}{n D} = 1.22 \frac{\lambda}{n D} = 1.22 \left( \frac{500 \times 10^{-9} \text{ m}}{1.33} \frac{1}{2.00 \times 10^{-3} \text{ m}} \right) = 2.29 \times 10^{-4} \text{ rad}
\]

(b) From \( s = r \theta \), the distance from the eye that two points separated by a distance \( s = 1.00 \text{ cm} \) will intercept this minimum angle of resolution is

\[
r = \frac{s}{\theta_{\text{min}}} = \frac{1.00 \text{ cm}}{2.29 \times 10^{-4} \text{ rad}} = 4.36 \times 10^{3} \text{ cm} = 43.6 \text{ m}
\]

25.41 The angular separation of the headlights when viewed from a distance of \( r = 10.0 \text{ km} \) is

\[
\theta = \frac{s}{r} = \frac{2.00 \text{ m}}{10.0 \times 10^{3} \text{ m}} = 2.00 \times 10^{-4} \text{ rad}
\]

If the headlights are to be just resolved, this separation must equal the limiting angle of resolution for the circular aperture, \( \theta_{\text{min}} = 1.22 \lambda/D \), so the diameter of the aperture is

\[
D = \frac{1.22 \lambda}{\theta} = \frac{1.22 \left( 885 \times 10^{-9} \text{ m} \right)}{2.00 \times 10^{-4} \text{ rad}} = 5.40 \times 10^{-3} \text{ m} = 54.0 \mu \text{m}
\]
25.42 Diffraction occurs when waves pass through an aperture, causing the intensity to go through maxima and minima as one goes from the center of the beam outward as illustrated in the figure at the right. The angular separation of the first minimum from the central maximum is a constant determined by the dimension of the aperture, the wavelength of the wave, and the shape of the aperture. For a circular aperture, this angular separation is given by \( \theta_{\text{min}} = 1.22 \lambda / D \), where \( D \) is the diameter of the aperture. The full angular width of the central maximum is then \( \alpha = 2 \theta_{\text{min}} = 2.44 \lambda / D \).

The lateral width of the central maximum, \( d \), increases as the distance \( r \) from the aperture increases. When a beam of laser light having wavelength \( \lambda = 632.8 \text{ nm} \) diffracts through a circular opening of diameter \( D = 0.200 \text{ cm} \), we estimate the diameter of the beam at distance \( r = 3.00 \text{ km} \) past the opening as equal to the diameter of the central maximum in the diffraction pattern at this location. This gives

\[
d = r \alpha = r \left( \frac{2.44 \lambda}{D} \right) = (3.00 \times 10^3 \text{ m}) \left[ \frac{2.44 (632.8 \times 10^{-9} \text{ m})}{0.200 \times 10^{-2} \text{ m}} \right] = 2.32 \text{ m}
\]

25.43 If just resolved, the angular separation of the objects is \( \theta = \theta_{\text{min}} = \frac{1.22 \lambda}{D} \)
and \( s = r \theta = (8.0 \times 10^3 \text{ km}) \left[ 1.22 \left( \frac{500 \times 10^{-9} \text{ m}}{5.00 \text{ m}} \right) \right] = 9.8 \text{ km} \)

25.44 If just resolved, the angular separation of the objects is \( \theta = \theta_{\text{min}} = \frac{1.22 \lambda}{D} \)
and \( s = r \theta = (200 \times 10^3 \text{ m}) \left[ 1.22 \left( \frac{550 \times 10^{-9} \text{ m}}{0.35 \text{ m}} \right) \right] = 0.38 \text{ m} = 38 \text{ cm} \)

25.45 The grating spacing is \( d = \frac{1 \text{ cm}}{6000} = 1.67 \times 10^{-4} \text{ cm} = 1.67 \times 10^{-6} \text{ m} \), and the highest order of 600 nm light that can be observed is

\[
m_{\text{max}} = \frac{d \sin 90^\circ}{\lambda} = \frac{(1.67 \times 10^{-6} \text{ m})(1)}{600 \times 10^{-9} \text{ m}} = 2.78 \rightarrow 2 \text{ orders}
\]

The total number of slits is \( N = (15.0 \text{ cm})(6000 \text{ slits/cm}) = 9.00 \times 10^4 \), and the resolving power of the grating in the second order is

\[
R_{\text{available}} = Nm = (9.00 \times 10^4)(2) = 1.80 \times 10^5
\]

The resolving power required to separate the given spectral lines is

\[
R_{\text{needed}} = \frac{\lambda}{\Delta \lambda} = \frac{600.000 \text{ nm}}{0.003 \text{ nm}} = 2.0 \times 10^7
\]

These lines cannot be separated with this grating.
25.46 The resolving power of a diffraction grating is \[ R = \frac{\lambda}{\Delta \lambda} = N m \]

