Surgical teams go to extraordinary lengths to avoid bacterial infection of a patient during surgery. Masks are donned, hands are meticulously cleaned and then gloved, and instruments are sanitized at high temperature and in alcohol baths. Recently a subtle source of bacteria was discovered in operating rooms, one that had been overlooked for years. It can be seen in this photograph.

How can modern electronics be the cause of bacterial contamination?

The answer is in this chapter.
You are surrounded by devices that depend on the physics of electromagnetism, which is the combination of electric and magnetic phenomena. This physics is at the root of computers, television, radar, telecommunications, household lighting, and even the ability of food wrap to cling to a container. This physics is also the basis of the natural world. Not only does it hold together all the atoms and molecules in the world, it also produces lightning, auroras, and rainbows.

The physics of electromagnetism was first studied by the early Greek philosophers, who discovered that if a piece of amber is rubbed and then brought near bits of straw, the straw will jump to the amber. We now know that the attraction between amber and straw is due to an electric force. The Greek philosophers also discovered that if a certain type of stone (a naturally occurring magnet) is brought near bits of iron, the iron will jump to the stone. We now know that the attraction between magnet and iron is due to a magnetic force.

From these modest origins with the Greek philosophers, the sciences of electricity and magnetism developed separately for centuries—until 1820, in fact, when Hans Christian Oersted found a connection between them: an electric current in a wire can affect a magnetic compass needle. Interestingly enough, Oersted made the discovery, a big surprise, while preparing a lecture demonstration for his physics students.

The new science of electromagnetism was developed further by workers in many countries. One of the best was Michael Faraday, a truly gifted experimenter with a talent for physical intuition and visualization. That talent is attested to by the fact that his collected laboratory notebooks do not contain a single equation. In the mid-nineteenth century, James Clerk Maxwell put Faraday's ideas into mathematical form, introduced many new ideas of his own, and put electromagnetism on a sound theoretical basis.

Our discussion of electromagnetism is spread through the next 16 chapters. We begin with electrical phenomena, and our first step is to discuss the nature of electric charge and electric force.

### 21-2 Electric Charge

In dry weather, you can produce a spark by walking across certain types of carpet and then bringing one of your fingers near a metal doorknob, metal faucet, or even a thread. You can also produce multiple sparks when you pull, say, a sweater from your back or clothes from a dryer. Sparks and the "static cling" of clothing (similar to what is seen in Fig. 21-1) are usually just annoying. However, if you happen to pull off a sweater and then spark to a computer, the results are more than just annoying.

These examples reveal that we have electric charge in our bodies, sweaters, carpets, doorknobs, faucets, and computers. In fact, every object contains a vast amount of electric charge. **Electric charge** is an intrinsic characteristic of the fundamental particles making up those objects that is, it is a property that comes automatically with those particles wherever they exist.

The vast amount of charge in an everyday object is usually hidden because the object contains equal amounts of the two kinds of charge, positive charge and negative charge. With such an equality—zero balance—of charge, the object is said to be electrically neutral; that is, it contains no excess charge. If the two types of charge are not in balance, then there is a net charge. We say that an object is **charged** to indicate that it has a charge imbalance, or net charge. The imbalance is always much smaller than the total amounts of positive charge and negative charge contained in the object.

Charged objects interact by exerting forces on one another. To show this, we first charge a glass rod by rubbing one end with silk. At points of contact between
the rod and the silk, tiny amounts of charge are transferred from one to the other, slightly altering its surface charge. This repel the electrons of each. Now rub the silk over the rod to increase the number of contact points and thus the amount, still tiny, of transferred charge.

Suppose we now suspend the charged rod from a thread in an electrically isolated room that is free of any influences that might change its charge. If we bring a second, similarly charged, glass rod near it (Fig. 21-2a), the two rods repel each other; that is, each rod experiences a force directed away from the other rod. However, if we rub a plastic rod with fur and then bring the rod near the suspended glass rod (Fig. 21-2b), the two rods attract each other; that is, each rod experiences a force directed toward the other rod.

We can understand these two demonstrations in terms of positive and negative charges. If a glass rod is rubbed with silk, the glass loses some of its positive charge and thereby has a small unbalanced positive charge (represented by the plus sign in Fig. 21-2a). When the plastic rod is rubbed with fur, the plastic gains a small unbalanced negative charge (represented by the minus sign in Fig. 21-2b). Our two demonstrations reveal the following:

- Charges with the same electrical sign repel each other, and charges with opposite electrical signs attract each other.

In Section 21-1, we shall put this rule into quantitative form as Coulomb’s law of electrostatic force (force of electric force) between charges. The term electrostatic is used to emphasize that, relative to each other, the charges are either stationary or moving only very slowly.

The “positive” and “negative” labels and signs for electric charge were chosen arbitrarily by Benjamin Franklin. He could easily have interchanged the labels on some other pair of substances to distinguish two kinds of charge. Franklin was a scientist of international reputation. It has even been said that Franklin’s triumphs in diplomacy in France during the American War of Independence were factorized, and perhaps even made possible, because he was so highly regarded as a scientist.

The attraction and repulsion between charged bodies have many industrial applications, including electrostatic paint sprayng and powder coating, fly-ash collection in chimneys, nonsmoke inkjet printing, and photocopying. Figure 21-2 shows a tiny carrier bead in a photocopying machine, covered with particles of black powder called toner, which stick to it by means of electrostatic forces. The negatively charged toner particles are eventually attracted from the carrier bead to a rotating drum where a positively charged image of the document being copied has formed. A charged sheet of paper then attracts the toner particles from the drum to itself, after which they are heat-fused permanently in place to produce the copy.

21-3 Conductors and Insulators

We can classify materials generally according to the ability of charge to move through them. Conductors are materials through which charge can move rather freely: examples include metals (such as copper in common lamp wire), the human body, and tap water. Nonconductors—also called insulators—are materials through which charge cannot move freely: examples include rubber (such as the insulation on common lamp wire), plastic, glass, and chemically pure water. Semiconductors are materials that are intermediate between conductors and insulators; examples include silicon and germanium in computer chips. Superconductors are materials that are perfect conductors, allowing charge to move without any hindrance. In these chapters we discuss only conductors and insulators.

Here is an example of how conduction can eliminate excess charge on an object. If you rub a copper rod with wool, charge is transferred from the wool to...
the rod. However, if you are holding the rod while also touching a friend, you cannot charge the rod in spite of the transfer. The reason is that you, the rod, and the friend are all conductors connected, via the plumbing, to Earth's surface, which is a huge conductor. Because the excess charges in the rod by the wood repel one another, they move away from one another by moving first through the rod, then through you, and then through the faucet and plumbing to reach Earth's surface, where they can spread out. The process leaves the rod electrically neutral. In this way setting up a pathway of conductors between an object and Earth's surface, we are said to ground the object, and in neutralizing the object (by eliminating an unbalanced positive or negative charge), we are said to discharge the object. If instead of holding the copper rod in your hand, you hold it by an insulating handle, you eliminate the conducting path to Earth, and the rod can then be charged by rubbing (the charge remains on the rod), as long as you do not touch it directly with your hand.

The properties of conductors and insulators are due to the structure and electrical nature of atoms. Atoms consist of positively charged protons, negatively charged electrons, and electrically neutral neutrons. The protons and neutrons are packed tightly together in a central nucleus. The charge of a single electron and that of a single proton have the same magnitude but are opposite in sign. Hence, an electrically neutral atom contains equal numbers of electrons and protons. Electrons are held near the nucleus because they have the electrical sign opposite that of the protons in the nucleus and thus are attracted to the nucleus.

When atoms of a conductor like copper come together to form the solid, some of their outermost (and so most loosely held) electrons become free to wander about within the solid, leaving behind positively charged atoms (positively ionized). We call the mobile electrons conduction electrons. There are few (if any) free electrons in a nonconductor.

