1: We have discussed the drag force on a moving object (sphere) in fluid. Let us perform the dimensional analysis with a slightly different set of governing variables:

\[ F_d = f(D, v, \rho, \mu) \]

where \( D, v, \rho, \) and \( \mu \) represent the diameter and speed of the sphere, fluid density, and viscosity, respectively.

(1) The motion of a viscous fluid is governed by the Navier-Stokes equation (equation of motion for fluid):

\[ \rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \nabla) \vec{v} = -\nabla p + \mu \nabla^2 \vec{v}. \]

Here, \( \vec{v} \) is the velocity of fluid component. Using this equation, extract the dimension of viscosity \( \mu \). This \( \vec{v} \) has nothing to do with \( v \), the object speed.

(2) Are the four governing variables dimensionally independent? Why?

(3) Construct a dimensionless variable \( \Pi_\mu \) from \( \mu \) using other three variables.

(4) Show that the drag force can be rewritten the completely dimensionless form:

\[ \frac{F_d}{\rho D^2 v^2} = \Phi (\Pi_\mu), \]

where \( \Phi(x) \) is an unknown function of a dimensionless variable \( x \).

It is important to note that the above equation implies that all flows with the same \( \Pi_\mu \) are dynamically similar. \( Re = \Pi_\mu^{-1} \) is called the Reynolds number, which is an important dimensionless number that determines the character of a flow such as laminar or turbulent flow. If you do not have the dimensionless form of the drag force, you will have to conduct experiments to determine \( F_d \) vs \( D \) keeping all other variables and then \( F_d \) vs \( v \) keeping other variables, and so on. However, the dimensionless form extracted through the dimensional analysis has effectively two variables, the reduced drag force and \( \Pi_\mu \). You do not have to vary viscosity and density. You can extract the above dependence in a single wind tunnel experiment.

HW 2: Perform the Taylor expansion of \( \tanh(x) \) to the order of \( x^3 \) around \( x = 0 \).

HW 3: Using Taylor expansion, express \( f(\beta) = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \) for \( \beta \ll 1 \).
HW 4: A particle of mass $m$ is moving in 1-dimension under potential energy given by

$$U(x) = -\frac{1}{3}x^3 + \frac{3}{2}ax^2 - 2a^2x + \frac{1}{3}a^3 \quad (a > 0).$$

(1) Without using the aid of graphing calculator, plot $U(x)$.

(2) You will find a minimum in $U(x)$. The particle is oscillating around the minimum point. What is the oscillation frequency $f_o$?

HW 5: Tipler 1-10.

HW 6: Consider an inelastic collision in which two particles collide and scatter into two other particles: $A + B \rightarrow C + D$. $m_A, m_B, m_C$, and $m_D$ represent the mass of each particle. Show that the law of conservation of classical linear momentum is invariant under the Galilean transformation if and only if total mass is conserved.