Gram–Schmidt orthogonalization:

Let $|f_1>, |f_2>, |f_3>, ...$ be linearly independent, but not necessarily orthogonal.
We can use this procedure to get a new set of vectors, $|g_1>, |g_2>, ...$, which are orthogonal.

$|g_1> = |f_1>$

$|g_2> = |f_2> - |g_1><g_1|f_2> / <g_1|g_1>$

$\rightarrow <g_1|g_2> = 0$

$|g_3> = |f_3> - |g_1><g_1|f_3> - |g_2><g_2|f_3> / <g_1|g_1> / <g_2|g_2>$

$\rightarrow 0 = <g_1|g_3> = <g_2|g_3>$

$\vdots$