The experiment of Fig. 21-4 demonstrates the mobility of charge in a conductor. A negatively charged plastic rod will attract either end of an isolated neutral copper rod. What happens is that many of the conduction electrons in the lower end of the copper rod are repelled by the negative charge on the plastic rod. Some of the conduction electrons move to the far end of the copper rod, leaving the near end depleted in electrons and thus with an unbalanced positive charge. This positive charge attracts to the negative charge in the plastic rod. Although the copper rod is still neutral, it is said to have an induced charge, which means that some of its positive and negative charges have been separated due to the presence of a nearby charge.

Similarly, if a positively charged glass rod is brought near one end of a neutral copper rod, conduction electrons in the copper rod are attracted to that end. Thus, one end becomes negatively charged and the other end positively charged, so again an induced charge is set up in the copper rod. Although the copper rod is still neutral, it rod the glass rod attracts each other.

Note that only conduction electrons, with their negative charges, can move: positive ions are fixed in place. Thus, an object becomes positively charged only through the removal of negative charges.

**Blue Flashes from a Wintergreen LifeSaver**

Indirect evidence for the attraction of charges with opposite sign can be seen with a wintergreen LifeSaver. If you adapt your eyes to darkness for about 15 minutes and then have a friend chomp on a piece of the candy in the darkness, you will see a faint blue flash from your friend's mouth with each chomp. Whenever a chomp breaks a sugar crystal into pieces, each piece will probably end up with a different number of electrons. Suppose a crystal breaks into pieces A and B, with A ending up with more electrons on its surface than B (Fig. 21-5). This
means that $B$ has positive ions (atoms that lost electrons to $A$) on its surface. Because the electrons on $A$ are strongly attracted to the positive ions on $B$, some of those electrons jump across the gap between the plates.

As $A$ and $B$ fall away from each other, air (primarily nitrogen, $N_2$) flows into the gap, and many of the jarring electrons collide with nitrogen molecules in the air, causing the molecules to emit ultraviolet light. You cannot see this type of light. However, the watergreen molecules on the surfaces of the candy pieces absorb the ultraviolet light and then emit blue light, which you can see—it is the blue light coming from your friend's mouth.

**Bacterial Contamination During Endoscopic Surgery**

In endoscopic surgery, a surgeon sees the interior of a patient's body on the viewing screen of a video monitor. The screen image is produced by electrons directed toward the screen from the back of the monitor. To attract these electrons, the screen is kept positively charged. The charged screen also attracts airborne particles floating around in the operating room, such as dust, dirt, and skin cells. If an airborne particle is negatively charged, it is pulled onto the screen's exterior surface. If, instead, it is electrically neutral, some of its conduction electrons can be pulled to the side of the particle near the screen, giving the particle an induced charge (Fig. 21-4a). Such a particle is then pulled to the screen's exterior surface just as the copper rod is pulled to the charged plastic rod in Fig. 21-4.

Because many of the particles collected on the screen's exterior surface carry bacteria, the screen becomes contaminated with bacteria. Suppose a surgeon's gloved fingers come within a few centimeters of the screen, pointing to a particular part of the image, say, in explaining a surgical concern to other medical staff. The negatively charged fingers then cause particles (airborne or on the screen) to collect on the gloves at the tips. When the surgeon next touches the patient with the contaminated gloves, the bacteria are carried over onto or (worse) inside the patient's body. To avoid this risk, surgeons are now warned not to bring fingers near a video monitor.

**Checkpoint 1** The figure shows five pairs of plates: $A$, $B$, and $D$ are charged plastic plates and $C$ is an electrically neutral copper plate. The electrostatic force between the pairs of plates is shown for three of the pairs. For the remaining two pairs, do the plates repel or attract each other? 

![Diagram showing electrostatic forces between charged plates](image)

**21-4 Coulomb's Law**

Let two charged particles (also called point charges) have charges $q_1$ and $q_2$, and be separated by a distance $r$. The electrostatic force of attraction or repulsion between them has the magnitude

$$F = k \frac{|q_1 q_2|}{r^2} \quad \text{(Coulomb's law)} \tag{21-1}$$

in which $k$ is a constant. Each particle exerts a force of this magnitude on the other particle: the two forces form a third-law force pair. If the particles repel each other, the force on each particle is directed away from the other particle (as
in Figs. 21-7a and b). If the particles attract each other, the force on each particle is directed toward the other particle (as in Fig. 21-7b). Equation 21-1 is called Coulomb's law. Augustin Coulomb, whose experiments in 1785 led him to it. Curiously, the form of Eq. 21-1 is the same as that of Newton's equation for the gravitational force between two particles with masses \( m_1 \) and \( m_2 \) that are separated by a distance \( r \):

\[
F = \frac{G m_1 m_2}{r^2}
\]

in which \( G \) is the gravitational constant.

The constant \( k \) in Eq. 21-2, by analogy with the gravitational constant \( G \) in Eq. 21-2, may be called the electrostatic constant. Both equations describe inverse square laws that involve a property of the interacting particles—the mass in one case and the charge in the other. The law's invariance in that gravitational forces may be either attractive or repulsive, depending on the signs of the charges. This difference arises from the fact that although there is only one kind of mass, there are two kinds of charge: positive and negative. The absolute value of the charge is the same in both cases, but the signs are different.

Coulomb's law has been observed in every experimental test to date, with no exceptions to it since it was first discovered. It holds in every universe that has been considered, together with the forces that bind atoms and molecules, to form solids and liquids.

The SI unit of charge is the coulomb. For practical reasons having to do with the accuracy of measurements, the coulomb is divided from the SI unit ampere for electric current. Current is the rate at which charge moves past a point or through a region. In Chapter 26 we shall discuss current in detail. Until then we shall use the relation

\[
j = \frac{dl}{dt} \quad \text{[electric current]}
\]

in which \( l \) is the current (in amperes) and \( dl \) (in coulombs) is the amount of charge moving past a point or through a region in time \( dt \) (in seconds). Rearranging Eq. 21-3 tells us that

\[
1 \text{ C} = 1 \text{ A} \times 1 \text{ s}
\]

For historical reasons (and because doing so simplifies many other formulas), the electrostatic constant \( k \) of Eq. 21-1 is usually written \( 1/4 \pi \epsilon_0 \). Then Coulomb's law becomes

\[
F = \frac{1}{4 \pi \epsilon_0} \frac{q_1 q_2}{r^2}
\]

(Coulomb's law). (21-4)

The constants in Eqs. 21-1 and 21-2 have the value

\[
k = \frac{1}{4 \pi \epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2.
\]

The quantity \( \epsilon_0 \), called the permittivity constant, sometimes appears separately in equations and is

\[
\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}.
\]

Still another parallel between the gravitational force and the electrostatic force is that both obey the principle of superposition. If we have a charged particle, they interact independently in pairs, and the force on any one of them let us say particle 1, is given by the vector sum

\[
F_{1} = F_{12} + F_{13} + F_{14} + \ldots + F_{1n}
\]
in which, for example, \( F \) is the force acting on particle 1 due to the presence of particle 4. An identical formula holds for the gravitational force.

Finally, the shell theorem tells us that forces in our study of gravitationally bound systems in electrostatics.

A shell of uniform charge appears as if all the shell's charge were concentrated at its center.

If a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

In the first theorem, we assume that the charge on the shell is much greater than that of the particle. Then any redistribution of the charge on the shell due to the presence of the particle's charge can be neglected.

**Spherical Conductors**

If excess charge is placed on a spherical shell that is made of conducting material, the excess charge spreads uniformly over the (external) surface. For example, if we place excess electrons on a spherical metal shell, those electrons repel one another and tend to move apart, spreading over the available surface until they are uniformly distributed. That arrangement minimizes the distances between all pairs of excess electrons. According to the first shell theorem, the shell then will attract or repel an external charge as if all the excess charge on the shell were concentrated at its center.

