Exam #2

άγεωμέτρητος μηδείς εισί τώ

Motto of Plato’s Academy

(1) Consider a spin 1/2 particle with gyromagnetic ratio $\gamma$ which interacts with a magnetic field,

$$\vec{B} = B_{tf} \cos(\omega t) \hat{x} - B_{tf} \sin(\omega t) \hat{y} + B_0 \hat{z}.$$ 

Recall that this system is described by a 2-component vector (top element gives the amplitude for spin up, bottom element gives the amplitude for spin down) upon which the Hamiltonian acts as the $2 \times 2$ matrix,

$$H = \frac{-\hbar}{2} \gamma B_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{\hbar}{2} \gamma B_{tf} \begin{pmatrix} 0 & e^{i\omega t} \\ e^{-i\omega t} & 0 \end{pmatrix}.$$ 

You should consider the first term as the unperturbed Hamiltonian and the second as the perturbation. Suppose the system is released at $t = 0$ with spin up.

a) What is the amplitude for the system to have spin down at time $t$ using first order perturbation theory? (20 points)

b) What is the exact amplitude for spin down at time $t$? (15 points)

(2) Suppose the one-dimensional harmonic oscillator (mass $m$, frequency $\omega$) is subjected to a driving force of the form $F(t) = m\omega^2 f(t)$, where $f(t)$ is some specified function which is zero for all times on or before $t = 0$.

a) What is the instantaneous Hamiltonian $H(t)$? (10 points)

b) What are the instantaneous eigenstates $\psi_n(t, x)$ and energies $E_n(t)$? (10 points)

c) What is the adiabatic approximation for the state $\Psi(t, x)$ which begins in the $n$-th excited state, $\Psi(0, x) = \psi_n(x)$? (15 points)

(3) Consider scattering from the soft-sphere potential

$$V(\vec{r}) = \begin{cases} V_0, & \text{if } r \leq a, \\ 0, & \text{if } r > a. \end{cases}$$

a) What is the scattering amplitude $f(\theta, \phi)$ in the Born approximation? (15 points)

b) What is the Born approximation scattering amplitude for $\theta = 0$? (10 points)

c) Write down, but do not attempt to evaluate, the total cross section $\sigma$ in the Born approximation. (10 points)