Due at the start of class on Friday, April 3. Half credit will be available for late homework submitted no later than the start of class on Monday, April 6.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. Here are several questions about using cross sections.

   a) A beam of particles is directed through a tank of liquid hydrogen. The tank’s length is 50 cm and the liquid density is 0.07 gram/cm$^3$. What is the target density (number/area) of hydrogen atoms seen by the incident particles?

   b) The cross section for scattering a certain nuclear particle by a nitrogen nucleus is 0.5 barns. If $10^{11}$ of these particles are fired through a cloud chamber of length 10 cm, containing nitrogen at 0°C and a pressure of $1.013 \times 10^5$ Pa (1 atm.), how many particles are scattered? (Use the ideal gas law. Each nitrogen molecule has two atoms. Scattering by atomic electrons is negligible.)

   c) Calculate the solid angles subtended by the moon and by the sun, both as seen from the earth. Comment on your answers. (The radii of the moon and sun are $R_m = 1.74 \times 10^6$m and $R_s = 6.96 \times 10^8$m. Their distances from the Earth are $d_m = 3.84 \times 10^8$m and $d_s = 1.50 \times 10^{11}$m.)

   d) The differential cross section for scattering 6.5-MeV alpha particles at 120° off a silver nucleus is about 0.5 barns/sr. If $10^{10}$ alphas impinge on a silver foil of thickness $1 \mu$m and if we detect the scattered particles using a counter of area 0.1 mm$^2$ at 120° and 1 cm from the target, about how many scattered alphas should we expect to count? (Silver has a specific gravity of 10.5, and atomic mass of 108.)

2. Consider a point projectile moving in a fixed, spherical force whose potential energy is

   \[ U(r) = \begin{cases} -U_0 & (0 < r < R) \\ 0 & (R < r) \end{cases} \]

   where $U_0$ is a constant. This spherical well represents a projectile which moves freely in each of the regions $r < R$ and $R < r$, but receives a radially inward impulse, when it crosses the boundary $r = R$, that changes its kinetic energy by $+U_0$ going inward, $-U_0$ going outward.

   a) Sketch the orbit of a projectile that approaches the well with momentum $p_0$ and impact parameter $b < R$. 
b) From energy conservation, find the particle's momentum $p$ inside the well ($r < R$).

c) Let $\zeta$ denote the momentum ratio $p_0/p$, and $d$ the projectile's distance of closest approach to the origin. From angular momentum conservation, show that $d = \zeta b$.

d) It can be shown that the scattering angle \( \theta \) is given as a function of \( b \) by:

\[
\theta = 2 \left( \arcsin \frac{b}{R} - \arcsin \frac{\zeta b}{R} \right).
\]

By differentiating \( \theta \) with respect to \( b \), find an expression for the differential cross section as a function of \( b \). What is the maximum angle of scattering?

e) By integrating \( d\sigma/d\Omega \) over all directions, find the total cross section.

3. The general appearance of the scattering orbit in a spherically symmetric potential $U(r)$ is shown in the figure. It is symmetric about the direction $u$ of closest approach. If $\psi$ is the projectile’s polar angle, measured from the direction $u$, then $\psi \Rightarrow \pm \psi_0$ as $t \Rightarrow \pm \infty$ and the scattering angle is $\theta = \pi - 2\psi_0$, where $\psi_0$ is equal to $\int \dot{\psi} dt$ taken from the time of closest approach to $\infty$.

Using the now-familiar trick, one can rewrite this integral as $\int (\dot{\psi}/\dot{r})dr$. Next $\dot{\psi}$ can be rewritten in terms of the angular momentum $l$ and $r$, and $\dot{r}$ can be rewrite in terms of the energy $E$ and the effective potential $U_{\text{eff}}$. Hence, it can be proved that:

\[
\theta = \pi - 2 \int_{r_{\text{min}}}^{\infty} \frac{(b/r^2)dr}{\sqrt{1 - (b/r)^2 - U(r)/E}}.
\]

Provided this integral can be evaluated, it gives $\theta$ in terms of $b$, and thus allows the differential cross section $d\sigma/d\Omega$ to be evaluated.

a) A particle with energy $E$ is scattered by a fixed, repulsive $1/r^3$ force field, with potential energy $U = \gamma/r^2$. Use the relation above to find $\theta$ in terms of $b$. [Hint: You will need to find $r_{\text{min}}$ for the case $E > 0$ in order to evaluate the integral.]

b) Solve your equation for $\theta$ in terms of $b$ to get $b$ in terms of $\theta$.

c) Hence show that the differential cross section is:

\[
\frac{d\sigma}{d\Omega} = \frac{\gamma}{E} \frac{\pi^2(\pi - \theta)}{\theta^2(2\pi - \theta)^2 \sin \theta}.
\]

d) Compare the singularity of this as $\theta \Rightarrow 0$ with the corresponding singularity in the Coulomb case (i.e., for Rutherford scattering).
e) Compare the dependence on $E$ with that in the Coulomb case.

4. This question concerns the Lorentz transformation from a stationary frame to a frame moving with velocity $V > 0$ in the $x$-direction.

   a) Consider two events located at $x_1, t_1$ and $x_2, t_2$ in the stationary frame. What are the coordinates of each event if the origin of the moving frame coincides with the $x, t$ origin.

   b) What is the invariant squared separation between the two events? Show that it is the same in both the moving and unmoving frames.

   c) Suppose that the invariant squared separation is positive. In a third frame moving so that both events occur at the same time, what are the coordinates of the two events, assuming its origin coincides with the other two.

   d) Suppose that, for a third event, $x_3, t_3$, the invariant squared separation from $x_1, t_1$ is negative. There is then a fourth frame in which both events occur at the same place. What are the coordinates of the two events in that frame, assuming its origin also coincides with the others?

5. This question involves working with space-time diagrams. Suppose a stick is moving along the $x$ axis with speed $V > 0$. Let $l$ be the length of the stick in its own rest frame.

   a) If the observer is hit by the front end of the stick at the $x, t$ origin, what are the coordinates of the other end of the stick, as seen in the observers rest frame at that time?

   b) At the instant of initial impact, what are the coordinates of the far end of the stick in its own rest frame? What are the $x, t$ coordinates which correspond to that?

   c) At the instant of initial impact, the observer sees the far end of the stick against a grid in his frame. What grid mark does he see against the far end of the stick and at what time in his frame did the light he sees leave the far end of the stick?

   d) At the instant of impact in the unmoving frame, a luckier observer is standing by the far end of the stick. As he looks up, he sees light coming from the front end of the stick. At what time and place in the unmoving frame did the light which he sees leave the front of the stick?