If we remove negative charge from a spherical metal shell, the resulting positive charge on the shell is also spread uniformly over the surface of the shell. For example, if we remove an electron, there are then a net of positive charge resides missing (one electron) that are spread uniformly over the shell. According to the first shell theorem, the shell will again attract or repel an external charge as if all the shell's excess charge were concentrated at its center.

**Checkpoint 2**

The figure shows two protons (symbol p) and one electron (symbol e) at rest. What is the direction of the electric force on the electron due to the other proton and (b) the electron? Draw the electric force on the electron due to the other proton.

**Sample Problem 2.1**

(a) Figure 21.4a shows two positively charged particles fixed in place on an x axis. The charge is \( q_1 = 6.00 \times 10^{-10} \text{ C} \) and \( q_2 = 3.20 \times 10^{-10} \text{ C} \) and the particles are separated by \( r = 0.0600 \text{ m} \). What are the magnitude and direction of the electric force \( F_{ee} \) on particle 1 from particle 2?

**Solution:** The key idea is that, because both particles are positively charged, particle 1 is repelled by particle 2, with a force magnitude given in Fig. 21.4a. Thus, the direction of force \( F_{ee} \) on particle 1 is away from particle 2, in the negative direction of the x axis, as indicated in the free-body diagram of Fig. 21.3b. Using Eq. 21.3 with separation 8 substituted for 4, we can solve the magnitude \( F_{ee} \) of the force as:

\[
F_{ee} = \frac{1}{4 \pi \varepsilon_0} \frac{q_1 q_2}{r^2} \\
= \frac{9.00 \times 10^9 \text{N} \cdot \text{m}^2 / \text{C}^2}{(0.0600 \text{ m})^2} \\
= 1.50 \times 10^{-7} \text{ N}
\]
Solution: One Key Idea is that the presence of particle 3 does not alter the electronic force on particle 1 from particle 2. Thus, force \( \mathbf{F}_{12} \) still acts on particle 1. Similarly, the force \( \mathbf{F}_{23} \) that acts on particle 3 due to particle 2 is not affected by the presence of particle 3. Because particles 1 and 3 have charge of opposite signs, particle 1 is attracted to particle 3. Thus, force \( \mathbf{F}_{12} \) is directed toward particle 3, as indicated in the free-body diagram of Fig. 21-8b.

To find the magnitude of \( \mathbf{F}_{12} \), we can rewrite Eq. 21-4 as

\[
\mathbf{F}_{12} = \frac{1}{4 \pi \varepsilon_0} \frac{q_1q_2}{r_{12}^2} \mathbf{r}_{12}
\]

\[
= \frac{9.00 \times 10^{-10} \text{ N} \cdot \text{m}^2/\text{C}^2}{(9.00 \times 10^{-10} \text{ C})^2} \cdot \frac{(3.29 \times 10^{-10} \text{ C})(1.60 \times 10^{-10} \text{ C})}{(3.29 \times 10^{-10} \text{ C})(1.60 \times 10^{-10} \text{ C})}
\]

\[
= 2.05 \times 10^{-10} \text{ N}
\]

We can also write \( \mathbf{F}_{12} \) in unit-vector notation:

\[
\mathbf{F}_{12} = 2.05 \times 10^{-10} \hat{n}_2
\]

A second Key Idea is that the net force \( \mathbf{F}_{13} \) on particle 3 is the vector sum of \( \mathbf{F}_{12} \) and \( \mathbf{F}_{23} \). That is, from Eq. 21-3b, we can write the net force \( \mathbf{F}_{13} \) on particle 3 in unit-vector notation as

\[
\mathbf{F}_{13} = \mathbf{F}_{12} + \mathbf{F}_{23}
\]

\[
= (2.05 \times 10^{-10} \hat{n}_2) + (1.25 \times 10^{-10} \hat{n}_3)
\]

\[
= 3.30 \times 10^{-10} \hat{n}_3
\]

Thus, \( \mathbf{F}_{13} \) has the following magnitude and direction relative to the positive direction of the \( x \) axis:

\[
9.08 \times 10^{-10} \text{ N} \quad \text{and} \quad 53.16^\circ
\]

(c) Figure 21-8d is identical to Fig. 21-8c except that particle 4 is now included. It has charge \( q_4 = -3.20 \times 10^{-10} \text{ C} \) at a distance \( R \) from particle 1, and lies on a line that makes an angle \( \theta = 60^\circ \) with the \( x \) axis. What is the net electronic force \( \mathbf{F}_{14} \) on particle 1 due to particles 2 and 4?

\[\text{Solution: The Key Idea is that the net force } \mathbf{F}_{14} \text{ is the vector sum of } \mathbf{F}_{12} \text{ and the net force } \mathbf{F}_{13} \text{ acting on particle 1 due to particle 4. Because particles 1 and 4 have charge of opposite signs, particle 1 is attracted to particle 4. Thus, force } \mathbf{F}_{14} \text{ on particle 1 is directed toward particle 4 at an angle } \theta = 60^\circ \text{, as indicated in the free-body diagram of Fig. 21-8d.}
\]

To find the magnitude of \( \mathbf{F}_{14} \), we can rewrite Eq. 21-4 as

\[
F_{14} = \frac{1}{4 \pi \varepsilon_0} \frac{q_1q_4}{r_{14}^2}
\]

\[
= 9.00 \times 10^{-10} \text{ C}^2/\text{N} \cdot \text{m}^2
\]

\[
= \frac{(3.29 \times 10^{-10} \text{ C})(-3.20 \times 10^{-10} \text{ C})}{(3.29 \times 10^{-10} \text{ C})(1.60 \times 10^{-10} \text{ C})}
\]

\[
= 2.05 \times 10^{-10} \text{ N}
\]

Then from Eq. 21-5, we can write the net force \( \mathbf{F}_{14} \) on particle 1 as

\[
\mathbf{F}_{14} = \mathbf{F}_{14} - \mathbf{F}_{13}
\]

To evaluate the right side of this equation, we need another Key Idea: Because the forces \( \mathbf{F}_{12} \) and \( \mathbf{F}_{23} \) are not directed along the same axes, we obtain the sum simply by combining their magnitudes. Instead, we can add them as vectors, using one of the following methods.

Method 1: Summing directly as a vector-calculator. For \( \mathbf{F}_{12} \), we enter the magnitude 1.13 × 10^{-10} \text{ N} and the angle 30°. For \( \mathbf{F}_{13} \), we enter the magnitude 2.05 × 10^{-10} \text{ N} and the angle 60°. Then we add the vectors.