(a) The number of lines the grating must have to resolve the H\(_\alpha\) line in the first order is
\[ N = \frac{R}{m} = \frac{\lambda}{\Delta \lambda} \]
\[ \left(1\right) \]
\[ = \frac{656.2 \text{ nm}}{0.18 \text{ nm}} = 3.6 \times 10^3 \text{ lines} \]

(b) In the second order \((m = 2)\), \[ N = \frac{R}{2} = \frac{656.2 \text{ nm}}{2 \times 0.18 \text{ nm}} = 1.8 \times 10^3 \text{ lines} \]

25.47 A fringe shift occurs when the mirror moves distance \(\lambda/4\). Thus, if the mirror moves distance \(\Delta L = 0.180 \text{ mm}\), the number of fringe shifts observed is
\[ N_{\text{shifts}} = \frac{\Delta L}{\lambda/4} = \frac{4 \left(0.180 \times 10^{-3} \text{ m}\right)}{550 \times 10^{-9} \text{ m}} = 1.31 \times 10^3 \text{ fringe shifts} \]

25.48 (a) When the central spot in the interferometer pattern goes through a full cycle from bright to dark and back to bright, two fringe shifts have occurred and the movable mirror has moved a distance of \(2(\lambda/4) = \lambda/2\). Thus, if \(N_{\text{cycles}} = 1700\) such cycles are observed as the mirror moves distance \(d = 0.382 \text{ mm}\), it must be true that
\[ d = N_{\text{cycles}} \left(\frac{\lambda}{2}\right) \quad \text{or} \quad \lambda = \frac{2d}{N_{\text{cycles}}} \]
and the wavelength of the light illuminating the interferometer is
\[ \lambda = \frac{2 \left(0.382 \times 10^{-3} \text{ m}\right)}{1700} = 4.49 \times 10^{-7} \text{ m} = 449 \text{ nm} \]
which is in the blue region of the visible spectrum.

(b) Red light has a longer wavelength than blue light, so fewer wavelengths would cover the given displacement, hence \(N_{\text{cycles}}\) would be smaller.

25.49 A fringe shift occurs when the mirror moves distance \(\lambda/4\). Thus, the distance moved (length of the bacterium) as 310 shifts occur is
\[ \Delta L = N_{\text{shifts}} \left(\frac{\lambda}{4}\right) = 310 \left(\frac{650 \times 10^{-9} \text{ m}}{4}\right) = 5.04 \times 10^{-5} \text{ m} = 50.4 \mu\text{m} \]

25.50 A fringe shift occurs when the mirror moves distance \(\lambda/4\). Thus, the distance the mirror moves as 250 fringe shifts are counted is
\[ \Delta L = N_{\text{shifts}} \left(\frac{\lambda}{4}\right) = 250 \left(\frac{632.8 \times 10^{-9} \text{ m}}{4}\right) = 3.96 \times 10^{-5} \text{ m} = 39.6 \mu\text{m} \]
25.51 When the optical path length that light must travel as it goes down one arm of a Michelson’s interferometer changes by one wavelength, four fringe shifts will occur (one shift for every quarter-wavelength change in path length).

The number of wavelengths (in a vacuum) that fit in a distance equal to a thickness \( t \) is \( N_{\text{vac}} = \frac{t}{\lambda} \). The number of wavelengths that fit in this thickness while traveling through the transparent material is \( N_t = \frac{t}{\lambda/n} = \frac{t}{\lambda/n} = m/\lambda \). Thus, the change in the number of wavelengths that fit in the path down this arm of the interferometer is

\[
\Delta N = N_t - N_{\text{vac}} = (n-1)\frac{t}{\lambda}
\]

and the number of fringe shifts that will occur as the sheet is inserted will be

\[
\# \text{ fringe shifts} = 4(\Delta N) = 4(n-1)\frac{t}{\lambda} = 4(1.40 - 1) \left( \frac{15.0 \times 10^{-6} \text{ m}}{600 \times 10^{-9} \text{ m}} \right) = 40
\]

25.52 A fringe shift will occur each time the effective length of the tube changes by a quarter of a wavelength (that is, for each additional wavelength fitted into the length of the tube, 4 fringe shifts occur). If \( L \) is the length of the tube, the number of fringe shifts observed as the tube is filled with gas is

\[
N_{\text{shifts}} = 4 \left[ \frac{L}{\lambda} - \frac{L}{\lambda/n_{\text{gas}}} \right] = 4 \left[ \frac{L}{\lambda/n_{\text{gas}}} - \frac{L}{\lambda} \right] = \frac{4L}{\lambda} (n_{\text{gas}} - 1)
\]

