8.1: Summary of strong scattering model

\[ \sigma_{xx} = \sigma_{xx}^{ss} + \sigma_{ss}^{s,s} \approx \sum \frac{1}{2} \gamma_0^2 N_r \tilde{\gamma} \]

\[ \sigma_{xy} = \sigma_{xy}^{ss} + \sigma_{xy}^{s,s} = \frac{\pi \lambda_0^1}{1 - \lambda_1^2} + \ln(\frac{1}{1 - \lambda_1^2}) \]

Quantum corrections, only ss important

\[ \Delta \sigma_{xx} = \ln(T) \]

\[ \Delta \sigma_{xy} = \ln(T) \]

8.2: UF exps on Fe-film; \( R_0 < 3 \mu \Omega \)

\[ \Delta^w \sigma_{xx} = \frac{\sigma_{xx}^{ss} + \sigma_{ss}^{s,w} + \sigma_{ss,w,l}}{L_0} \]

\[ \Delta^w \sigma_{xy} = \frac{\sigma_{xy}^{ss} + \sigma_{xy}^{s,w} + \sigma_{xy,w,l}}{L_0} \]

\[ \Delta \sigma_{xx} = \frac{\sigma_{xx}^{ss} + \sigma_{xy}^{s,w} + \sigma_{xy,w,l}}{L_0} + \ln(T) \]

\[ \Delta \sigma_{xy} = \frac{\sigma_{xy}^{ss} + \sigma_{xy}^{s,w} + \sigma_{xy,w,l}}{L_0} + \ln(T) \]

Where \( R_\gamma = \frac{\sigma_{xy}^{s,w}}{\sigma_{xy}^{ss}} \rightarrow \) a non universal quantity.

Compare exp:

\[ \Delta^w \sigma_{xx} = A_R \frac{\ln(T)}{T_0} \]

\[ \Delta^w \sigma_{xy} = (2A_R - A_{AH}) \frac{\ln(T)}{T_0} \]

\[ A_R = 1 + h_{xx} \]

Glass substrate: \( h_{xx} \rightarrow 0 \), \( R_{xy} \rightarrow 0 \)

Sapphire/By: \( h_{xx} \rightarrow 0 \), \( R_{xy} \rightarrow 1 \)

Note: Contradicts by explanation that \( \mu \) is interaction correction as opposed to WL correction.
8.3: Fe-film: $R_0 \gg 3k_B$

Experimental observation: film becomes granular

- $R_{xx}$ dominated by tunneling: $R_{xx} \approx R_{xx}^T$
- No current along $y = 0$ grains coupled as capacitors

$$= R_{xy} \approx R_{xy}^9$$ (of a grain independent of $R_{xx}^T$

$$\begin{pmatrix} R_{xx}^T & R_{xy}^9 \\ R_{xy}^9 & R_{xx} \end{pmatrix} \approx \begin{pmatrix} \delta R_{xx}^9 & \delta R_{xy}^9 \\ \delta R_{xy}^9 & \delta R_{xx} \end{pmatrix} \approx \begin{pmatrix} \delta R_{xy}^9 \\ \delta R_{xx} \end{pmatrix}$$

$$\approx \text{independent of } R_{xx}^T \times R_0$$

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![Graph](image)

FIG. 4 (color online). $\ln(T)$ dependence of relative changes in the AH resistance for type A films with three different $R_0$.

Prob: Show that in this granular model

$$\frac{\delta R_{xy}^9}{R_{xy}^9} = \frac{\delta R_{xy}^9}{R_{xy}^9} + 2 \frac{\delta R_{xx}^T}{R_{xx}^T} - 2 \frac{\delta R_{xx}^9}{R_{xx}^9}$$

Note: AHE calculations remain valid within single grains.

$$= 0 \quad A_R^9 = 1 \quad A_{AH}^9 = 1$$

Using

$$\frac{\delta R_{xy}^T}{R_{xx}^T} = \frac{\delta R_{xy}^T}{R_{xx}^T} A_R \ln(T/T_H)$$

this gives
\[ \Delta_n (E_{\text{w}}) = (2A_R - \frac{R_o^3}{R_T^2}) \propto T/T_0 \]

Consistent with expt.

Also, \( A_R \) involves only tunneling resistances, while \( A_{\text{HH}} \sim \frac{R_o^3}{R_T^2} \ll 1 \), so expect \( A_{\text{HH}} < A_R \), again consistent with expt.

8.4: Phase relaxation rates, in Fe and spin waves:

In order to have WL corrections in Fe, we need large \( \frac{1}{T_\Phi} \) s.t.

\[ \max \left( \frac{1}{T_\Phi}, \frac{1}{T_5}, \frac{1}{T_0} \right) \ll \frac{1}{T_\Phi} \ll \frac{1}{T_5} \]

where \( T_0 = \frac{eB_{\text{Int}}}{m^*c}, B_{\text{Int}} \) in the internal field

\[ T_5 = \frac{eB_{\text{Int}}}{m^*c} \] in Tesla

It turns out that \( \frac{1}{T_\Phi} = \frac{T}{E_p T_K} \), but \( \frac{E_p T_K}{2} \) is too small, and hit from WL should not be observable.