Method 2: Summing in unit-vector notation. First, we rewrite \( \mathbf{F}_{12} \) as

\[
\mathbf{F}_{12} = (F_{12} \cos 0^\circ \hat{n}_2) + (F_{12} \sin 0^\circ \hat{n}_3)
\]

Substituting 2.05 × 10^{-10} \text{ N} for \( F_{12} \) and 60° for \( \theta \), this becomes

\[
\mathbf{F}_{12} = (4.02 \times 10^{-10} \hat{n}_2) + (1.775 \times 10^{-10} \hat{n}_3)
\]

Then we sum:

\[
\mathbf{F}_{13} = \mathbf{F}_{12} + \mathbf{F}_{13}
\]

\[
= (4.02 \times 10^{-10} \hat{n}_2) + (1.775 \times 10^{-10} \hat{n}_3)
\]

\[
+ (1.25 \times 10^{-10} \hat{n}_3) + (1.775 \times 10^{-10} \hat{n}_2)
\]

\[
= 5.79 \times 10^{-10} \hat{n}_3
\]

(Answer)

Method 3: Summing components along axes. The sum of the \( x \) components gives us

\[
F_{1x} = F_{12x} + F_{13x} = 1.13 \times 10^{-10} \text{ N} + 1.25 \times 10^{-10} \text{ N}
\]

\[
= 2.38 \times 10^{-10} \text{ N}
\]

The sum of the \( y \) components gives us

\[
F_{1y} = F_{12y} + F_{13y} = 2.05 \times 10^{-10} \text{ N} + 1.775 \times 10^{-10} \text{ N}
\]

\[
= 3.83 \times 10^{-10} \text{ N}
\]

The net force \( \mathbf{F}_{14} \) has the magnitude

\[
F_{14} = \sqrt{F_{1x}^2 + F_{1y}^2} = 5.79 \times 10^{-10} \text{ N}
\]

(Answer)

To find the direction of \( \mathbf{F}_{14} \), we take

\[
\theta = \tan^{-1} \left( \frac{F_{1y}}{F_{1x}} \right) = 76.7^\circ
\]

(Answer)
PROBLEM-SOLVING TACTICS

TACTIC 1: Symbols Representing Charge

Here is a general guide to the symbols representing charge. If the symbol $q$ with or without a subscript, is used in a sentence when no electrical sign has been specified, the charge can be either positive or negative. Sometimes the sign is explicitly shown, as in the notation $+q$ or $-q$.

Sample Problem 21-2

Figure 21-6 shows two particles fixed in place: a particle of charge $q_1 = -q_2$ at the origin and a particle of charge $q_3 = -2q_2$ at $x = L$. At what point (other than infinitely far away) can a proton be placed so that it is in equilibrium (the net force on it is zero)? Is that equilibrium stable or unstable?

Solution: The key idea here is that if $F_1$ is the force on the proton due to charge $q_2$ and $F_2$ is the force on the proton due to charge $q_3$, then the point we seek is where $F_1 + F_2 = 0$.

This condition requires that

$$F_1 = -F_2.$$  \hspace{1cm} (21-8)

This tells us that at the point we seek, the forces are in the same direction.

and the force must have opposite directions.

Because a proton has a positive charge, the proton and the particle of charge $q_2$ are of the same sign, and force $F_1$ on the proton must point away from $q_2$. Also, the proton and the particle of charge $q_3$ are of opposite signs, so force $F_2$ on the proton must point toward $q_3$. "Away from $q_2"$ and "toward $q_3"$ can be in opposite directions only if the proton is located on the x-axis.

If the proton is on the x-axis at any point between $q_2$ and $q_3$, such as point $P$ in Fig. 21-6b, then $F_1$ and $F_2$ are in the same direction and not in opposite directions as required. If the proton is at any point on the x-axis to the left of $q_2$, such as point S in Fig. 21-9c, then $F_1$ and $F_2$ are in opposite directions. However, Eq. 21-4 tells us that $F_1$ and $F_2$ cannot have equal magnitudes there; $F_1$ must be greater than $F_2$, because $F_1$ is produced by a closer charge (with lesser $r$) of greater magnitude ($2q_2$) than $F_2$.

Finally, if the proton is at any point on the x-axis to the right of $q_2$, such as point R in Fig. 21-9d, then $F_1$ and $F_2$ are again in opposite directions. However, because now the charge of greater magnitude ($2q_2$ is farther away from the proton than the charge of lesser magnitude, there is a point at which $F_1$ is equal to $F_2$. Let us be the coordinate of this point, and let $q_4$ be the charge of the proton. Then with the aid of

When more than one charged object is being considered, their charges might be given as multiples of a charge magnitude. As examples, the notation $+2q$ means a positive charge, with magnitude twice that of some reference charge magnitude $q$; and $-3q$ means a negative charge, with magnitude three times that of the reference charge magnitude $q$.

Fig. 22-9c) Two particles of charges $q_4$ and $q_5$ are fixed in place on an axis, with separation $L$. (b) to (d) Three possible locations $S$, $P$, and $R$ for a proton. At each location, $F_1$ is the force on the proton from particle 1 and $F_2$ is the force on the proton from particle 2.

Eq. 21-4, we can rewrite Eq. 21-9 as

$$\frac{1}{4\pi\varepsilon_0} \frac{q_4 q_5}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{2q_2 q_5}{L^2}.$$  \hspace{1cm} (21-10)

(Note that only the charge magnitudes appear in Eq. 21-10.) Rearranging Eq. 21-10 gives us

$$\left(\frac{x - L}{x}\right)^2 = \frac{1}{2},$$

After taking the square roots of both sides, we have

$$\frac{x - L}{x} = \frac{1}{2}$$

which gives us

$$x = 2L.$$  \hspace{1cm} (Answer)

The equilibrium at $x = 2L$ is unstable, that is, if the proton is displaced leftward from point $R$, then $F_1$ and $F_2$ both increase but $F_1$, increases more (because $q_2$ is closer to $q_4$), and a net force will drive the proton further leftward. If the proton is displaced rightward, both $F_1$ and $F_2$ decrease but $F_1$, decreases more, and a net force will drive the proton further rightward. In a stable equilibrium, if the proton is displaced slightly, it returns to the equilibrium position.
Problem-Solving Tactics

Tactic 2: Drawing Electrostatic Force Vectors

When you are given a diagram of charged particles, such as Fig. 21-6a, and are asked to find the net electrostatic force on one of them, you should usually draw a free-body diagram showing only the particle of concern and the forces that particle experiences, as in Fig. 21-6b. If, instead, you choose to superimpose those forces on the given diagram showing all the particles, be sure to draw the force vectors on the other charges (preferably) or on their heads on the particle of concern. If you draw the vectors elsewhere in the diagram, you invite confusion—and confusion is guaranteed if you reverse the vectors on the particles causing the forces on the particle of concern.

Sample Problem 21-3

In Fig. 21-6b, the two identical, electrically isolated conducting spheres A and B are separated by a centerto-center distance that is large compared to the spheres. Sphere A has a positive charge of +Q and sphere B is electrically neutral. Initially, there is no electrostatic force between the spheres. Assume that there is no induced change on the spheres because of their large separation.

(a) Suppose the spheres are connected by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?

Solution: A Key Idea here is that when the spheres are wired together, the negatively charged electrons on sphere A transfer to sphere B. The wire is thin enough so that any net charge on it is negligible.

Fig. 21-10 Two small conducting spheres A and B. (a) To start, sphere A is charged positively. (b) Negative charge is transferred from A to B through a connecting wire. (c) Both spheres are then charged positively. (d) Negative charge is transferred through a grounding wire to sphere A. (e) Sphere A is then neutral.

After the charge has been removed (Fig. 21-10b), we can assume that the charge on either sphere does not disturb the uniformity of the charge distribution on the other sphere, because the spheres are small relative to their separation. Thus, we can apply the first shell theorem to each sphere. But, in Fig. 21-10d with \( q = -q = -Q/2 \) and \( r = 0 \), the electrostatic force between the spheres has a magnitude of

\[
F = 4\pi\varepsilon_0 \frac{(Q/2)^2}{r^2} \left( \frac{1}{r_A} + \frac{1}{r_B} \right)
\]

Answer.

The spheres, now positively charged, repel each other.

(b) Next, suppose sphere A is grounded momentarily and then the ground connection is removed. What is the electrostatic force between the spheres?

Solution: The Key Idea here is that the ground connection allows electrons, with a total charge of \( +Q/2 \), to move from the ground to sphere A. (Fig. 21-10b). Neutralizing that sphere (Fig. 21-10e), with no charge on it, there is no electrostatic force between the two spheres (just as initially, in Fig. 21-6a).

