Quiz 2

An infinite square well is located between $0 < x < a$.

1. At $t = 0$ the wave function satisfies $\psi(x, 0) = 0$ for $0 < x < a/2$ and $\psi(x, 0) = C$ for $a/2 < x < a$. What is the normalization constant $C$?

$$1 = \int_{0}^{a} |\psi(x, 0)|^2 \, dx = \int_{0}^{a/2} |C|^2 \, dx = \frac{a}{2} |C|^2$$

$$\rightarrow \quad C = \sqrt{\frac{2}{a}}$$

2. Determine the $c_n$ needed to express $\psi(x, 0)$ in terms of the eigenstates of the infinite square well:

$$\psi(x, 0) = \sum_{n=1}^{\infty} c_n \frac{\sqrt{2}}{\sqrt{a}} \sin(n \pi x / a).$$

Set $a = 1$,

$$c_n = \int_{0}^{1} \sqrt{2} \sin(n \pi x) \psi(x, 0) \, dx$$

$$= \int_{0}^{1/2} 2 \sqrt{2} \sin(n \pi x) \, dx$$

$$= \frac{2}{n \pi} \cos(n \pi x) \bigg|_{1/2}^{1}$$

$$c_n = \frac{2}{n \pi} \left( \cos(n \pi / 2) - \cos(n \pi) \right)$$

3. Based on your results for part 2, write down an expression for $\psi(x, t)$.

$$\psi(x, t) = \sum_{n=1}^{\infty} \frac{2}{n \pi} \left( \cos(n \pi / 2) - \cos(n \pi) \right) \sqrt{2} \sin(n \pi x)$$

$$\times e^{-i \frac{\hbar}{2m} \left( \frac{n \pi}{a} \right)^2 t}$$

$a = 1$