6-2 Calculate the entropy change for the following processes

a) 1 kg of water is heated reversibly by an electric heating coil from 20 \(^{\circ}\)C to 80 \(^{\circ}\)C. Assume constant pressure

\[ \Delta S = \int_{T_i}^{T_f} \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{1}{T} n c_p dT = n c_p \ln \frac{T_f}{T_i} \]

\[ = 4.18 \times 10^3 \times \ln \frac{353.15}{293.15} \]

\[ = 778 \]

b) 1 kg of ice melts at 0 \(^{\circ}\)C and 1 atm.

\[ \Delta S = \frac{\Delta Q}{T} = \frac{\ell}{T} = \frac{3.34 \times 10^5}{273.15} = 1222 \]

c) 1 kg of steam condenses at 100 \(^{\circ}\)C and 1 atm.

\[ \Delta S = \frac{\Delta Q}{T} = \frac{\ell}{T} = \frac{2.26 \times 10^6}{373.15} = -6057 \]

6-7 Derive an expression for the entropy of an ideal gas

a) As a function of \(T\) and \(V\)

\[ T dS = dU + P dV = \left( \frac{\partial U}{\partial T} \right) V dT + \left( \frac{\partial U}{\partial V} \right) T dV + P dV \]

\[ = n c_V dT + P dV \]

\[ S = n c_V \int_T^T \frac{dT}{T} + n R \int_V^V \frac{dV}{V} = n c_V \ln T + n R \ln V + \text{constant} \]

b) As a function of \(T\) and \(P\)

\[ S = n c_V \ln T + n R \ln V + \text{constant} \]

\[ = n c_V \ln T + n R \ln \frac{n R T}{P} + \text{constant} \]

\[ = n c_p \ln T - n R \ln P + \text{constant} \]

6-11 Consider a van der Waals gas

a) Show that \(c_v\) is a function of \(T\) only. From (6.26)

\[ \left( \frac{\partial u}{\partial v} \right)_T = T \left( \frac{\partial P}{\partial T} \right)_v - P \]

\[ = T \left( \frac{\partial}{\partial T} \left( \frac{RT}{v - b} - \frac{a}{v^2} \right) \right) \]

\[ = \frac{a}{v^2} \]
Integrating this gives
\[ u = -\frac{a}{v} + f(T) \]
and so
\[ c_v = \left( \frac{\partial u}{\partial T} \right)_v = \frac{\partial f(T)}{\partial T}. \]

b) The specific internal energy is
\[
u = -\frac{a}{v} + f(T) = -\frac{a}{v} + \int_T^T \frac{\partial f(T)}{\partial T} dT + \text{const} \\
= -\frac{a}{v} + \int c_v dT + \text{const}
\]

c) The specific entropy is
\[
s = \int \frac{du}{T} + \int \frac{P}{T} dv = \int_T^T c_v \frac{dT}{T} + \int \left( \frac{1}{T} \left( \frac{\partial u}{\partial v} \right)_T + \frac{R}{v - b} \right) dv \\
= \int_T^T c_v \frac{dT}{T} + RT \ln (v - b) + \text{const}
\]