Quiz 6

Three Hermitian operators are defined below in a two dimensional space, and their eigenvalues and eigenvectors are given.

\[
S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ with } +\frac{\hbar}{2} : \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } -\frac{\hbar}{2} : \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\
S_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ with } +\frac{\hbar}{2} : \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \text{ and } -\frac{\hbar}{2} : \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \\
S_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ with } +\frac{\hbar}{2} : \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } -\frac{\hbar}{2} : \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

Initially the system is in the state

\[
|\psi\rangle = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}
\]

1. A measurement of \(S_x\) is made. What are the possible outcomes and the probabilities of those outcomes?

\[
+\hbar/2 \quad \text{with prob. } = \left| \frac{1}{\sqrt{2}} (1, 1) \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} \right|^2 = \frac{1.44}{2} = 0.92
\]

\[
-\hbar/2 \quad \text{with prob. } = \left| \frac{1}{\sqrt{2}} (-1, 1) \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} \right|^2 = \frac{0.25}{2} = 0.02
\]

2. The result of the \(S_x\) measurement is \(-\hbar/2\). What is the state of the system immediately after this measurement?

\[
\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}
\]

3. Assuming the system is in the state of part 2, a \(S_y\) measurement is made. What are the possible outcomes and the probabilities of those outcomes?

\[
+\hbar/2 \quad \text{with prob. } = \left| \frac{1}{\sqrt{2}} (1, -i) \frac{1}{\sqrt{2}} (1) \right|^2 = \frac{1 + i \cdot i}{4} = \frac{1}{4}
\]

\[
-\hbar/2 \quad \text{with prob. } = \left| \frac{1}{\sqrt{2}} (1, i) \frac{1}{\sqrt{2}} (-1) \right|^2 = \frac{1 - i \cdot i}{4} = \frac{1}{4}
\]