21-5 Charge Is Quantized

In Benjamin Franklin's time, electric charge was thought to be a continuous fluid—an idea that was useful for many purposes. However, we now know that fluids themselves, such as air and water, are not continuous but are made up of atoms and molecules; matter is discrete. Experiment shows that "electrical fluid" is also not continuous but is made up of multiples of a certain elementary charge. Any positive or negative charge \( q \) that can be detected can be written as

\[
q = ne
\]

where \( e \) is the elementary charge, and \( n \) is an integer. (21-11)

The elementary charge \( e \) is one of the important constants of nature. The electron...
and proton, both have a charge of magnitude $e$ (Table 21-1). Quarks, the constituent particles of protons and neutrons, have charges of $\pm \frac{2}{3} e$ or $\pm \frac{1}{3} e$, but they apparently cannot be detected individually. For this and for historical reasons, we do not take their charges to be the elementary charge.

You often see physics—such as "the charge on a sphere," "the amount of charge transferred," and "the charge carried by the electron"—that suggest that charge is a substance. Indeed, such statements have already appeared in this chapter. You should, however, keep in mind what is intended. Particles are the substance, and charge happens to be one of their properties, just as mass is.

When a physical quantity such as charge can have only discrete values rather than any value, we say that the quantity is quantized. It is possible, for example, to find a particle that has no charge at all or a charge of $-6e$ or $6e$, but not a particle with a charge of say, $3.5e$.

The quantum of charge is small. In an ordinary 100-W lightbulb, for example, about $10^{32}$ elementary charges enter the bulb every second and just as many leave. However, the graminities of electricity does not show up in such large-scale phenomena (the bulb does not flicker with each electron), just as you cannot feel the individual molecules of water with your hand.

**CHECKPOINT 6** Initially, sphere A has a charge of $-5e$ and sphere B has a charge of $+2e$. The spheres are made of conducting materials and are identical in size. If the spheres then touch, what is the resulting charge on sphere A?

---

**Sample Problem 21-4**

The nucleus of an iron atom has a radius of about $4.3 \times 10^{-15}$ m and contains 26 protons. (a) What's the magnitude of the repulsive electrostatic force between two of the protons that are separated by $4.0 \times 10^{-15}$ m? 

**Solution:** The key idea here is that the protons can be treated as charged particles, so the magnitude of the electrostatic force on one from the other is given by Coulomb's law. Table 21-1 tells us that their charge is $+e$. Thus, Eq. 21-4 gives us

$$F = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{(1.602 \times 10^{-19} \text{ C})^2}{(4.0 \times 10^{-15} \text{ m})^2} \quad \text{N}. 

\text{(Answer)}$$

This is a small force to be acting on a macroscopic object like a cat's tail, but an enormous force to be acting on a proton. Such forces should be kept in mind when building molecules. Hydrogen has only one proton in its nucleus; however, they don't live in mines with a great storm of protons. Therefore, there must be some enormous attractive force to hold over such a vast series of purely electrostatic forces.

(b) What is the magnitude of the gravitational force between those two protons?

---

**Solution:** The key idea here is that in part (a) because the protons are particles, the magnitude of the gravitational force on one from the other is given by Newton's equation for the gravitational force (Eq. 21-21). With $m_1 = 1.67 \times 10^{-27}$ kg representing the mass of a proton, Eq. 21-2 gives us

$$F = \frac{G m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.67 \times 10^{-27} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{(4.0 \times 10^{-15} \text{ m})^2} \quad \text{N}. 

\text{(Answer)}$$

This result tells us that the attractive gravitational force is far too weak to counter the repulsive electrostatic forces between protons in a nucleus. Instead, the protons are bound together by an enormous force called (with the strong nuclear force) a force that acts between protons (and neutrons) when they are close together, as in a nucleus.

Although the gravitational force is many times weaker than the electrostatic force, it is more important in large-scale situations because it is always attractive. This means that it can collect many small bodies into huge bodies with huge masses, such as planets and stars, that then exert large gravitational forces. The electrostatic force, on the other hand, is repulsive for charges of the same sign, so it is unable to collect either positive charge of negative charge into large concentrations that would then exert large electrostatic forces.

---

**21-6 Charge Is Conserved**

If you rub a glass rod with silk, a positive charge appears on the rod. Measurement shows that a negative charge of equal magnitude appears on the silk. This suggests that rubbing does not create charge but only transfers it from one body to another, upsetting the electrical neutrality of each body shifting the protons.
This hypothesis of conservation of charge, first put forward by Benjamin Franklin, has stood up under close examination. Both for large-scale charged bodies and for atoms, nuclei, and elementary particles, no exceptions have ever been found. Thus, we add electric charge to our list of quantities—including energy and both linear and angular momentum—that obey a conservation law.

Important examples of the conservation of charge occur in the radioactive decay of nuclei, in which a nucleus transforms into (becomes) a different type of nucleus. For example, a uranium-238 nucleus ($^{238}\text{U}$) transforms into a thorium-234 nucleus ($^{234}\text{Th}$) by emitting an alpha particle. Because that particle has the same makeup as a helium-4 nucleus, it has the symbol $^{4}\text{He}$. The number used in the name of a nucleus and as a superscript in the symbol for the nucleus is called the mass number and is the total number of the protons and neutrons in the nucleus. For example, the total number in $^{234}\text{U}$ is 238. The number of protons in a nucleus is the atomic number $Z$, which is listed for all the elements in Appendix F. From that list we find that in the decay

$$^{238}\text{U} \rightarrow ^{234}\text{Th} + ^{4}\text{He},$$

(21-13)

the parent nucleus $^{238}\text{U}$ contains 92 protons (a charge of $+92e$), the daughter nucleus $^{234}\text{Th}$ contains 90 protons (a charge of $+90e$), and the emitted alpha particle $^{4}\text{He}$ contains 2 protons (a charge of $+2e$). We see that the total charge is $+92e$ before and after the decay; thus, charge is conserved. (The total number of protons and neutrons is also conserved: 238 before the decay and 234 + 4 = 238 after the decay.)

Another example of charge conservation occurs when an electron $e^{-}$ (whose charge is $-e$) and its antiparticle, the positron $e^{+}$ (whose charge is $+e$), undergo an annihilation process in which they transform into two gamma rays (high-energy light): $e^{-} + e^{+} \rightarrow \gamma + \gamma$ (annihilation).

(21-14)

In applying the conservation-of-charge principle, we must add the charges algebraically, with due regard for their signs. In the annihilation process of Eq. 21-14 then, the net charge of the system is zero both before and after the event. Charge is conserved.

In pair production, the converse of annihilation, charge is also conserved. In this process a gamma ray transforms into an electron and a positron: $\gamma \rightarrow e^{-} + e^{+}$ (pair production).

(21-15)

Figure 21-11 shows a pair-production event that occurred in a bubble chamber. A gamma ray entered the chamber from the bottom and at one point transformed into an electron and a positron. Because those new particles were charged and moving, each left a trail of tiny bubbles. (The trails were curved because a magnetic field had been set up in the chamber.) The gamma ray, being electrically neutral, left no trail. Still, you can tell exactly where it underwent pair production—at the tip of the curved $\gamma$, which is where the trails of the electron and positron began.

**Review & Summary**

**Electric Charge** The strength of a particle’s electrical interaction with objects around it depends on its electric charge, which can be either positive or negative. Charges with the same sign repel each other, and charges with opposite signs attract each other. An object with equal amounts of the two kinds of charge is electrically neutral, whereas one with an imbalance is electrically charged.

Conductors are materials in which a significant number of charged particles (electrons in metals) are free to move. The charged particles in nonconductors, or insulators, are not free to move.

The Coulomb and Ampere The SI unit of charge is the coulomb (C). It is defined in terms of the unit of current, the ampere (A), as the charge passing a particular point in 1 second when there is a current of 1 ampere at that point: $1 \text{C} = 1 \text{A} \times 1 \text{s}$. 