Hence, \( n_{\text{gas}} = 1 + \left( \frac{\lambda}{4L} \right) N_{\text{shifts}} = 1 + \left[ \frac{600 \times 10^{-9} \text{ m}}{4 \left( 5.00 \times 10^{-3} \text{ m} \right)} \right] (160) = 1000.5
\]

25.53 (a) For a refracting telescope, the magnification is \( m = f_o/f_e \), where \( f_o \) and \( f_e \) are the focal lengths of the objective lens and the eyepiece, respectively. Thus, when the Yerkes telescope uses an eyepiece with \( f_e = 2.50 \text{ cm} \), the magnification is

\[
m = \frac{f_o}{f_e} = \frac{20.0 \text{ m}}{2.50 \times 10^{-2} \text{ m}} = 8.00 \times 10^2 = 800
\]

(b) Standard astronomical telescopes form inverted images. Thus, the observer Martian polar caps are upside down.

25.54 When viewed from a distance of 50 meters, the angular length of a mouse (assumed to have an actual length of \( 10 \text{ cm} \)) is

\[
\theta = \frac{s}{r} = \frac{0.10 \text{ m}}{50 \text{ m}} = 2.0 \times 10^{-3} \text{ radians}
\]

Thus, the limiting angle of resolution of the eye of the hawk must be

\[
\theta_{\text{min}} \leq \theta = 2.0 \times 10^{-3} \text{ rad}
\]
The resolving power of the grating is \( R = \lambda / \Delta \lambda = N m \). Thus, the total number of lines needed on the grating to resolve the wavelengths in order \( m \) is

\[
N = \frac{R}{m} = \frac{\lambda}{m(\Delta \lambda)}
\]

(a) For the sodium doublet in the first order,

\[
N = \frac{589.30 \text{ nm}}{(1)(0.59 \text{ nm})} = 1.0 \times 10^3
\]

(b) In the third order, we need \( N = \frac{589.30 \text{ nm}}{(3)(0.59 \text{ nm})} = 3.3 \times 10^2 \)

25.56

(a) Since this eye can already focus on objects located at the near point of a normal eye (25 cm), no correction is needed for near objects. To correct the distant vision, a corrective lens (located 2 cm from the eye) should form virtual images of very distant objects at 23 cm in front of the lens (or at the far point of the eye). Thus, we must require that \( q = -23 \text{ cm} \) when \( p \to \infty \). This gives

\[
\varphi = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = 0 + \frac{1}{-0.23 \text{ m}} = -4.3 \text{ dioptrers}
\]

(b) A corrective lens in contact with the cornea should form virtual images of very distant objects at the far point of the eye. Therefore, we require that \( q = -25 \text{ cm} \) when \( p \to \infty \), giving

\[
\varphi = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = 0 + \frac{1}{-0.25 \text{ m}} = -4.0 \text{ dioptrers}
\]

When the contact lens \( \left( f = \frac{1}{\varphi} = -25 \text{ cm} \right) \) is in place, the object distance which yields a virtual image at the near point of the eye (that is, \( q = -16 \text{ cm} \)) is given by

\[
p = \frac{qf}{q - f} = \frac{(-16 \text{ cm})(-25 \text{ cm})}{-16 \text{ cm} - (-25 \text{ cm})} = 44 \text{ cm}
\]

25.57

(a) The lens should form an upright, virtual image at the near point of the eye \( q = -75.0 \text{ cm} \) when the object distance is \( p = 25.0 \text{ cm} \). The thin lens equation then gives

\[
f = \frac{pq}{p + q} = \frac{(25.0 \text{ cm})(-75.0 \text{ cm})}{25.0 \text{ cm} - 75.0 \text{ cm}} = 37.5 \text{ cm} = 0.375 \text{ m}
\]

so the needed power is \( \varphi = \frac{1}{f} = \frac{1}{0.375 \text{ m}} = +2.67 \text{ dioptrers} \).

(b) If the object distance must be \( p = 26.0 \text{ cm} \) to position the image at \( q = -75.0 \text{ cm} \), the actual focal length is

\[
f = \frac{pq}{p + q} = \frac{(26.0 \text{ cm})(-75.0 \text{ cm})}{26.0 \text{ cm} - 75.0 \text{ cm}} = 0.398 \text{ m}
\]

and \( \varphi = \frac{1}{f} = \frac{1}{0.398 \text{ m}} = +2.51 \text{ dioptrers} \)

The error in the power is

\[
\Delta \varphi = (2.67 - 2.51) \text{ dioptrers} = 0.16 \text{ dioptrers too low}
\]
25.58  (a) If \( q = 2.00 \text{ cm} \) when \( p = 1.00 \text{ m} = 100 \text{ cm} \), the thin lens equation gives the focal length as
\[
f = \frac{pq}{p + q} = \frac{(100 \text{ cm})(2.00 \text{ cm})}{100 \text{ cm} + 2.00 \text{ cm}} = 1.96 \text{ cm}
\]

