We control matter because we control the mind. Reality is inside the skull. You will learn by degrees, Winston. There is nothing that we could not do. Invisibility, levitation — anything. I could float off this floor like a soap bubble if I wished to. I do not wish to, because the Party does not wish it. You must get rid of those nineteenth-century ideas about the laws of nature. We make the laws of nature.

After all the progress in M-theory over the past year only two things remain to be done:
1. Find the Model; and
2. Solve it exactly.

Witten (1996)

(1) A particle of mass $m$ is initially in the ground state of the (one-dimensional) infinite square well. At time $t = 0$ a “brick” is dropped into the well, so that the potential becomes

$$V(x) = \begin{cases} 
  V_0, & \text{if } 0 \leq x \leq a/2, \\
  0, & \text{if } a/2 < x \leq a, \\
  \infty, & \text{otherwise,}
\end{cases}$$

where $V_0 \ll E_1$. After a time $T$, the brick is removed, and the energy of the particle is measured. Find the probability (in first-order perturbation theory) that the result is now $E_2$. (30 points)

(2) Suppose the one-dimensional harmonic oscillator (mass $m$, frequency $\omega$) is subjected to a driving force of the form $F(t) = m\omega^2 f(t)$, where $f(t)$ is some specified function which is zero for all times on or before $t = 0$.

a) What is the instantaneous Hamiltonian $H(t)$? (10 points)

b) What are the instantaneous eigenstates $\psi_n(t, x)$ and energies $E_n(t)$? (10 points)

c) What is the adiabatic approximation for the state $\Psi(t, x)$ which begins in the $n$-th excited state, $\Psi(0, x) = \psi_n(x)$? (10 points)

(3) Use the Born approximation to determine the total cross-section for scattering from a gaussian potential

$$V(\vec{r}) = Ae^{-\mu r^2}.$$  

Express your answer in terms of the constants $A$, $\mu$ and $m$ (the mass of the incident particle), and $k \equiv \sqrt{2mE}/\hbar$, where $E$ is the incident energy. (30 points)