![Fig. 21-11. A photograph of trails of bubbles left in a bubble chamber by an electron and a positron. The pair of particles was produced by a gamma ray that entered the chamber directly from the bottom. It is electrically neutral, the gamma ray did not generate a visible trail of bubbles along its path, as the electron and positron did.](image-url)
Questions

1. Figure 21-12 shows three situations involving a charged particle and a uniformly charged spherical shell. The charges are given, and the radii of the shells are indicated. Rank the situations according to the magnitude of the force on the particle due to the presence of the shell, greatest first.

![Figure 21-12](image)

**Fig. 21-12** Question 1.

2. Figure 21-13 shows three pairs of identical spheres that are to be stuck together and then separated. The initial charges on them are indicated. Rank the pairs according to (a) the magnitude of the charge transferred during touching and (b) the charge left on the positively charged sphere, greatest first.

![Figure 21-13](image)

**Fig. 21-13** Question 2.

3. Figure 21-14 shows six situations in which charged particles are fixed in place on an axis. In which situations is there a point to the left of the particles where an electron will be in equilibrium?

![Figure 21-14](image)

**Fig. 21-14** Question 3.

4. Figure 21-15 shows two charged particles on an axis. The charges are free to move. However, a third charged particle can be placed at a certain point such that all three particles are in equilibrium. (a) Is that point to the left of the first two particles, to their right, or between them? (b) Should the third particle be positively or negatively charged? (c) Is the equilibrium stable or unstable?

![Figure 21-15](image)

**Fig. 21-15** Question 4.

5. Figure 21-16 shows four situations in which five charged particles are evenly spaced along an axis. The charge values are indicated except for the central particle, which has the same charge in all four situations. Rank the situations according to the magnitude of the net electrostatic force on the central particle, greatest first.

![Figure 21-16](image)

**Fig. 21-16** Question 5.

6. In Fig. 21-17, a central particle of charge $-q$ is surrounded by two circular rings of charged particles. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (Hint: Consideration of symmetry can greatly reduce the amount of work required here.)

![Figure 21-17](image)

**Fig. 21-17** Question 6.
Figure 21.19 shows four arrangements of charged particles. Rank the arrangements according to the magnitude of the net electrostatic force on the particle with charge +q, greatest first.

Fig. 21.19 Question 9.

Problems

- **Problem 2**: A particle of charge $+5 \times 10^{-9}$ C is 12.0 cm distant from a second particle of charge $-1.0 \times 10^{-9}$ C. Calculate the magnitude of the electrostatic force between the particles.

- **Problem 4**: Identical isolated conducting spheres 1 and 2 have equal charges and are separated by a distance that is large compared with their diameters (Fig. 21.21a). The electrostatic force acting on sphere 2 due to sphere 1 is $F$. Suppose now that a third identical sphere 3, having an unsplitting handle and initially neutral, is placed first in sphere 1 (Fig. 21.21b), then to sphere 2 (Fig. 21.21c), and finally removed (Fig. 21.21d).
electrostatic force that now acts on sphere 2 has magnitude $F$. What is the ratio $F/F^*$?

**Fig. 21-23** Problem 4.

**Fig. 21-24** Problem 11.

**Fig. 21-25** Problem 12.

**Fig. 21-26** Problem 13.

**Fig. 21-27** Problem 14.

---

**Fig. 21-23** Problem 4.

**Fig. 21-24** Problem 11.

**Fig. 21-25** Problem 12.

**Fig. 21-26** Problem 13.

**Fig. 21-27** Problem 14.

---

**Fig. 21-23** Problem 4.

**Fig. 21-24** Problem 11.

**Fig. 21-25** Problem 12.

**Fig. 21-26** Problem 13.

**Fig. 21-27** Problem 14.
**21** Five particles are fixed on an x-axis. Particle 1 of charge $q_1 = 4.0 \mu C$ is located at $x = -2.0 \text{ cm}$. Particle 2 of charge $q_2 = 2.0 \mu C$ is located at $x = 3.0 \text{ cm}$. Particle 3 of charge magnitude $20 \mu C$ is released from rest at the x-axis at $x = 2.0 \text{ cm}$. What is the value of $q$ if the initial acceleration of particle 3 is in the positive direction of the (a) x-axis and (b) the y-axis?

**22** In Fig. 21-26, particle 1 of charge $-q$ and particle 2 of charge $+4q$ are held at separation $L = 300 \text{ cm}$ on the x-axis. If particle 3 of charge $q_3$ is to be located such that the three particles remain in place when released, what must be (a) the ratio $q_3/q$ and (b) the (x,y) coordinates of particle 3?

**23** Figure 21-29 shows an arrangement of four charged particles, with angle $\theta = 30^\circ$ and distance $d = 200 \text{ cm}$. Particle 2 has charge $q_2 = -4.0 \times 10^{-9} \text{ C}$. Particles 3 and 4 have charges $q_3 = q_4 = 1.0 \times 10^{-9} \text{ C}$. (a) What is the distance $d$ between the origin and particle 2 if the net electrostatic force on particle 1 due to the other particles is zero? (b) If particles 3 and 4 were moved closer to the x-axis but maintained their symmetry about that axis, would the required value of $d$ be greater than, less than, or the same as in part (a)?

**24** A nonconducting spherical shell, with an inner radius of 4.0 cm and an outer radius of 6.0 cm, has charge spread uniformly throughout its volume between its inner and outer surfaces. The volume charge density $\rho$ is the charge per unit volume, with the unit coulomb per cubic meter. For this shell $\rho = \rho_0$, where $\rho_0$ is the distance in meters from the center of the shell and $h = 3.0 \text{ cm}$. What is the net charge in the shell?

**25** In Fig. 21-29, particles 1 and 2 of charge $q_1 = q_2 = +3.20 \times 10^{-9} \text{ C}$ are on a y-axis at distance $d = 15.0 \text{ cm}$ from the origin. Particle 3 of charge $q_3 = -1.60 \times 10^{-9} \text{ C}$ is moved gradually along the x-axis from $x = 0$ to $x = 5.0 \text{ cm}$. At what value of $x$ is the magnitude of the electrostatic force on the third particle from the other two particles to (a) minimum and (b) maximum? What are the (c) minimum and (d) maximum magnitudes?

**26** Charge is Quantized

**27** The magnitude of the electrostatic force between two identical ions that are separated by a distance of $5.0 \times 10^{-10} \text{ m}$ is $3.7 \times 10^{-9} \text{ N}$. (a) What is the charge of each ion? (b) How many electrons are "missing" from each ion (thus giving the ion its charge imbalance)?

**28** What is the magnitude of the electrostatic force between a singly charged sodium ion (Na$^+$) and a charge $-e$ on a salt crystal if they are separated by $3.28 \times 10^{-10} \text{ m}$?

**29** How many electrons would have to be removed from a coin to leave it with a charge of $+1.0 \times 10^{-10} \text{ C}$?

**30** Two tiny, spherical water drops, with identical charges of $-1.00 \times 10^{-9} \text{ C}$, hover at-concrete separation of $1.00 \text{ cm}$ above a concrete surface. (a) What is the magnitude of the electrostatic force acting between them? (b) How many excess electrons are on each drop, giving it its charge imbalance?

**31** Earth’s atmosphere is constantly bombarded by cosmic ray protons that originate somewhere in space. If the protons all passed through the atmosphere, each square meter of Earth’s surface would intercept protons at the average rate of 1500 protons per second. What would be the electric current intercepted by the total surface area of the planet? (Use)

**32** Calculate the number of coulombs of positive charge in 750 cm of (neutral) sodium. (Hint: A hydrogen atom contains one proton; an oxygen atom contains eight protons.)