(b) The \( f \)-number of a lens aperture is the focal length of the lens divided by the diameter of the aperture. Thus, the smallest \( f \)-number occurs with the largest diameter of the aperture. For the typical eyeball focused on objects 1.00 m away, this is
\[
(f\text{-number})_{\text{min}} = \frac{f}{D_{\text{max}}} = \frac{1.96 \text{ cm}}{0.600 \text{ cm}} = 3.27
\]

(c) The largest \( f \)-number of the typical eyeball focused on a 1.00-m-distance object is
\[
(f\text{-number})_{\text{max}} = \frac{f}{D_{\text{min}}} = \frac{1.96 \text{ cm}}{0.200 \text{ cm}} = 9.80
\]

25.59  (a) The implanted lens should give an image distance of \( q = 22.4 \text{ mm} \) for distant \( (p \to \infty) \) objects. The thin lens equation then gives the focal length as \( f = q = 22.4 \text{ mm} \), so the power of the implanted lens should be
\[
P_{\text{implant}} = \frac{1}{f} = \frac{1}{22.4 \times 10^{-3} \text{ m}} = +44.6 \text{ dipters}
\]

(b) When the object distance is \( p = 33.0 \text{ cm} \), the corrective lens should produce parallel rays \( (q \to \infty) \). Then the implanted lens will focus the final image on the retina. From the thin lens equation, the required focal length is \( f = p = 33.0 \text{ cm} \), and the power of this lens should be
\[
P_{\text{correction}} = \frac{1}{f} = \frac{1}{0.330 \text{ m}} = +3.03 \text{ dipters}
\]

25.60  We use \( \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_1 - n_2}{R} \), with \( p \to \infty \) and \( q \) equal to the cornea to retina distance. Then,
\[
R = q \left( \frac{n_2 - n_1}{n_2} \right) = (2.00 \text{ cm}) \left( \frac{1.34 - 1.00}{1.34} \right) = 0.507 \text{ cm} = 5.07 \text{ mm}
\]

25.61  When a converging lens forms a real image of a very distant object, the image distance equals the focal length of the lens. Thus, if the scout started a fire by focusing sunlight on kindling 5.00 cm from the lens, \( f = q = 5.00 \text{ cm} \).

(a) When the lens is used as a simple magnifier, maximum magnification is produced when the upright, virtual image is formed at the near point of the eye \( (q = -15 \text{ cm} \text{ in this case}) \). The object distance required to form an image at this location is
\[
p = \frac{qf}{q - f} = \frac{(-15 \text{ cm})(5.0 \text{ cm})}{-15 \text{ cm} - 5.0 \text{ cm}} = 15 \text{ cm}
\]

and the lateral magnification produced is \( M = -\frac{q}{p} = -\frac{-15 \text{ cm}}{15 \text{ cm}/4.0} = +4.0 \)

*continued on next page*
(b) When the object is viewed directly while positioned at the near point of the eye, its angular size is \( \theta_0 = h/15 \text{ cm} \). When the object is viewed by the relaxed eye while using the lens as a simple magnifier (with the object at the focal point so parallel rays enter the eye), the angular size of the upright, virtual image is \( \theta = h/f \). Thus, the angular magnification gained by using the lens is

\[
m = \frac{\theta}{\theta_0} = \frac{h/f}{h/15 \text{ cm}} = \frac{15 \text{ cm}}{f} = \frac{15 \text{ cm}}{5.0 \text{ cm}} = 3.0
\]

25.62 The angular magnification is \( m = \theta/\theta_0 \), where \( \theta \) is the angle subtended by the final image, and \( \theta_0 \) is the angle subtended by the object as shown in the figure. When the telescope is adjusted for minimum eyestrain, the rays entering the eye are parallel. Thus, the objective lens must form its image at the focal point of the eyepiece.

From triangle ABC, \( \theta_o \approx \tan \theta_o = h'/q_o \), and from triangle DEF, \( \theta \approx \tan \theta = h'/f_e \). The angular magnification is then

\[
m = \frac{\theta}{\theta_o} = \frac{h'/f_e}{h'/q_o} = \frac{q_o}{f_e}
\]

From the thin lens equation, the image distance of the objective lens in this case is

\[
q_o = \frac{p_1 f_1}{p_1 - f_1} = \frac{(300 \text{ cm})(20.0 \text{ cm})}{300 \text{ cm} - 20.0 \text{ cm}} = 21.4 \text{ cm}
\]

With an eyepiece of focal length \( f_e = 2.00 \text{ cm} \), the angular magnification for this telescope is

\[
m = \frac{q_o}{f_e} = \frac{21.4 \text{ cm}}{2.00 \text{ cm}} = 10.7
\]