In some sort of crude sense which no vulgarity, no humour, no overstatement can quite extinguish, the physicists have known sin; and this is a knowledge which they cannot lose.

J. Robert Oppenheimer

(1) The wave function \( \psi(x) = A \exp\left[-\frac{b^2 x^2}{2}\right] \), where \( A \) and \( b \) are real constants, is a normalized eigenfunction of the Schrödinger equation for a particle of mass \( M \) and energy \( E \) in a one dimensional potential \( V(x) \) such that \( V(x) = 0 \) at \( x = 0 \). Which of the following is correct? **(20 points)**

(A) \( V = \frac{\hbar^2 b^4}{2M} \)

(B) \( V = \frac{\hbar^2 b^4 x^2}{2M} \)

(C) \( V = \frac{\hbar^2 b^6 x^4}{2M} \)

(D) \( E = \hbar^2 b^2 (1 - b^2 x^2) \)

(E) \( E = \frac{\hbar^2 b^4}{2M} \)

(2) In perturbation theory, what is the first order correction to the energy of a hydrogen atom (Bohr radius \( a_0 \)) in its ground state due to the presence of a static electric field \( E \)? **(20 points)**

(A) Zero

(B) \( eEa_0 \)

(C) \( 3eEa_0 \)

(D) \( \frac{8\varepsilon_2 E a_0^3}{3} \)

(E) \( \frac{8\varepsilon_2 E a_0^3}{3} \)

(3) Two ions, 1 and 2, at fixed separation, with spin and angular momentum operators \( \vec{S}_1 \) and \( \vec{S}_2 \), have the interaction Hamiltonian \( H = -J \vec{S}_1 \cdot \vec{S}_2 \), where \( J > 0 \). The values of \( ||\vec{S}_1||^2 \) and \( ||\vec{S}_2||^2 \) are fixed at \( S_1(S_1 + 1) \) and \( S_2(S_2 + 1) \), respectively. Which of the following is the energy of the ground state of the system? **(25 points)**

(A) 0

(B) \( -JS_1S_2 \)

(C) \( -J[S_1(S_1 + 1) - S_2(S_2 + 1)] \)

(D) \( -(J/2)[(S_1 + S_2)(S_1 + S_2 + 1) - S_1(S_1 + 1) - S_2(S_2 + 1)] \)

(E) \( -J \left[ \frac{S_1(S_1+1)+S_2(S_2+1)}{(S_1+S_2)(S_1+S_2+1)} \right] \)