Effects of Inhomogeneities on Cosmic Expansion

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Outline:

• Motivation – explain accelerated expansion

• Examine gravitational inhomogeneities
  • Sub-horizon scales
  • Super-horizon scales

• What is their impact on cosmological expansion?

• *Can they explain away dark energy?*
What's wrong with this picture?
Why is the universe's expansion rate accelerating?

- Cosmological Constant ($\Lambda$)
- Dark Energy / Quintessence
- Modified Friedmann Equations (Cardassian)
- Gravitational Inhomogeneities???
\( \Omega_m = 1 \) Without Gravitational Inhomogeneities

- From an isotropic and homogeneous universe

\[
ds^2 = a^2(\tau)(-d\tau^2 + dx^2) \]

- Obtain the standard Friedmann Equation from \( G^0_0 \)

\[
3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi G \rho
\]
Gravitational Inhomogeneities change everything!

- From an inhomogeneous universe:

\[
s^2 = a^2(\tau)\left[-(1 + 2\psi)d\tau^2 + (1 - 2\phi)dx^2\right]
\]

- Obtain a slightly more complicated equation for \( G^0_0 \)

\[
3\left(\frac{\dot{a}}{a}\right)^2(1 - 2\psi) + (2 + 6\phi)\frac{1}{a^2}\nabla^2\phi + \frac{1}{a^2}(\nabla\phi)^2 = 8\pi G\bar{\rho}(1 + \delta)(1 + v^2)
\]
Wind up with a new equation governing cosmological expansion:

\[ 3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi G \bar{\rho} \left( 1 - 5 W + 2 K \right) \]

\[ W = \frac{1}{2} \left\langle (1 + \delta) \phi \right\rangle \quad K = \frac{1}{2} \left\langle (1 + \delta) v^2 \right\rangle \]

Valid for fully nonlinear density contrasts, \( \delta \equiv \delta \rho / \rho \gg 1 \).
The Gravitational Potential Energy $W$

- Binding energy in the (Newtonian) potential $\phi$
- Can compute from the power spectrum, $P(k)$

\[
W = -\frac{1}{2} 4\pi G \bar{\rho} a^2 \int \frac{d^3k}{(2\pi)^3} \frac{P(k)}{k^2} = - \int \frac{dk}{k} \Delta^2_W(k)
\]

- Defines the dimensionless spectral density, $\Delta^2_W(k)$
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Spectral Density vs. Wavenumber
What do we learn from $W$?

- $W$ contributes most on scales of $\mathcal{O}(1 \text{ Mpc})$
- $\Delta^2_W(k)$ falls off as $\sim k^2$ for small and large scales
  - True for both linear and nonlinear power spectra
- *We also learn about $K$!*
How to find the Kinetic Energy?

- **K** is related to **W** through the *Cosmic Energy Equation*:

\[
\left( \frac{d}{dt} + \frac{2\dot{a}}{a} \right) K = - \left( \frac{d}{dt} + \frac{\dot{a}}{a} \right) W
\]

- With **K** and **W**, can now compute the contribution to Cosmological Expansion, from

\[
3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi G \bar{\rho} (1 - 5W + 2K)
\]
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Contribution for linear $\delta$
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Contribution for second-order $\delta$
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Contribution for fully nonlinear $\delta$
Net Inhomogeneity Effects

- Grow as curvature in the linear regime
- Behave as dust in the deeply nonlinear regime
- Never induce an accelerated expansion
- Never contribute significantly to cosmic expansion
Super-Horizon Scale Fluctuations

- Contributions from $W$ are small, but what about the variance of $W$, $\Delta W$?

$$\langle (\Delta W)^2 \rangle = \frac{1}{V^2} \int d^3 x \, d^3 x' \, \frac{1}{4} \langle \phi(x) \phi(x') \rangle$$

$$= (2\pi G \bar{\rho} a^2)^2 \int \frac{d^3 k}{(2\pi)^3} \frac{P(k)}{k^4} W^2(kR)$$

- Has a divergent contribution as $k \to 0$
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$\Delta W$ vs. large-scale cutoff
Conclusions:

- Sub-horizon inhomogeneities have a negligibly small effect on cosmic expansion.
- Super-horizon inhomogeneities can have a large effect if you extrapolate far enough (~ $10^{10^9}$ decades).
  - still behave as curvature only, not dark energy.
- Inhomogeneities should not be responsible for the accelerating expansion of the universe.