**33** Figure 21-30 shows charged particles 1 and 2 that are fixed in place on an x-axis. Particle 1 has a charge with a magnitude of $q_1 = 3.00 \mu C$. Particle 3 of charge $q_2 = -5.00 \mu C$ is initially on the x-axis near particle 2. Then particle 3 is gradually moved in the positive direction of the x-axis. As a result, the magnitude of the net electrostatic force $F_{\text{net}}$ on particle 2 due to particles 1 and 3 changes. Figure 21-30B gives the x component of that net force as a function of the position x of particle 3. The plot has an asymptote of $F_{\text{net}} = 1.5 \times 10^{-9} \text{ N}$ as $x \to \infty$. As a multiple of $q_2$, and including the sign, what is the charge $q_1$ of particle 2?

**34** Figure 21-31 shows electrons 1 and 2 on an x-axis and charged ions 3 and 4 of identical charge $-q_1$ and $q_2$. (a) What is the direction of the force $F_{\text{net}}$ on electron 2? (b) Are the two new forces $F_{\text{net}}$ and $F_{\text{def}}$ on electron 2 balanced? (c) The electron 2 and are intended to hold electron 2 in place. Is physically possible for $q_1 = 5e$? What are the (a) smallest, (b) second smallest, and (c) third smallest values of $q_2$ for which electron 2 is held in place?

**35** In crystals of the salt cesium chloride, cesium ions Cs$^+$ form the edge corners of a cube and a chlorine ion Cl$^-$ is at the cube’s center (Fig. 21-32). The edge length of the cube is 0.400 nm. The Cs$^+$ ions are each deficient by one ele-
ion (and thus each has a charge of $+e$), and the Cl ion has one excess electron (and thus has a charge of $-e$). (b) What is the magnitude of the net electrostatic force exerted on the Cl ion by the six carbon atoms at the corners of the cube? (If one of the Cl ions is missing, the crystal is said to have a defect; what is the magnitude of the net electrostatic force exerted on the Cl ion by the seven remaining Cs?)(a) 10.

**Fig. 21-32** Problem 29.

### 21-3 Change Is Conserved

30. Electrons and positrons are produced by the nuclear transformations of protons and neutrons known as beta decay. If a proton transforms into a positron, is an electron or a positron produced? (b) If a neutron transforms into a positron, is an electron or a positron produced? (c) If a positron transforms into a neutron, is an electron or a positron produced?

31. Identify X in the following nuclear reactions: (a) $^1H + ^1H \rightarrow X$, (b) $^1H + ^3H \rightarrow X$, (c) $^1H + ^6C \rightarrow X + X$.

### Additional Problems

32. Figure 21-33 shows an arrangement of three charged particles separated by a distance $d$. Particles A and C are fixed on a horizontal axis, but particle B can be moved along a straight line centered on particle A. During the movement, a radial line between A and B makes an angle $\theta$ relative to the positive direction of the x-axis. For two situations, the magnitude $F_{AB}$ of the net electrostatic force on particle A due to the other two particles. What is the force as given as a function of angle $\theta$ and as a multiple of a basic amount $F_0$. For example, on curve 1, at $\theta = 0^\circ$, we see that $F_{AB} = 2F_0$. (a) For the situation corresponding to curve 1, what is the ratio of the charge of particle C to the charge of particle B? (b) For the situation corresponding to curve 2, what is that ratio?

**Fig. 21-33** Problem 32.

33. In Fig. 21-34, we charged particles surround particle 2 at radial distances of either $d = 1.0$ cm or $2d$, as shown. The charges are $q_1 = +2e$, $q_2 = -4e$, $q_3 = +e$, $q_4 = +4e$, $q_5 = -2e$, $q_6 = -6e$, $q_7 = +e$, with $r = 1.0 \times 10^{-10}$ m. What is the magnitude of the net electrostatic force on particle 7? (a) Particle 7 is fixed at the origin of an x-y coordinate system. (b) Particle 7 is located on the x-axis at $x = 20$ cm, moving with a speed of 50 m/s in the positive x-direction. For what value of $v$ will the moving particle cease circular motion? (c) If the gravitational force on the particle (g) is not negligible, what is the net electrostatic force on particle 7? (d) If the gravitational force on particle 7 is negligible, what is the net electrostatic force on particle 7 if particle 2 is due to the other particles? (e) Figure 21-36 shows four identical conducting spheres that are actually well-separated from one another. Sphere W, with an initial charge of zero, is touched to sphere A and then to sphere B (with a small initial charge of $+1e$), and then they are separated. Find the final charge on each sphere. Was the initial charge on sphere A? (f) In Fig. 21-37, particle 1 of charge $+2e$ is free to move along a horizontal line at distance $d = 6.00$ m horizontally from the particle 2. What is the component of the electrostatic force on particle 2 due to particle 1? (g) In Fig. 21-38, particles 2 and 4 of charge $-e$ are fixed in place on a x-y axis, at $x_2 = -10.0$ cm and $y_2 = 5.00$ cm. Particle 1 of charge $+e$, can be moved along the x-axis. Particle 3 of charge $-e$ is fixed at the origin. Initially, particle 1 is at $x_1 = -10.0$ cm and particle 3 at $x_3 = 10.0$ cm. (a) To what x-value must particle 1 be moved to rotate the direction of the net electrostatic force $F_{13}$ on particle 3 by 90° counterclockwise? (b) With particle 1 fixed at its new position, to what value must you move particle 3 to rotate $F_{13}$ back to its original direction? (c) Three charged particles form a triangle: particle 1 with charge $Q_1 = 90.0$ nC at the coordinates $(0, 0, 000)$, particle

**Fig. 21-34** Problem 33.

**Fig. 21-35** Problem 35.

**Fig. 21-36** Problem 36.

**Fig. 21-37** Problem 37.

**Fig. 21-38** Problem 38.
weight W at a distance s from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charge q and 2q respectively. A distance d distant between these spheres is a fixed sphere with positive charge q. (a) Find the distance s when the rod is horizontal and balanced. (b) What voltage should be set on the line between particles 1 and 2 so that they produce a net electrostatic force on particle 2, as shown in Fig. 21-29. (c) What conditions should particle 3 be placed to minimize the magnitude of that force? (d) What is the minimum segmentary charge?

41 In Fig. 21-30, what are the (a) magnitude and (b) direction of the net electrostatic force on particle 4 due to the other three particles? All four particles are fixed in the xy plane, and q₁ = -3.2 nC, q₂ = 7.2 nC, q₃ = -16 nC, q₄ = 7.2 nC. What is the magnitude and direction of the net electrostatic force on particle 4 due to particles 1 and 2 alone?

42 A charged non-conducting rod, with a length of 2.00 m and a cross-sectional area of 4.00 cm², lies along the positive side of an axis with one end at the origin. The uniform charge density ρ₀ is a constant throughout the rod. How many excess electrons are on the rod if (a) uniform with a value of -4.00 μC/m, and (b) non-uniform, with a value given by ρ = bx², where b = 2.00 μC/m²?

43 Two point charges of 30.0 nC and -40.0 nC are held fixed on an axis at the origin and at x = 27 cm, respectively. A particle with a charge of 42 nC is released from rest at x = 28 cm. If the initial acceleration of the particle for a magnitude of 100 km/s², what is the particle’s mass?

44 A charge of 8.0 μC is to be split into two parts that are then separated by 0.05 m. What is the maximum possible magnitude of the electrostatic force between those two parts?

45 In Fig. 21-22, four particles from a square. The charges are q₁ = 2.00 μC, q₂ = -2.00 μC, q₃ = -2.00 μC, and q₄ = 2.000 μC. What is q if the net electrostatic force on particle 1 is zero?

46 What charges of 0.010 μC and -0.010 μC are placed on an axis, at x = 50 m and x = 16 m, respectively. What charge must be placed at x = 32 m so that an axis placed at the origin would experience an electrostatic force?

