1. Show that

\[ e^\Omega = \lim_{n \to \infty} \left( 1 + \frac{1}{n} \Omega \right)^n \]  

where \( \Omega \) is a linear operator.

2. Show that if \( U \) is a linear operator on a vector space \( V \) and if for all vectors \( V \in V \)

\[ <V'|V> = <V|V> \]  

where \( |V'> = U|V> \), then \( U \) must be a unitary operator on \( V \).

3. If \( X \) and \( P \) are canonically conjugate observables, we have \( X = X^\dagger \), \( P = P^\dagger \) and \( [X, P] = \hbar i \). Using these equations, show that

a. \( [X, F(P)] = \hbar i \frac{dF}{dP} \), where \( F(P) \) is any function of the operator \( P \).

b. \( <x|P^n|\Psi> = \left( \frac{1}{i} \right)^n \frac{d^n}{dx^n} <x|\Psi> \) where the \( |x> \) are the eigenstates of \( X \).

c. \( <x|p> = Ne^{ixp/\hbar} \) where the \( |p> \) are the eigenstates of \( P \) and \( N \) is a normalization constant.