1. A donut-shaped space station (outer radius $R$) arranges for artificial gravity by spinning on the axis of the donut with angular velocity $\omega$. Sketch the forces on, and accelerations of, an astronaut standing in the station:

   a) as seen from an inertial frame outside the station, and

   b) as seen in the astronaut’s personal rest frame (which has a centripetal acceleration $A = \omega^2 R$ as seen in the inertial frame).

   c) What angular velocity is needed if $R = 40$ meters and the apparent gravity is to equal the usual value of about 10 m/s$^2$?

   d) What is the percentage difference between the perceived $g$ at a six-foot astronaut’s feet ($R = 40$ m) and at his head ($R = 38$ m)?

2. In a non-inertial frame on the surface of the Earth, the Centrifugal Force is perpendicular to the axis of rotation. Let $g_0$ be the purely gravitational force at the surface of the Earth and assume that the Earth is exactly spherical.

   a) Show that, at the Poles, $g_P = g_0$ and points toward the Earth’s center.

   b) Show that, at the Equator, $g_E = g_0 - \Omega^2 R_e$ and points toward the Earth’s center.

   c) Calculate the percentage difference, assuming $g_0 \equiv 9.8$ m/s$^2$ and $R_e \approx 6,400$ km.

   d) At an arbitrary point on the Earth’s surface, the net force on a stationary mass does not point directly toward the Earth’s center. Using an appropriate cartesian frame at the surface of the Earth, find the various components of the net force on a stationary mass due to gravitational and centrifugal effects.

   e) Show that the magnitude of the net force at an angle $\theta$ from the Pole can be written as $g_\theta = \sqrt{g_P^2 \cos^2 \theta + g_E^2 \sin^2 \theta}$.

3. The Coriolis Force only acts on masses when they are in motion. Consider a mass of air which has initially been set in motion by a pressure difference.
a) Suppose the air-mass is no longer subject to a horizontal pressure force, and that it remains at a fixed distance above the surface of the earth. Write down the equations governing its motion.

b) Solve these (coupled) equations for generic initial data.

c) If the air-mass is originally moving East at latitude 45° N, show that in less than 4.25 hours the airmass will be moving in a southerly direction.

d) Is the effect of the Coriolis force greater at the Equator or at the Poles? Give an explanation for your answer.

4. In class we solved the Foucault Pendulum problem by assuming the the oscillatory period of the pendulum was much less that a day. Here we will examine some effects of relaxing that assumption. First, let \( \omega_0 \) be the natural frequency of the pendulum, and let \( \omega_1 = \sqrt{\omega_0^2 + \Omega_z^2} \).

a) Write down the equations governing a Foucault pendulum with small angular oscillations. Identify \( \Omega_z \) in terms of \( \Omega \) and the angle \( \theta \) from the Pole.

b) Write down the generic solution in terms of arbitrary constants of integration.

c) Solve for the integration constants for the following initial data: \( x(0) = x_0, y(0) = 0, \dot{x}(0) = 0 \) and \( \dot{y}(0) = 0 \).

d) Show that your answer can be written as:

\[
x(t) + iy(t) = x_0 \left( \cos \omega_1 t + \frac{i\Omega_z}{\omega_1} \sin \omega_1 t \right) e^{-i\Omega_z t}.
\]

e) The result in d) differs in two ways from the solution discussed in class: i) \( \omega_1 \neq \omega_0 \), and ii) by the presence of the second term in the brackets. Explain the relative significance of these two differences.

5. The theoretical calculation for the height of the tides depends on considering the surface of the sea as being an equipotential.

a) Write down an expression for the potential due to the Earth's gravity and the lunar tidal force.

b) For a point on the surface at an arbitrary angle \( \theta \) from the \( y \)-axis, write the potential to quadratic order in the small quantity \( R_e/d_0 \), fully including its \( \theta \) dependence.
c) At an angle $\theta_0 = \arcsin 1/\sqrt{3}$, the height of the tide can be presumed to be zero. Calculate the height of the tide on the $x$-axis relative to the unchanged equipotential at $\theta_0$, i.e., with $h_0 = 0$.

d) By how much does the tide on the $y$-axis fall relative to the equipotential at $\theta_0$, with $h_0 = 0$?

e) Check your results by showing that the difference between c) and d) gives exactly the known height difference of the lunar tides.