47 How many coulombs of positive charge are in 1.00 m and of neutral potassium-hydrogen gas (KH)?

48 Figure 21-40 shows a long, non-conducting, massless rod of length L, placed at its center and balanced with a block of weight W at a distance s from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges q and 2q respectively. A distance d distant between these spheres is a fixed sphere with positive charge q. Using the given data, determine the distance s when the rod is horizontal and balanced. What voltage should be set on the line between particles 1 and 2 so that they produce a net electrostatic force on particle 2, as shown in Fig. 21-29. What conditions should particle 3 be placed to minimize the magnitude of that force? What is the minimum segmentary charge?

49 In Fig. 21-29, particle 1 of charge -300 pC and particle 2 of charge +400 μC are held at separation L = 200 cm on an axis. Using vector notation, what is the net electrostatic force on particle 3, of charge q₃ = 2.0 μC, if particle 3 is placed at (x = 100 cm and y = -400 cm)? What should be the (x) and (y) coordinates of particle 3 for the electrostatic force on it to be directed away from particles 1 and 2 on a zero.

50 In the radioactive decay of 213Bi, a 213Bi nucleus transforms into 213Po and an α-particle (helium nucleus, two protons, and two neutrons) is emitted. A lead sample is used to accumulate the α-particle. If the distance between the sets of data points is 10 mm, what is the minimum magnitude of the electrostatic force between them?

51 A neutron consists of one "up" quark of charge +2/3 and two "down" quarks each having charge -1/3. If we assume that the down quark's core radius is 10⁻³ m in the neutron, what is the magnitude of the electrostatic force between them?

52 In Fig. 21-41, three identical conducting spheres form an equilateral triangle of side length L = 200 cm. The sphere radius is much smaller than L, and the sphere charges are q₁ = -2.00 μC, q₂ = +4.00 μC, and q₃ = +4.00 μC. What is the magnitude of the electrostatic force between spheres A and C? The following signs and colors are connected by a thin wire and then disconnected. It is grounded by the wire, and the wire is then removed, B and C are connected by the wire and then disconnected. What now are the magnitudes of the electrostatic force (a) between spheres A and C and (b) between spheres B and C?

53 Two small, positively charged spheres have a combined charge of 2.0 μC and 3.0 μC, each where is suspended from the other by an electrostatic force of 1.0 N when the spheres are 2.0 m apart. What is the charge on the smaller sphere?

54 In the electron stroke of a typical lightning bolt, a current of 2.5 x 10⁶ A exists for 20 μs, how much charge is transferred in this event?

55 What would be the magnitude of the electrostatic force between two 600 nC point charges separated by a distance of (a) 1.00 m and (b) 1.00 km if such point charges existed? How might such a configuration be set up?

56 A current of 1.00 A through your chest can stop your heart in about 1 second. If a current of 1.00 A passes through your brain, would disrupting the flow of blood and thus oxygen to your brain, and current passes for 200 m, how many conductive electrons pass through your chest?
The initial charges on the three identical metallic spheres in Fig. 21-42 are the following: sphere A, 120 mC; sphere B, 240 mC; and sphere C, 140 mC. Where Q = 240 mC to the left, sphere A and B are placed so that the inter-sphere separation d = 1.3 m, which is much larger than the spheres. Sphere C is brought first to sphere A and then to sphere B and is then removed. What then is the magnitude of the electrostatic force between spheres A and B? 

In Fig. 21-42 three identical conducting spheres initially have the following charges: sphere A, 120 mC; sphere B, 240 mC; and sphere C, 140 mC. Spheres A and B are fixed in place, with a center-to-center separation of 1.3 m, which is much larger than the spheres. Two experiments are conducted. In experiment 1, sphere C is brought to sphere A and then (separately) to sphere B, and then it is removed. In experiment 2, starting with the same initial states, the procedure is reversed. Sphere C is brought to sphere B and then (separately) to sphere A, and then it is removed. What is the ratio of the electrostatic force between A and B in the end of experiment 2 to that at the end of experiment 1?

We know that the negative charge on the electron is the same as the positive charge on the proton and equal, but opposite. However, that these magnitudes differ from each other by 0.00001%. What force would two copper coins, placed 10 m apart, repel each other? Assume that each coin contains 3 x 10^23 copper atoms. (Here, a metal copper coin contains 29 protons and 29 electrons.) What do you conclude?

An electron is in a vacuum near Earth's surface and located y = 0 m on a car's surface. As what value of x should a second electron be placed such that its electrostatic force on the first electron balances the gravitational force on the first electron?

How far apart must two protons be if the magnitude of the electrostatic force acting on one proton due to the other is equal to the magnitude of the gravitational force on a proton at Earth's surface?

Two engineering students, John with a mass of 90 kg and Mary with a mass of 45 kg, are 30 m apart. Suppose each has a 0.01% inclination in the amount of positive and negative charge, one student being positive and the other negative. Find the ratio of the magnitude of the electrostatic force of repulsion between them by replacing each student with a sphere of copper having the same mass as the student.

If an especially sticky against your cotton sticks out a bit of lint, the charge to reduce the lint and the cotton can be removed by an excess charge of 1.2 mC. Ask how many electrons are transferred between you and the cat?

You will gradually discharge a rubber via the floor, but if placed on a surface, you immediately reach toward a faucet, a paintball gun can suddenly appear as your fingers on the faucet. (a) In this case, what is the flow direction? (b) What is the flow direction? (c) What is the flow direction? (d) Instead, the cat studied a pair toward the larger, which way do electrons flow in the resulting spark? (e) If you strike a match with a hair brush on a dry day, you should take care not to bring your fingers near the e's by contact or you will burn with a spark. Considering that our hair is an insulator, explain how the spark-aided reaction.

The charge Q on a single sphere, a function x to be translated to a second, nearby sphere. The sphere can be treated as a particle. (a) What value of x minimizes the magnitude of the electrostatic force between the two spheres? What are the (b) smaller and (c) largest values of x for that force at the maximum magnitude?

What equal positive charges would have to be placed on Earth and on the Moon to neutralize their gravitational attraction? (b) Why don't we want to know the lunar distance to solve this problem? (c) How many kilograms of hydrogen ions resulting in one proton would be needed to provide the positive charge calculated in (a)?

In Fig. 21-43, two ray conducting balls of identical mass m and identical charge q hang from non-contacting threads of length L. Assume that it is so small that can be replaced by its approximate equal. (a) Show that:

\[ x = \frac{q^2}{4 \pi \varepsilon_0 (2mg)^{1/2}} \]

gives the equilibrium separation x of the balls. (b) If L = 12 cm; m = 10 g and q = 5.0 \text{ C}, what is x? (c) Explain what happens so the balls of Problem 68 hit one of them is discharged (resists its charge q) to say, the ground. (d) Find the new equilibrium separation x, using the given values of L and m and the computed value of x.

If the particles 1 and 2 are fixed in place, but particle 3 is free to move. If the net electrostatic force of particle 3 due to particles 1 and 2 is zero and \( q_3 = 2q_1 \). What is the ratio \( q_1/q_2 \)?

Particles 1 and 2, each of positive charge q, are fixed in place and at a distance of 5 m apart. Particle 3 of positive charge Q is to be placed along that axis at location given by x = y. (a) While expressing in terms of n, that gives the net electrostatic force F acting on particle 3 when it is in the three regions x < 0, 0 < x < a, and x > a. The expressions should give a positive equal when F is in the negative direction. (b) Graph F vs. x for the range -2 < x < 3.

What is the total charge in coulombs of 75.0 kg of electrons?

In Fig. 21-26, particle 1 of charge 50.0 mC and particle 2 of charge 200 mC are held at separation x on an axis. If particle 3 of unknown charge q is to be located such that the net electrostatic force on it from particles 1 and 2 is zero, what must be the (a) x and (b) coordinate of particle 3?