However, a much longer contribution comes from spin waves:

\[ \frac{1}{T_\Phi} \approx \frac{J}{E_p d g} T, \text{ for } J = 160 \text{ K, exchange energy of s-electrons} \]

\[ \Delta_{\text{ gw}} \approx 1k \left( \frac{m^*}{m} \right) B_{\text{Int}} : \text{spin wave gap} \]

For \( 150m < R_0 < 3 \text{ k} \), \( \epsilon_p T_K < 1 \)

\[ \text{in } \omega_{\text{ph}} \epsilon_p < 1 \] can be satisfied down to 5 K.
9.1: UF Expts on Gd

In order to understand the linear T-dependence
of Gd, we consider the spin-wave contributions, because
Gd has a much smaller Curie temp. ($293$K) compared
to Fe ($\sim 1000$K) and also a small spin-wave gap ($\Delta \sim 30$meV.

9.2: Model Hamiltonian

$$H = \sum_i (\varepsilon_i - \frac{1}{2} B) \hat{c}_i^+ \hat{c}_i + \sum_{i,j} V_{ij} \hat{c}_i^{+} \hat{c}_j + \sum_{\mathbf{q}} \omega_{\mathbf{q}} a_{\mathbf{q}}^{+} a_{\mathbf{q}} + \sum_{q,k} J \sum_{i \neq k} \left[ a_{\mathbf{q}}^{+} c_{\mathbf{k}} + c_{\mathbf{k}}^{+} a_{\mathbf{q}} + h.c. \right]$$

where $c_i, c_i^+$ are electron field operators and $a_\mathbf{q}, a_\mathbf{q}^+$
are the spin-wave operators. $J$ is the effective spin-exchange interaction, and the spin-wave is characterized
by $\omega_\mathbf{q} = \Delta_g + \Delta_g^2$, where $\Delta_g = \mu_B B_{ext} \approx 1$K/Tesla is the
spin-wave gap and $\Lambda = J/k_f^2$ is the spin stiffness.
$B \approx J k_f^2$ is the exchange splitting.

Note 1: $\Delta_g < T \quad \Rightarrow$ we can set $\Delta_g = 0$

Note 2: $B = 750$ meV at 20K and $\epsilon_F = 3.4$ eV

$B \gg 1 \quad \Rightarrow \quad \frac{B}{\epsilon_F} < 1$

9.3: Spin-wave propagator

$$S_{pq}(q, \omega_n) = \frac{1}{i \omega_n - \alpha \omega_q} = [S_{pq}]^*$$
Spin \( p-h \) propagator:

\[
\Delta^\text{\( p-h \)} = \frac{1}{k+k'} + \frac{1}{k-k'} \sum_k G_{k+} (\xi_k) G_{k-} (\xi_{k'}+\omega_m) \Gamma_{\text{\( p-h \)}}
\]

Define \( \sum_k G_{k+} (\xi_k) G_{k+} (\xi_{k'}+\omega_m) = X_{\text{\( p-h \)}}(\xi, \xi_{k'}+\omega_m) \)

\[
\Gamma (1-\frac{\chi}{2\pi N\omega_\tau}) = \frac{1}{2\pi N\omega_\tau}
\]

\[
\Rightarrow \Gamma = \frac{\chi}{2\pi N\omega_\tau} \quad \text{and} \quad 1+\Gamma \chi = \frac{1}{2\pi N\omega_\tau} \Gamma
\]

It is easy to show that for \( q=0 \), \( X_{\text{\( p-h \)}}^+ \) is given by

\[
\sum_{\xi_k} G_{k+} (\xi_k+\omega_m) G_{k+} (\xi_k) = \frac{2\pi N\omega_\tau}{\chi}
\]

where

\[
\frac{\chi}{\tau} \equiv \frac{\tau}{2} + \omega_m - iB
\]

where we have used \( \xi_k^+ = \xi_k - \frac{B}{2}, \quad \xi_k^- = \xi_k + \frac{B}{2} \)

Given the result for \( q=0 \), we expand for small \( q \):

\[
X_{\text{\( p-h \)}}^+ = \sum_{\xi_k} G_{k+} (\xi_k+\omega_m) \left[ 1 + \frac{(\xi_k \cdot v_p)}{\xi_k^+} G_{k+}^+ + \frac{(\xi_k \cdot v_p)}{\xi_k} G_{k+}^2 + \cdots \right] G_{k+} (\xi_k)
\]

\[
= 2\pi N\omega_\tau \left( 1 - \hat{D} q^2 \right)
\]

where \( \hat{D} \equiv \frac{1}{2} v_p \frac{\xi^+}{\tau} = D \left( \frac{\xi}{\tau} \right)^2 \)

This leads to the \( p-h \) propagator:

\[
\Gamma_{\text{\( p-h \)}} (\xi, \omega_m) = \frac{1}{2\pi N\omega_\tau} \frac{1}{\omega_m - iB + \hat{D} q^2}
\]
Similarly:

\[ \mathcal{E}^{\mathbf{k},\mathbf{k}'}(2\Omega) = \frac{1}{2\pi N_0} \frac{1}{\frac{i}{2}} \frac{1}{\omega_n + iB + \hat{\mathbf{q}}^2} , \quad \text{where} \]

\[ \hat{\mathbf{q}}^2 = 2v_F^2 \frac{\tau^2}{2}, \quad \frac{1}{\tau} = \frac{1}{2} + \omega_m + iB \]

\[ q_F = \frac{1}{v_F} \]

\[ \omega_c = \frac{1}{\tau} \]

9.4: Connection to conductivity

\[ \delta \mathcal{E}^{\mathbf{k},\mathbf{k}'}(2\Omega) = \frac{1}{2\pi N_0} \frac{1}{\omega_m + iB + \hat{\mathbf{q}}^2} \]