HOMEWORK ASSIGNMENT 2  
Problems Chapter 2

2-2  Assume an ideal gas whose equation of state gives 

\[ PV = nRT \]

a) How many moles? 

\[ n = \frac{PV}{RT} = \frac{1.5 \times 10^6 \times 0.5}{8.314 \times 10^3 \times (273.15 + 20)} = 0.308 \]

b) How many kilograms? The atomic weight of oxygen is 16, but it is a molecular gas with two oxygens per molecule, so 

\[ m = 32n = 32 \times 0.308 = 9.86 \]

c) What is the pressure if the temperature is increased to 500 °C? 

\[ P' = P \frac{T'}{T} = 1.5 \times 10^6 \frac{(273.15 + 500)}{(273.15 + 20)} = 3.96 \times 10^6 \]

d) How many kilomoles can be withdrawn at 20 °C before the pressure decreases to 10%? 

\[ \frac{n - \Delta n}{n} = \frac{P'}{P} = .1 \]

\[ \Delta n = 0.9 \times n = 0.9 \times 0.308 = 0.277 \]

2-3  Ideal gas equation of state 

\[ P = \frac{RT}{v} = Av \]

The second equality is true only because of the process considered. 

a) The constant A can be evaluated at the beginning point 

\[ A = \frac{RT_1}{v_1^2} \]

b) The curve in the \( P - v \) plane is a straight line with slope A. 

c) What is the temperature if \( v = 2v_1 \), given \( T_1 = 200K \)? Since A is constant 

\[ T = T_1 \frac{v^2}{v_1^2} = 200 \times 4 = 800K \]

2-8  The Dieterici equation of state is 

\[ P = \frac{RT}{v - b} e^{-a/RTv} \]
a) Find the expansivity \( \beta \).

\[
\beta = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_P
\]

This is difficult since we are not given a simple relationship \( v = v(T, P) \). Instead use the cyclic relation

\[
\left( \frac{\partial v}{\partial T} \right)_p \left( \frac{\partial P}{\partial v} \right)_T \left( \frac{\partial T}{\partial P} \right)_v = -1
\]

to get

\[
\left( \frac{\partial v}{\partial T} \right)_p = - \left( \frac{\partial v}{\partial P} \right)_T \left( \frac{\partial P}{\partial T} \right)_v
\]

From the equation of state

\[
\left( \frac{\partial P}{\partial v} \right)_T = - \frac{P}{v-b} + \frac{a}{RTv^2} P, \quad \left( \frac{\partial P}{\partial T} \right)_v = \frac{P}{T} + \frac{a}{RT^2v} P
\]

Finally then

\[
\beta = -\frac{1}{v} \left( \frac{\partial v}{\partial P} \right)_T \left( \frac{\partial P}{\partial T} \right)_v = -\frac{1}{v} \frac{P}{T} + \frac{a}{RTv^2} P
\]

\[
= \frac{(v-b)}{Tv} \frac{1 + \frac{a}{RTv} (v-b)}{1 - \frac{a}{RTv} (v-b)}
\]

b) For large \( v \) and large \( T \) the result of part a) becomes

\[
\beta \rightarrow \frac{1}{T}
\]

which it the expected ideal gas result.

2-13 Assume that the pressure is constant and that \( \beta \) is only weakly dependent on the temperature (valid for the liquid phase). Then

\[
V = V_0 (1 + \beta (T - T_0))
\]

The amount that spills over is

\[
V - V_0 = V_0 \beta (T - T_0) = 250 \times 0.21 \times 10^{-3} (50 - 20) = 1.58 \text{ cm}^3
\]